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# Models of motivation, satisfaction and successful planning in R & D

by

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The paper deals with modelling of motivation and satisfaction (i.e. maximum attainable utility) of research staff and its impact on successful planning in R & D systems. The planning involves choosing tasks and activities as well as assigning tasks to researchers. It also takes into account research supporting activities such as teaching, consulting, writing papers etc. It is assumed that each researcher is acting within an organization, i.e. he, or she, has a boss or sponsor, subordinates and competing fellow-researchers. The paper describes the deterministic as well as probabilistic choice and utility maximizing models. It takes into account possible conflicts when researchers are competing for the research funds, and the conflict-resolution methods. The methodology proposed makes it possible to improve planning and decision making by using the computerized support systems.

## 1. Introduction

Many people believe that management of R & D is more an art than a science. They argue that there is lack of good normative models, taking into account behavioral factors, such as motivations, utilities, satisfaction etc., which influence creativity, innovations and which could be used for research planning purposes.

In the present paper an attempt has been done to construct formal models of individual research activity including as far as possible the behavioral factors. For that purpose one assumes that each researcher belongs to an organization (university, research institute etc.). He, or she, plays a certain role within organization. That means that he, or she, has superiors (e.g. chairman of department, rector of university, sponsors etc.); fellow researchers , who represent competition, when the access to some values (e.g. financial funds, authority, having rightness etc.) is concerned and - subordinates (younger research assistants, students etc.).

According to his role the researcher is supposed to carry on research activity at the required level, and he is supposed to conduct research support activity, such as teaching, consulting, supervising younger staff, attending seminars etc.

The research, regarded as the main activity, can be viewed as a process which involves discovering laws of nature and which is based on observations and experiments, theoretical analysis and comparison of particular phenomena or physical processes. This activity can employ the inductive, plausible reasoning and heuristic approaches (see e.g. Polya (1965)). It requires that guesses be made as to how the nature works and demonstrations provided that the guesses were right. One can also start by constructing a model of the process studied and verify the model by experiments or - check it "in historical runs"

It is generally acknowledged that a fruitful research activity requires a lot of researcher's perseverance, persistency and endurance. It is impossible, as well, to get original results and creativity without the properly set research motivations.

The role of supportive research activities should not be overlooked. Though these activities may not produce (immediately) research results they prepare the fertile soil for future success.

In the present formulation research is regarded as a continuous activity with nonstrictly verifiable results or objectives. It is, however, believed that out of this activity concrete tasks with verifiable objectives can emerge. For example, after a series of theoretical studies, experiments and computations the task of constructing a prototype of a new engine, in the given time period, can be formulated.

It can be assumed that the higher the motivation for particular research area, the larger the probability of concrete achievement in this area in the form of new ideas, innovations or concrete tasks (implementation of the ideas and innovations).

A task with verifiable objectives may also follow as a result of grants or contracts signed by the organization and ordered or commissioned by superiors.

Activity, generally speaking, can be also regarded as a collective name for a set of actions or tasks. For example, teaching can be regarded as a set of lectures, checking of homeworks of students, conduct of examinations etc. The verifiable tasks can be regarded as deterministic, i.e. characterized by a given performance time, completion date and the outcome probability close to unity. The research activities, to the contrary have no definite completion time and the probability of getting the desired outcome can be less than unity.

The researchers are supposed to accept, or propose for approval of the superiors, a program of tasks or activities. Since the researchers and superiors may have a different preference structure a negotiation process may be necessary at that point. When a consensus is reached in negotiation the funds necessary for carrying on the research are also approved.

Since each individual has limited time resources the completion of bigger tasks requires organization of a team composed of several researchers. The leader, being a superior for the team members, is supposed to organize, i.e.

to choose best people and assign them to the concrete subtasks. He must also control, supervise, grant awards and negotiate with subordinates his decisions, if necessary.

It should be noted that sometimes, instead of negotiating, the team-leaders prefer to act in a autocratic manner. Generally, they do not achieve better results than the leaders who give the subordinates some research initiative and a freedom of choice. The reason is that the research outcomes are usually strongly correlated with researchers motivations and individual satisfaction.

The motivation in the models studied depends on a number of intrinsic factors (individual abilities, preferences with respect to the particular activities) and - as well - on the extrinsic factors (rewards, research facilities, access to financial resources).

One can show that the problem of maximizing researcher's utility is equivalent to maximizing motivation subject to the research performance level constraint, which is set by research organization and accepted by the researchers. In such a model the individual researcher pursues his individual objectives (utility) and at the same time contributes to the organization objectives, which is keeping a high level of scientific performance and at the same time - maximizing the probability of achievements (i.e. outcomes in the form of new ideas and theories, innovations, inventions etc.).

It is also shown that the role of sponsors or managers of an organization is not a passive one. To the opposite, by the proper strategy of allocation of research resources (time, financial funds) one can influence the motivations of the researchers and direct them to attain the desired objectives.

The methodology proposed creates the possibility of resolving the problems of selection of tasks and activities, and of allocating the research resources in an optimum manner. The computerized support systems can be also applied to increase efficiency and improve planning in the R & D organizations.

## 2. Choice of tasks in deterministic models

In the present section the problem of choice of a best subset of tasks, out of m tasks possible, will be studied.

It is convenient to express the outcome $Y$ of a task in monetary units, using the concept of economic production function $\varphi$, i.e. $Y = p\varphi(T, K)$, where $p$ =outcome price, $T$ =labour (expressed by production time) and $K$ =capital used. It is assumed that $\varphi$ is strictly concave and increasing.

Suppose the cost $Q$ of production factors $(T, K)$ are fixed, i.e.

$$\omega T + \kappa K = Q, \qquad (1)$$

where $\omega$ and $\kappa$ are factor prices.

By solving the problem $\max \varphi(T, K)$, subject to (1) it is possible to find the best factors ratio $u = \widehat{K}/\widehat{T}$.

In a concrete case of production function (called Cobb - Douglas)

$$\varphi = AT^{\beta}K^{1-\beta},$$

where $A$ and $\beta$ are positive constants, $0 < \beta < 1$,
one obtains

$$\widehat{T} = \frac{\beta}{\omega}Q, \qquad \widehat{K} = \frac{1-\kappa}{\kappa}Q,$$

and

$$u = \frac{1-\beta}{\beta}\frac{\omega}{\kappa} \tag{2}$$

It should be noted that there are two main financial systems used in R & D business. First, called profitable, with the outcomes sold at a market, at a price $p$, and wages ($\omega$) paid in proportion to the outcome value. The second, called nonprofitable, with the research cost $Q$ paid by a sponsor and wages fixed, depending on the researchers' position or titles. The position in turn depends on researchers' qualification or past achievements. The nonprofitable system is used mostly in the (so called) basic research, while the profitable one - in applications (there are also organizations where a combination of both systems is used).

In the present paper one deals mainly with nonprofitable organizations, though some references to profitable system are given.

Since in nonprofitable system $Y = Q$, one gets by (2)

$$Y = \omega\widehat{T} + \kappa\widehat{K} = \omega\widehat{K}(1 + u\frac{\kappa}{\omega}) = \frac{\omega\widehat{T}}{\beta} \tag{3}$$

where $1/\beta$ can be intepreted as a measure of research (technological) productivity ($\beta = \frac{d\varphi}{\varphi} : \frac{dT}{T}$). By using a research technology with smaller $\beta$, or increased capital/labour, one can get bigger $Y$.

Besides the technology the outcome depends also on researcher's individual productivity denoted by $b$. One can say that the researcher indexed "$i$" is $b_i$ times more productive (compared to the average productivity equal 1) when he completes a task in $T_i = T/b_i$, where $T$ = average completion time.

In recognition of researcher's productivity he is rewarded proportionally to $b_i$, so his wage is $b_i\omega T$, where $\omega$ = average wage in the organization.

When one deals with a model with n researchers, characterized by given $b_i$, $\forall i$, and $m$ alternative tasks, characterized by $\beta_j$, $\forall j$, it is also necessary to assume that each task has, generally, a different relative value or importance, denoted by $d_j$, $\forall j$. In other words the $j$-th task outcome value can be $d_j$ times bigger than its average monetary cost value $\omega b_i T/\beta_j$, $\forall i, j$. It is also assumed that the average time intervals (for different researchers) necessary to complete each task $T_j \leq T$, $\forall j$, or the numbers $a_j = T_j/T \leq 1$, $\forall j$ are given.

One can observe that, generally, the outcome of the $j$-th task, performed by $i$-th researcher can be written as the product $Y_{ij} = I_i J_j$, $\forall i, j$, of factors related to individuals ($I_i$) and tasks ($J_j$) respectively, where $I_i = \omega b_i T$, $J_j = a_j d_j / \beta_j$ . In the case of profitable systems one can write $Y_{ij} = a_j p_j \overline{b}_j T$, $\forall i, j$, where $\overline{b}_j$ is the "productivity of time" (for Cobb - Douglas) $\overline{b}_j = A_j {u_j}^{1-\beta_j}$ ). The parameters $a_j$, $\forall j$ can be determined by researchers, while $d_j, \beta_j, b_j, \overline{b}_j$ - by an independent body, e.g. by the experts.

It will be also assumed that the research organization sets for each position a level of performance (called "the research efficiency") which the researcher is supposed to accept (or join another organization). For example a university may require that a professor writes a given number of papers and teaches a given number of students per year.

One can say that a program composed of $m$ tasks is accepted when the relative outcome $\sum_{j=1}^{m} Y_{ij}/\omega b_i T$ is not less than the given (for the concrete position) research efficiency $l$, i.e.

$$\sum_{j=1}^{m} Y_{ij}/\omega b_i T = \sum_{j=1}^{m} a_j d_j / \beta_j \geq l, \ \forall i \tag{4}$$

According to (4) the ambitious (large $a_j$) tasks with large values $d_j$ and productivity $1/\beta_j$ (i.e. good research equipment and facilities), which yield research efficiency not less than the value $l$, should be accepted first of all.

The individual task selection problem can be formulated as a decision problem. For that purpose one can drop for a moment the index "$i$" and introduce the discrete strategy vector $x \equiv (x_1, x_2, \dots x_m) \in [0,1]^m$ ( which consists of discrete components $x = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\}$, $\forall j$) and the outcome

$$Y(x) = \sum_{j=1}^{m} Y_j x_i, \qquad Y_j = I J_j = b \omega T a_j d_j / \beta_j, \tag{5}$$

as the goal function.

Problem 1 *Find the strategy $x = \widehat{x}$, such that*

$$Y(\widehat{x}) = \max Y(x) \tag{6}$$

*subject to the constraints:*

*a. time of completion*

$$\sum_{j=1}^{m} a_j x_j \leq b \tag{7}$$

*b. research efficiency*

$$\sum_{j=1}^{m} \frac{a_j d_j}{\beta_j} x_j \geq l, \quad x_j \in [0,1]^m \tag{8}$$

REMARK *In the case of profitable system the constraint (8) can be replaced by (compare sec.4 or Kulikowski (8) ):*

$$\frac{b\omega T}{\sum_{j=1}^{m} Y_j} \geq v; \quad b\omega T = \sum_{j=1}^{m} Y_j - c_j T \tag{9}$$

*where* $c_j$ *=costs/unit time,* $v$ *=equitable reward rate.*

The solution of Problem 1, which belongs to the class of "knapsack problems" or portfolio selection, yields maximum of outcomes in the planning horizon $T$ with the given research efficiency $l$.

In the case of tasks which require significant time resources it is necessary to organize (by a proper assignment of tasks to researchers) a research team. In such a case let the $i$-th team member be characterized by the outcomes $Y_{ij} = b_i \omega T a_j d_j / \beta_j$, $\forall i, j$. Then the goal function becomes

$$Y = \sum_{j=1}^{m} \sum_{i=1}^{n} Y_{ij} x_{ij}, \qquad x_{ij} = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\}, \ \forall i, j \tag{10}$$

PROBLEM 2 *Find the strategy* $x = \widehat{x} \in [0,1]^{nm}$, *such that*

$$Y(\widehat{x}) = \max Y(x) \tag{11}$$

*subject to the constraints:*
*a. time of completion*

$$\sum_{j=1}^{m} a_j x_{ij} \leq b_i, \quad \forall i \tag{12}$$

*b. research efficiency*

$$\sum_{j=1}^{m} \frac{a_j d_j}{\beta_j} x_{ij} \geq l, \quad \forall i \tag{13}$$

The optimum solution $\widehat{x}$ enables one to assign the most efficient team members (large $b_i$) to the tasks with largest $\frac{a_j d_j}{\beta_j}$ values.

It should be noted that there exist presently quite effective methods for solving large knapsack problems ( see e.g. Balas, Zemel (1980)), i.e. Problems 1, 2. The computerized decision support systems can be also easily implemented for the problems formulated.

## 3. Probabilistic choice models

In the deterministic case, studied in sec.2, the outcomes of the chosen tasks are known in advance. In many R & D problems the situation is quite different. In the best case one can only describe outcomes with a given probability. In such cases the task-choice problem should rather be formulated in the probabilistic sense.

Assume for that purpose that the individual researcher chooses among alternative tasks according to a set of, generally, conflicting criteria. When he chooses a research task or research activity the applicable criteria might include e.g. the well known Alderfer's (1972) basic needs (Existence, Relatedness, Growth or ERG). Existence needs include various forms of material desires, such as financial support, research equipment, working conditions etc. Relatedness needs are those involving support or competition of research associates, bosses and subordinates, friends or enemies. Growth needs are those that drive the individual to have creative attitudes, knowledge, skill etc.

Suppose the individual must choose one of $m$ given research tasks, indexed $j = 1, 2, \dots m$ and there are $K$ criteria, indexed $k = 1, 2, \dots K$ that are applicable. Each criterion applied leads to a ranking of tasks, in the form of the probability vector:

$$\Pi_k = (\Pi_{k1}, \Pi_{k2}, \dots \Pi_{km}), \ \Pi_{kj} \geq 0, \ \forall k, j, \ \sum_{j=1}^{m} \Pi_{kj} = 1, \ \forall k, \tag{14}$$

$\Pi_{kj}$ is the probability that a researcher would choose task $j$ if he (or she) relied solely on criterion $k$.

Obviously, the rankings of tasks described by vectors $\Pi_k$ for different $k = 1, 2, \dots K$, are, generally, different and one may wonder how to derive (using a certain rule) the probability vector

$$a = (a_1, a_2, \dots a_m), \quad a_j \geq 0, \ \forall j, \ \sum_{j=1}^{m} a_j = 1, \tag{15}$$

which describes the resulting task rankings.

As shown by Intriligator (1982) such a rule, called weighted average, exists and is unique under the following assumptions (which he calls axioms):

a. the probability vector $a$ exists
b. When $a$ task is chosen with zero probability according to all criteria, then it is chosen with probability zero by the individual
c. The sensitivity of $a_j$ with respect to slight changes of $\Pi_{kj}$, i.e. $\partial a_j / \partial \Pi_{kj} = w_k > 0, \forall j$, where the weights coefficients w are normalized to sum up to unity, $\sum_k w_k = 1$. k The rule mentioned becomes

$$a_j = \sum_k w_k \Pi_{kj}, \ \forall j \quad \text{or} \quad a = w\Pi \tag{16}$$

where $w = (w_1, \dots w_K)$, $\Pi = m \times K$ matrix of $\Pi_{kj}$, $\forall k, j$.

To give an example of weighted average rule, consider the case of a researcher choosing among three tasks as alternatives and three criteria: existence, relatedness and growth. Assume also

$$\Pi = \begin{bmatrix} 0.5, & 0.4, & 0.1 \\ 0.1, & 0.8, & 0.1 \\ 0.3, & 0.2, & 0.5 \end{bmatrix}.$$

It means that according to existence the first task would be chosen with probability 0.5, task 2 would be chosen with probability 0.4 and task 3 would be chosen with probability 0.1. According to relatedness the second task has the highest ranking. Finally, according to growth the third task has the highest ranking.

In order to derive the resulting ranking it is necessary to specify the weights applied to the three (ERG) criteria. Suppose in this example, that

$$w = (0.4, 0.2, 0.4).$$

That means that $E$ and $G$ have equal importance, and they are twice as important as $R$.

The resulting probability vector a, derived by (16) becomes

$$a = (0.34, 0.40, 0.26).$$

So the second task has the highest choice probability.

According to the present probabilistic model of choice the individual selects a task using a two-stage mechanism. The first stage is to derive the choice vector $a$. Then, in the second stage, a random mechanism (e.g. a lottery in the form of a circle with unit circumference subdivided into arcs of lengths $a_1, a_2, \dots a_m$ and a fair pointer which spins around and stops at an arc) is used to select a particular alternative.

The probabilistic model can be also used to deal with the social choice (see Intriligator (1973)). For that purpose consider a society of $n$ individuals (indexed $i = 1, 2, \dots n$) who must choose one of $m$ given alternative options (indexed $j = 1, 2, \dots m$) labelled $A_j$, $\forall j$. Each individual has preferences over the options, characterized by an individual probability vector

$$a_i = (a_{i1}, \dots a_{im}), \quad a_{ij} \geq 0, \quad \forall i, j, \qquad \sum_{j=1}^{m} a_{ij} = 1, \quad \forall i$$

where $a_{ij}$ is the probability that individual $i$ would choose the option $A_j$ if he could act alone in deciding among the options.

In the probabilistic approach an option is chosen in two steps. The first step is to derive the social probability vector

$$p = (p_1, p_2, \dots, p_m), \quad p_j \geq 0, \quad \forall j, \quad \sum_j p_j = 1,$$

where $p_j$ is the probability that society chooses $A_j$.
The second step is then to use $p_j$ in a random mechanism to choose $A_j$, $\forall j$.

As shown in Intriligator (1973) under the three assumptions given (equivalent to a,b,c for the individual choice model) the social probabilities are the averages of individual probabilities, i.e.;

$$P_j = \frac{1}{n}\sum_{i=1}^{n} a_{ij}, \quad \forall j. \tag{17}$$

The rule (17) is the only rule consistent with the assumptions adopted.

## 4. Motivation and Utility Maximizing Models

The main drawback of the probabilistic choice model, described in sec. 3, is its passive character. Indeed, after determining the vector of preferences (a) the choice is left to a lottery. However, most people feel that choices they make depend rather on some motives and willingness to perform certain tasks and reject the others. In the field of management such a view is supported, in particular, by the known "Expectancy Theory,", usually traced to Victor Vroom, who presented the essentials of the theory in 1964.

According to that theory motivation $M$ ( i.e. the strength of individual motivation ) equals the product of " expectancy " $E$ ( i.e. the perceived probability that the particular action will lead to a desired outcome ) and a "valence" $V$ (i.e. the strength of an individual's preference for an outcome) or $M = EV$.

In the present section the outcome probability ($E$) is assumed to depend on the probability of choice of the research activity ($a$) and the achievement probability ($\overline{a}$). Depending on the formulation of research objectives $\overline{a}$ can be close to or less than 1. Indeed, suppose the research objective is to prove that a theory is right and the perceived probability of that achievement is less than unity. When one formulates the objective in the form: "find out if the theory is right or wrong" the achievement probability is, obviously, equal unity (unless one is unable to solve the problem at all). This example indicates that there are, generally two possible models of formulation of research outcomes.

**A.** The probabilities of outcomes $E_j$, $\forall j$, coincide with the choice probabilities $E_j = a_j$, $\forall j$ and $\sum_{j=1}^{m} E_j = 1$ (see section 3).

**B.** The probabilities of outcome $E_j \leq a_j$, $\forall j$, $\sum_{j=1}^{m} \leq 1$.

It should be observed that model A can be used, in particular, in nonprofitable research organizations, where the reward or utility does not depend much on the negative outcomes (i.e. the outcomes that do not produce the desired objectives).

For that reason in the present, nonprofitable utility maximizing model, one deals with the expected outcome[1]

$$Y = \sum_{j=1}^{m} Y_j = \sum_{j=1}^{m} a_j V_j, \qquad V_j = b\omega T d_j/\beta_j, \ \forall j$$

[1] compare sections 2,3

where

$$\sum_{j=1}^{m} a_j = 1$$

$a_j$ = subjective probability that the activity indexed $j$ would be chosen and would produce the desired outcome.

Regarding $a_j$ as expectancy and $V_j$ as valence of $j$-th activity one can see that the expected outcome $Y_j = a_j V_j$ is similar to Vroom's notion of motivation. For that reason one can assume that the expected outcomes equal motivations: $Y_j = M_j$, $\forall j$, where $M_j$ is called motivation of the activity indexed $j$.

It should be noted that Porter and Lawler (1968) extended the basic expectancy model by introducing the rewards. That extension breaks with earlier views that satisfaction leads to performance. They proposed that motivation makes a person perform. Performance leads to reward, which, if perceived as satisfactory, will cause satisfaction and this will eventually lead again to motivation.

In order to construct a formal model dealing with satisfaction the researcher's utility function should be introduced. The satisfaction is understood here as a state of researcher's mind, which follows, when the set of activities (tasks) carried out is perceived as the best, i.e. when it maximizes the researcher's expected utility subject to the constraints (such as time and research efficiency).

According to that definition the researcher is dissatisfied with his work when he is forced to act not in agreement with his motivations (which decreases his maximum attainable utility) or when he has to work too long or too hard (which violates his constraints).

In the present model of alternative activities the researcher may find out that the accomplishment of an outcome is equivalent to gambling at a lottery. When the pointer of the lottery stops at the arc of length $a_j$ the prize $V_j$ is won. Since $a_j$ is the probability of the $j$-th outcome the expected value of outcome is $Y_j = a_j V_j$. Following von Neumann's and Morgenstern's axiomatic theory of utility (see e.g. Luce and Raiffa (1959)), one can prove that the expected utility of the lottery equals:

$$U(Y_1, Y_2, \dots Y_m) = \sum_{j=1}^{m} a_j U(V_j) \tag{18}$$

where $U$ is called the utility function, $0 < a_j \leq 1$, $\forall j$, $\sum_{j=1}^{m} a_j = 1$.

Thus, whenever the assumptions of that theory hold, there exists a utility function $U(\cdot)$, preserving order and satisfying the expectation principle (18): the utility of a lottery equals expected utility of its outcomes. Moreover, the utility scale is uniquely determined except for the origin and the unit of measurement.

It should be also noted that the function $U(V_i)$ is generally unknown in the explicit form. Certain properties of $U(\cdot)$ are, however, postulated. First of all it is increasing and strictly concave, negatively accelerated (especially when $V_j$ is monetary). The last property makes it possible to explain many paradoxes of gambling or insurance business. The $U$ function, with the properties formulated, is also called "risk averse".

It can be observed that utility function (18) depends on the expected outcomes only. In order to take into account the factors which produce satisfaction it is necessary to introduce into (18) in addition to outcome $Y_j = M_j$, $\forall j$, the variable $b\omega z_j$ dealing with awards. For this reason, regarding (18) as the starting point in the theory presented, one can assume that researcher's utility function (depending parametrically on $b\omega z_j$) exists[2] such that

$$U(M_1, \ldots M_m, b\omega z) = \sum_{j=1}^{m} U(M_j, b\omega z_j), \tag{19}$$

where $z = (z_1, z_2, \ldots z_m)$, $z_j$ being the part of total time resource $T$, spent on $j$-th activity,

$$\sum_{j=1}^{m} z_j = T. \tag{20}$$

The function $U$ is assumed to be risk averse in both variables and, in addition, it should be homogeneous, degree one, called also "constant return to scale". Such a function has the following property: when one changes units of measurements of input variables $(M_j, z_j)$ the value of output $(U)$ (expressed in the same monetary units) does not change. Such a requirement is obviously innocuous, unless one can make people happy by paying award in cents instead of \$.

A typical function with required property is the Cobb-Douglas function

$$\mathrm{U} = \overline{u} M_j^{\alpha} (b\omega z_j)^{1-\alpha}, \quad 0 < \alpha < 1 \tag{21}$$

where $\overline{u}$, $\alpha$ = given positive constants. Another example is the "constant elasticity of substitution" – or – CES function.

The constant return to scale property enables one to write (19) in an equivalent form

$$U = \sum_{j=1}^{m} M_j f\left(\frac{b\omega z_j}{M_j}\right), \quad M_j = a_j V_j, \quad \forall j, \tag{22}$$

where $f$ is risk averse.

Analyzing the perceived motivations the individual may wonder what should be his best strategy of allocation of time $T$ among $m$ activities given ? The answer to that problem can be formulated in the form of the following assertion.

[2](the von Neumann and Morgenstern existence axioms are tacitly assumed to hold)

ASSERTION 1 *An individual with utility (22), conducting m given activities in time T, with motivations $M_j$, $\forall j$, attains maximum utility:*

$$S = \max U = Mf\left(\frac{b\omega T}{M}\right) = U(M, b\omega T), \tag{23}$$

*(called satisfaction), for the unique time allocation-strategy*

$$\widehat{z}_j = \frac{M_J}{M}T, \qquad M = \sum_{j=1}^{m} M_j, \quad \forall j \tag{24}$$

PROOF. One has to find the vector $z = (z_1, \ldots z_m)$ such that for $z = \widehat{z}$, $U(\widehat{z}) = \max U(z)$; subject to the constraint (20). The Lagrangian of the present problem becomes

$$\Phi(z, \lambda) = U(z) + \lambda\left(T - \sum_j z_j\right),$$

where $\lambda$ – Lagrange multiplier.

The necessary conditions of optimality can be written as follows

$$\frac{\partial \Phi}{\partial z_j} = b\omega f'\left(\frac{b\omega z_j}{M_j}\right) - \lambda = 0, \quad \forall j, \quad \frac{\partial \Phi}{\partial \lambda} = T - \sum_j z_j = 0, \tag{25}$$

Introducing (24) into (25) one can prove that the necessary conditions of optimality hold. Since $f$ is strictly concave the strategy $\widehat{z}$ exists and it is unique. ∎

Since strategy (24) does not depend on the form of utility function $f(\cdot)$ this strategy is universal, i.e. the strategy (24) is the same for all individuals. The level of satisfaction each individual enjoys is, of course, different.

One can also say that satisfaction $S$ an individual enjoys, equals his discounted motivation, with $f\left(\frac{b\omega T}{M}\right)$ playing the role of the discount factor.

For the fixed motivation (award) the satisfaction increases along with award (motivation) but – with a negative acceleration.

It should be observed that such an approach to utility maximizing problems has many advantages. Using that approach one is not required to know the explicit form of utility function, which is generally unknown. The experimental identification of utilities, proposed by some authors, is difficult and unpopular (especially among decision makers). Besides, the utility function is, generally, different for each individual and may change with his age, status etc.

When regarding the variables $\omega bT$, $M$ as independent one can wonder what is the best ratio $l \triangleq \frac{M}{b\omega T}$, which can be called research efficiency ?

The researcher's superiors can argue that the interests of research organization require the research efficiency (for all scientists occupying the given position) to be equal, at least, given number $l$, i.e.

$$\sum_{j=1}^{m} \frac{M_j}{b\omega T} = \sum_{j=1}^{m} \frac{a_j d_j}{\beta_j} \geq l \tag{26}$$

REMARK *In profitable organizations (see (9) and Kulikowski (1991))*

$$M_j = a_j \overline{b}_j p_j T, \ \forall j; \qquad b\omega T = \sum_{j=1}^{m}(M_j - c_j T),$$

*where* $c_j$ *= capital and overhead costs, a researcher is rather interested in maximization of his award in proportion to the total outcome* $\sum_{j=1}^{m} M_j$. *This can be written as :*

$$\frac{b\omega T}{\sum_j M_j} \geq v \tag{27}$$

*where* $v$ *is a given number called "the equitable reward rate" (according to formulations of Porter and Lowler).*

In order to determine $l$ analytically one can maximize $U(M, \omega bT)$ subject to the constraint

$$Mw_1 + b\omega T w_2 = c = const,$$

where $w_1$, $w_2$ are positive weights attached to motivation (outcome) and rewards, respectively.

For this purpose one can introduce the Lagrangian

$$\Phi = U(M, b\omega T) + \lambda(Mw_1 + b\omega T w_2 - c)$$

and write down the necessary conditions of optimality:

$$\frac{\partial \Phi}{\partial M} = 0, \quad \frac{\partial \Phi}{\partial b\omega T} = 0, \quad \frac{\partial \Phi}{\partial \lambda} = 0.$$

Due to strict concavity of $U$ the present problem has a unique solution. Assume e.g.

$$U = \overline{u} M^{\alpha} (b\omega T)^{1-\alpha}$$

and derive

$$\frac{1}{bT}\frac{\partial \Phi}{\partial \omega} = (1-\alpha)\frac{U}{b\omega T} + \lambda w_2 = 0 \quad \frac{\partial \Phi}{\partial M} = \alpha \frac{U}{M} + \lambda w_1 = 0$$

After eliminating $\lambda$ one gets

$$l = \frac{\alpha}{1-\alpha}\frac{w_2}{w_1}.$$

One can observe that $l$ depends on the utility parameter a and reward/outcome weights ratio $w_2/w_1$.

Since $\alpha$ is the elasticity of $U$ with respect to $M$, i.e.

$$\alpha = \frac{\mathrm{d}U}{U} : \frac{\mathrm{d}M}{M}$$

an ambitious researcher ($\alpha$ large) will accept higher value of research efficiency than his less ambitious fellow researcher. He may also demand higher award.

In order to find numerical value of $l$ in practical situations it is also possible to inquire the staff members what research efficiency is required in the organization . It should be also noted that for an individual the research efficiency he accepts depends on his experience, position, title, age, etc.

When $l$ is fixed one can solve also the problem of choice of a set of activities (compare Problem 1). Indeed, since

$$S = Mf\left(\frac{b\omega T}{M}\right) = Mf\left(\frac{1}{l}\right), \tag{28}$$

where $M = b\omega T \sum_{j=1}^{m} \frac{a_j d_j}{\beta_j}$, one can formulate the following Problem 3, which is a probabilistic version of Problem 1.

PROBLEM 3 *Find the strategy $x = \widehat{x}$, such that*

$$M(\widehat{x}) = \max_{x} M(x) \tag{29}$$

*where*

$$M(x) = \sum_{j=1}^{m} a_j(x)V_j, \quad a_j(x) = \frac{a_j x_j}{\sum_{k=1}^{m} a_k x_k}, \quad V_j = b\omega T d_j/\beta_j, \quad \forall j$$

*subject to the constraints:*

*a. number of activities:*

$$\sum_{k=1}^{m} x_k \geq \overline{m} \tag{30}$$

$$\overline{m} \;=\; \textit{given integer}, \; \overline{m} \leq m$$

*b. research efficiency*

$$\sum_{j=1}^{m} \frac{a_j(x)d_j}{\beta_j} \geq l, \qquad x_j \in [0,1]^m \tag{31}$$

By solving Problem 3 one can find at least $\overline{m}$ of activities which produce the maximum of motivation $M$. The number $\overline{m}$ can represent the minimum (required) number of research projects, teaching subjects, administration duties etc.

It is also possible to formulate a probabilistic version of Problem 2.

The difference between the deterministic and probabilistic versions of these problems consists not in the different interpretations of preference coefficients $a_j$, $\forall j$ only.

The probabilistic models are generally more suitable for activities with a long planning horizon $T$ rather. In such a situation the probabilistic approach is quite natural. When e.g. a professor is asked what he would do on a particular day or hour a year ahead he may say that with probability 0.5 he would do research, with probability 0.3 - teaching and with probability 0.2 - consulting etc.

The solution of Problems $1 \div 3$ can be implemented in the form of computerized Satisfaction (Success) Support System ($S^3$). The $S^3$ system can be regarded as a management technique (such as PERT, Management By Objectives or MBO, etc.) which helps to plan R & D activities.

In order to implement the $S^3$ system it is necessary to collect and exchange all relevant information (such as $a_j$, $d_j$, $\beta_j$, $l$ coefficients) between the sponsor and the researchers. A simple version of such a decision support system was checked experimentally and applied to allocate funds at Systems Research Institute of Polish Academy of Sciences (see Kulikowski, Jakubowski, Wagner (1986)).

It is also important that the decision maker (who wants to use or consult the system) should understand and accept the system's methodology.

## 5. The Model of Competition and Access to Research Funds

In the present section let us consider the access model for research funds. Suppose a sponsor allocates research fund $Q$ among m research activities or branches of science in such a way that $j$-th activity gets $Q_j = q_j Q$, $j = 1, \ldots m$, $\sum_{j=1}^{m} q_j = 1$.

There are also $n$ researchers who apply to the sponsor with their proposals in order to get access to the funds $Q_j$, $\forall j$. If the researcher indexed $i$ got the total fund $q_j Q$ his reward would be $b_i \omega T_i = \beta_j q_j Q$, and his motivations

$$M_{ij} = b_i \omega T_i \frac{a_j d_j}{\beta_j} = a_j d_j q_j Q \triangleq M_j, \quad \forall i, j \tag{32}$$

However, due to competition from other researchers indexed $\nu$, who offer $z_{\nu j}$, $n \neq i$, of research time to conduct activity number $j$, his expected outcomes (motivations) become

$$\bar{M}_{ij} = M_j \frac{z_{ij}}{\sum\limits_{\nu=1}^{n} z_{\nu j}}, \quad \forall i, j$$

In the present situation researcher's utility can be written down as :

$$U_i(z_i) = \sum_{j=1}^{m} \bar{M}_{ij} f_i \left( \frac{b_i \omega z_{ij}}{\bar{M}_{ij}} \right), \quad \forall i \tag{33}$$

where strategy $z_i = (z_{i1}, z_{i2}, \ldots z_{im})$ should satisfy the constraint

$$\sum_{j=1}^{m} z_{ij} = T_i, \quad \forall i \tag{34}$$

ASSERTION 2 *$n$ researchers (competing in $m$ research areas) with risk averse utilities (33), attain maximum utilities (satisfactions):*

$$S_i = \max U_i(z_i) = \frac{MT_i}{nT} f_i \left( \frac{n}{M} b_i \, \omega T \right), \quad \forall i, \tag{35}$$

$$M = \sum_{j=1}^{m} M_j,$$

*for the unique time allocation strategies*

$$\widehat{z}_{ij} = \frac{M_j}{M} T_i, \quad \forall i, j, \qquad T = \frac{1}{n} \sum_{\nu=1}^{m} t_\nu. \tag{36}$$

SKETCH OF THE PROOF. The proof of Assertion 2 is similar to the proof of Assertion 1 (see also Kulikowski (1991)).
Construct $n$ Lagrangians

$$\Phi_i = U_i + \lambda_i \sum_{j=1}^{m} (T_i - z_{ij}), \quad \forall i.$$

Since for each fixed $i$ and $j = 1, \ldots m$

$$\Phi'_{z_{ij}} = M_j \left[ \frac{\sum\limits_{\nu \neq i} z_{\nu j}}{\left( \sum\limits_{\nu} z_{\nu j} \right)^2} f_i \left( \frac{b_i \omega}{M_j} \sum_{\nu=1}^{n} z_{\nu j} \right) \right.$$

$$\left. + \frac{z_{ij}}{\sum\limits_{\nu} z_{\nu j}} f'_i \left( \frac{b_i \omega}{M_j} \sum_{\nu=1}^{n} z_{\nu j} \right) \frac{b_i \omega}{M_j} \right] - \lambda_i \tag{37}$$

and for $z_{ij} = \widehat{z}_{ij}$, $\forall i, j$ the value (37) is a constant, which does not depend on index $j$, the necessary conditions of optimality hold. Existence of $\widehat{z}_{ij}$, $\forall i, j$ and uniqueness follows from the strict concavity of $f_i$, $\forall i$. ∎

REMARK *The equality constraints (20), (34) which are present in the formulation of Assertions 1,2, can be replaced by the*

$$\sum_{j=1}^{m} z_{ij} \leq T_i, \qquad z_{ij} \geq 0, \quad \forall i, j$$

*without changing the formulations (see Kulikowski (1991)).*
*The proofs require, however, application of Kuhn-Tucker conditions instead of the Lagrangian technique.*

It can be observed that optimum strategies $\widehat{z}_{ij}$, $\forall i$, do not depend on the individual functions $f_i$. These strategies are Pareto optimal. For each individual they yield the maximum possible utility and satisfaction under competition. Such strategies correspond to the cooperative relations among the competing scientists. It is possible to show that when at least one member of the competition becomes "risk fond" the unstable phenomena (bifurcation of strategies) follow (see Kulikowski (1990)). An organization composed of individuals who use optimum time-allocation strategy (in the sense of Assertion 1) is called efficient. When, in addition, these strategies are cooperative (in the sense of Assertion 2) and stable one can call them effective. Effectiveness is understood here as the ultimate measure of organization quality.

## 6. Models of Funds Allocation and Conflict Resolution

The research models studied so far reflected mainly the researcher's point of view. In the present section we are trying to look at the funds allocation problems from the sponsor's point of view rather.

It can be assumed that the sponsor is interested in allocating financial resources according to his preferences or desires among m given research domains or areas. The sponsor's preferences can be expressed by a probabilistic vector of desires. Suppose e.g. that there are 3 proposal areas: Electronics ($E$), Computer Sciences ($CS$), System Sciences ($SS$), while the sponsor is a computer manufacturer. He would like each dollar of expected research outcome to support computer manufacturing, i.e. to pertain to one of the three sciences mentioned, in the following proportion : $P = \{P_E, P_{CS}, P_{SS}\} = \{0.3; 0.5; 0.2\}$. The vector $P$ can be called the sponsor's probability of desires. In other words the sponsor desires the outcome to be produced with probability 0.3 by electronic scientists, with 0.5 by computer scientists and with probability 0.2 by system scientists.

Using the methodology of probabilistic choice model (3) one can construct also the probabilistic desire model:

$$P_j = \sum_{k=1}^{K} w_k p_{kj}, \quad \forall j, \quad w_k > 0, \quad \sum_k w_k = 1,$$

where

$P_j$ is the probability that sponsor would desire the outcome to pertain to the $j$-th research area,

$p_{kj}$ = probability that sponsor would desire the outcome to pertain to the $j$-th research area when he (or she) relied solely on criterion $k$,

$w_k$ = given weights of criteria $k = 1, \dots K$.

It should be noted that sponsor's set of criteria is generally different from the set which is used by the researchers to determine $a_j$, $\forall j$. It can reflect e.g. sponsor's concern with development of national, regional or local industry etc.

In some R & D systems, in particular in the system existing in Poland, the main sponsor is the Government. It is, however, represented by the Committee for Scientific Research, which is a collective body, with members who were partly elected by scientists and partly nominated by the Government. In such a system one should rather use the probabilistic social choice model (17).

Suppose the sponsor's desire probabilities $P_j$, $\forall j$, $\sum_j P_j = 1$ to be given. Since $M_j$, $\forall j$ depend on the sponsor's financing strategy $q = (q_1, q_2, \dots q_m)$, one can find $q = \widehat{q}$, such that the relative expected outcomes $Y_j/Y = M_j/M$, $\forall j$ (see 32)), equal $P_j$, $\forall j$, i.e.

$$P_j = a_j d_j q_j / \sum_{\nu=1}^{m} a_\nu d_\nu q_\nu, \quad \forall j.$$

Since

$$P_{j+1}/P_1 = \frac{a_{j+1} d_{j+1}}{a_1 d_1} \frac{q_{j+1}}{q_1}, \quad j = 2, \dots m$$

one gets

$$\frac{\widehat{q}_{j+1}}{\widehat{q}_1} = \frac{a_1 d_1}{a_{j+1} d_{j+1}} \frac{P_{j+1}}{P_1}, \quad j = 2, \dots m \tag{38}$$

Using (38) and $\sum_j \widehat{q}_j = 1$, one can derive the best of sponsor's strategies of allocation of research funds $(\widehat{q}_j, \forall j)$ among $m$ given research areas. In other words in the model formulated the sponsor is able to control the research motivations (outcomes) by changing the supply of funds $\widehat{q}_j Q$, $\forall j$.

In the case when the sponsor is autocratic in pursuing his objectives, and these objectives are far from what the researchers demand, a loss of research motivations (outcomes) follows.

The sponsor can, as well, be "liberal" and give the scientists a high degree of independence in choosing the areas of activity, requiring however, that they maximize the expected outcome. In such a case the sponsor can apply the strategy

$$\tilde{q}_j = a_j d_j / \sum_{\nu=1}^{m} a_\nu d_\nu, \quad \forall j \tag{39}$$

rather.

Indeed, he wants to maximize the outcome (motivation);

$$Y(q) = b\omega T \sum_{j=1}^{m} a_j d_j q_j,$$

subject to

$$\sum_{j=1}^{m} q_j = 1. \tag{40}$$

By using Schwartz inequality he gets

$$Y(q) \leq b\omega T \left\{ \sum_j (a_j d_j)^2 \right\}^{1/2} \left\{ \sum_j q_j^2 \right\}^{1/2},$$

where the equality sign appears for

$$\tilde{q}_j = c a_j d_j, \quad \forall j.$$

The constant $c$ becomes, by (40)

$$c = 1 / \sum_{\nu=1}^{m} a_\nu d_\nu.$$

Then, the liberal strategy (39) (with r.m.s of $\tilde{q} = const$) maximizes $Y(q)$.

Obviously, the liberal strategy $\tilde{q}$ satisfies the researchers the best, while the sponsor may favour the autocratic strategy $\widehat{q}$. When these two strategies are confronted controversy and hot discussion may follow. In order to avoid possible conflicts, negotiation and conflict resolution approaches should be used.

A possible approach, when the funds allocation problem is decided by the committee, is to use the probabilistic social choice algorithm (17).

Suppose that all, say $L$, members of the committee have "equal rights and weights" so the required assumptions of the social choice method hold. According to that method the social choice probabilities $q_j$, $\forall j$ are obtained by averaging the individual choice probabilities $q_{jl}$, $l = 1, \ldots L$ (see (17)).

Obviously, one can also find $q_{jl}$, $\forall j, l$, experimentally using the individual choice model (16) and the questionnaires filled by all the committee members (the information concerned with $a_j$, $d_j$, $\forall j$ can be supplied by researchers and experts).

Using this model of negotiations one should be aware of the exaggeration tactics, which can be ued by certain committee members who want to increase preferences given by committee to a particular research area.

In order to detect if there is a bias towards a definite research area one can derive the numbers (which indicate "how far" from the average, or consensus, each individual is):

$$\delta_{jl} = q_{jl} - \frac{1}{L-1} \sum_{i \neq l}^{L} q_{ji}, \quad \forall jl$$

When there are numbers $l = l_0$, $j = j_0$, such that $\delta_{j_0 l_0}$ is larger than the given number, the committee may punish the individual indexed $l_0$ by deleting his participation in social choice, (i.e. in the averaging formula (17)).

It should be noted that the idea of "punishment" can be also realised in the so called "double payments" algorithms, proposed by Groves, Ledyard (1972) to avoid the "free rider" effect. It should be observed, however, that no good (i.e. strategy-proof) algorithms have been devised yet for the case when the coalitions among committee members are possible.

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