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Consensus degrees under fuzzy majorities and fuzzy preferences using OWA (ordered weighted average) operators

by

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The use of Yager's (1988) OWA (ordered weighted average) operators is proposed for handling fuzzy linguistic quantifiers (many, most, almost all, ...) used for the formalization of a fuzzy majority in the derivation of a degree of consensus under fuzzy preferences. The idea of a degree of consensus proposed in the authors' works (Fedrizzi, 1988; Kacprzyk, 1987; Kacprzyk and Fedrizzi, 1986, 1988, 1989) is employed in which the classical Zadeh's (1983) and Yager's (1983) fuzzy-logic-based calculi of linguistically quantified proposition have been employed. The use of the OWA operators makes it possible to redefine these degrees of consensus with simplicity and intuitive appeal.

Keywords: fuzzy logic, linguistic quantifier, fuzzy preference relation, fuzzy majority, group decision making, social choice, consensus, OWA (ordered weighted average operator).

1. Introduction

This paper is a continuation of the authors' previous works (Fedrizzi and Kacprzyk, 1988; Kacprzyk, 1987a; Kacprzyk and Fedrizzi, 1986, 1988, 1989 - see also Kacprzyk, Fedrizzi and Nurmi, 1990, 1992a, 1992b, and Kacprzyk and Nurmi, 1989) in which new definitions of degrees of consensus in a group of experts under fuzzy preferences and fuzzy majorities have been proposed.

The process of group decision making, including that of the reaching of consensus, is centered on human beings, with their inherent subjectivity and imprecision in the articulation of opinions (e.g., preferences). To account for this, a predominant research direction is based on the introduction of *individual* and *social fuzzy preference relations* (see, e.g., Kacprzyk and Fedrizzi, 1990, Nurmi, 1981, 1988, Nurmi, Fedrizzi and Kacprzyk, 1990, for a comprehensive account).

Here, assuming fuzzy preference relations as in most works on the fuzzification of group decision making and consensus, we proceed further. Namely, one of basic, inherent elements of the group decision making and consensus reaching problem is also the concept of a *majority* as both solutions in group decision making and the essence of consensus have much to do with what a majority of the individuals (decisionmakers) accepts. As opposed to virtually all works in the area of group decision making and consensus reaching under fuzziness, in which fuzzy preferences but a nonfuzzy majority (e.g., a half, at least 2/3, ...) have been adopted, the authors have proposed and advocated the use of a *fuzzy majority* expressed by a fuzzy linguistic quantifier exemplified by *most, almost all, much more than 50%, ...* (Fedrizzi and Kacprzyk, 1988; Kacprzyk, 1984, 1985, 1986, 1987a, b, Kacprzyk and Fedrizzi, 1986, 1988, 1989; Kacprzyk, Fedrizzi and Nurmi, 1990, 1992a, b, Kacprzyk and Nurmi, 1989, 1991; Nurmi, 1981, 1988; Nurmi, Fedrizzi and Kacprzyk, 1990).

Fuzzy majority is commonly used by the humans, and not only in everyday discourse. A good example in a biological context may be found in Loewer and Laddaga (1985):

> ...It can correctly be said that there is a *consensus* among biologists that Darwinian natural selection is an important cause of evolution though there is currently *no consensus* concerning Gould's hypothesis of speciation. This means that there is a *widespread agreement* among biologists concerning the first matter but *disagreement* concerning the second ...

A rigid majority as, e.g., more than 75% would not evidently reflect the very essence of the above statement. It should be noted that there are naturally situations when a strict majority is necessary, for obvious reasons, as in, e.g., political elections. Anyway, the ability to accomodate a fuzzy majority in consensus formation models should help make them more human consistent hence easier implementable.

A natural manifestations of a fuzzy majority are the so-called *linguistic quantifiers* exemplified by *most, almost all, much more than a half, ...* Though they cannot be handled by conventional logical calculi, fuzzy logic provides here simple and efficient tools (cf. Zadeh and Kacprzyk, 1992 for a comprehensive exposition of diverse aspects of fuzzy logic). What is particularly important for our considerations, fuzzy logic has made it possible to devise calculi of linguistically quantified propositions. Among the most relevant one can mention the ones due to Yager (1983) and Zadeh (1983) – see also Kacprzyk, 1987b for an account of applications in diverse fields.

These fuzzy-logic-based calculi of linguistically quantified propositions have been applied by the authors to introduce a fuzzy majority for measuring (a degree of) consensus and deriving new solution concepts in group decision making (Fedrizzi and Kacprzyk, 1988; Kacprzyk, 1984, 1985, 1986, 1987; Kacprzyk and Fedrizzi, 1986, 1988, 1989; Nurmi, 1981; Nurmi and Kacprzyk, 1991).

The degrees of consensus proposed in those works have proved to have much conceptual and intuitive appeal. Moreover, they have been found useful and implementable in a decision support system for consensus reaching (Fedrizzi, Kacprzyk and Zadrożny, 1988; Kacprzyk, Fedrizzi and Zadrożny, 1988; Fedrizzi, Kacprzyk, Owsiński and Zadrożny, 1994).

Basically, this degree of consensus is meant to overcome some "rigidness" of the conventional concept of consensus (cf. Bezdek, Spillman and Spillman, 1978, 1979; Spillman, Bezdek and Spillman, 1979; Spillman, Spilman and Bezdek, 1980) in which (full) consensus occurs only when "*all* the individuals agree as to *all* the issues". This may often be counterintuitive, and not consistent with a real human perception of the very essence of consensus (see, e.g., the citation from a biological context given in the beginning of this paper). The new degree of consensus can be therefore equal to 1, which stands for full consensus, when, say, "*most* of the (important) individuals agree as to *almost all* (of the relevant) options". This new degree of consensus has been proposed by Fedrizzi and Kacprzyk (1988), Kacprzyk (1987), and Kacprzyk and Fedrizzi (1986, 1988) using Zadeh's (1983) calculus of linguistically quantified propositions, and by Kacprzyk and Fedrizzi (1989) using Yager's (1983) calculus of linguistically quantified propositions.

In the derivation of these degrees of consensus using the two calculi of linguistically quantified proposition, the one due to Zadeh (1983) and the one due to Yager (1983), there is some difficulty. Namely, Zadeh's calculus is much simpler but may lead to unacceptable results mainly in case of "not fuzzy enough" fuzzy majorities (e.g, a little bit more than a half). On the other hand, Yager's calculus seems to be more general and to give "better" results but, in its original version, is not really operational for larger problems (cf. Kacprzyk and Fedrizzi, 1989).

A solution to overcome this problem may be the use of Yager's (1988) ordered weighted average (OWA) operators for the representation of fuzzy linguistic quantifiers. This seems to work very well though some deeper works on the semantics of the OWA operators in the context of group decision making and consensus formation (cf. Kacprzyk and Yager, 1990 for a similar analysis within multicriteria decision making).

For clarity, and to provide a point of departure for our further discussion, we will first review basic elements of Zadeh's calculus of linguistically quantified propositions (for simplicity, Yager's calculus will not be presented, but it leads to similar problems). Then, a relation between this calculus and the OWA operators is shown, and finally we proceed to the reformulation of degrees of consensus proposed by the authors in terms of the OWA operators.

Our notation related to fuzzy sets will be standard. Basically, a fuzzy set A in a universe of discourse $U=\{u\}=\{u_1, \ldots, u_n\}$ will be represented by a set of pairs $\{\mu_A(u), u\}$, $\forall u \in U$, where $\mu_A : U \longrightarrow [0,1]$ is the membership function of A, and $\mu_A(u) \in [0,1]$ is the membership degree of u in A, from full membership (=1) to full nonmembership (=0) through all intermediate values. For brevity, fuzzy sets will be equated with their membership functions. Moreover, in the case of a finite universe of discourse $U = \{u_1, \ldots, u_n\}$ assumed here, the fuzzy set A represented by the set of pairs $\{\mu_A(u), u\}$, $\forall u \in U = \{u_1, \ldots, u_n\}$, will be denoted as $A = \mu_A(u_1)/u_1 + \cdots + \mu_A(u_n)/u_n$, where "+" is in the set-theoretic sense.

The basic operations on fuzzy sets are defined in a standard way, i.e.:

- the complementation
$$\mu_{\neg A}(u) = 1 - \mu_A(u), \forall u \in U \tag{1}$$
- the union
$$\mu_{A+B}(u) = \mu_A(u) \vee \mu_B(u) = \max(\mu_A(u), \mu_B(u)), \forall u \in U \tag{2}$$
where $\vee$ may be replaced by, e.g., an s-norm;
- intersection
$$\mu_{A \cup B}(u) = \mu_A(u) \wedge \mu_B(u) = \min(\mu_A(u), \mu_B(u)), \forall u \in U \tag{3}$$
where $\wedge$ may be replaced by, e.g., a t-norm (cf. Kacprzyk, 1987 for more information on s- and t-norms.

2. Lingustic quantifiers and OWA (ordered weighted average) operators

2.1. Linguistic quantifiers and a fuzzy-logic-based calculus of linguistically quantified prpositions

A *linguistically quantified proposition* may be exemplified by "most experts are convinced", and may be generally written as

$$Qy\text{'s are } F \tag{4}$$

where Q is a *linguistic quantifier* (e.g., most), $Y = \{y\}$ is a *set of objects* (e.g., experts), and F is a *property* (e.g., convinced). Importance can be added leading to "QBY's are F", but this will not be considered here.

For our purposes, the main problem is how to find the truth of such a linguistically quantified proposition, i.e. truth(Qy's are F) knowing truth(y is F), $\forall y \in Y$, which can be done by using two basic calculi: due to Zadeh (1983) and Yager (1983). They have their strong and weak points as already mentioned.

It may be illustrative, and to some extent expedient for further considerations, to briefly review the simpler Zadeh's (1983) approach.

It is assumed that property F is a fuzzy set in Y, truth(y_i is F) $= \mu_F(y_i)$, $\forall y_i \in Y = \{y_1, \ldots, y_p\}$, and a linguistic quantifier Q is represented as a fuzzy set in $[0,1]$ as, e.g.,

$$\mu_{\text{"}most\text{"}}(x) = \begin{cases} 1 & \text{for } x \geq 0.8 \\ 2x - 0.6 & \text{for } 0.3 < x < 0.8 \\ 0 & \text{for } x \leq 0.3 \end{cases} \tag{5}$$

Then

$$\text{truth}(Qy\text{'s are } F) = \mu_Q(\frac{1}{p}\sum_{i=1}^{p}\mu_F(y_i)) \tag{6}$$

For examples and details on this calculus as well as on other calculi of linguistically quantified propositions, see, e.g., Kacprzyk (1987b).

2.2. OWA (ordered weighted average) operators

The OWA (ordered weighted average) operators (Yager, 1988) seems to provide some alternative and attractive means for handling fuzzy linguistic quantifiers.

An *OWA (ordered weighted average) operator* of dimension p is a mapping $F : [0,1]^p \rightarrow [0,1]$ if associated with F is a weighting vector $W = [w_n]^T$ such that: $w_1 \in [0,1]$, $w_1 + \cdots + w_n = 1$, and

$$F(x_1, \ldots, x_n) = w_1 b_1 + \cdots + w_n b_n \tag{7}$$

where b_i is the i-th largest element among $\{x_1, \ldots, x_n\}$. B is called an ordered argument vector if for each $b_i \in [0,1]$, $j > i$ implies $b_i \geq b_j$, $i = 1, \ldots, p$.

Then

$$F(x_1, \ldots, x_n) = W^T B \tag{8}$$

Example 1. Let $W^T = [0.2\ 0.3\ 0.1\ 0.4]$, and calculate $F(0.6, 1.0, 0.3, 0.5)$. Thus, $B^T = [1.0\ 0.6\ 0.5\ 0.3]$, and $F(0.6, 1.0, 0.3, 0.5) = W^T B = 0.55$; and $F(0.0, 0.7, 0.1, 0.2) = 0.43$.

Some hints as to how to determine the w_i's are given in Yager (1988). For our purposes relations between the OWA operators and fuzzy linguistic quantifiers are relevant. Basically, under some mild assumptions (cf. Yager, 1988, Kacprzyk and Yager, 1990), a linguistic quantifier Q has the same properties as the F aggregation function, so that it is our conjecture that the weighting vector W is a manifestation of a quantifier underlying the process of aggregation of pieces of evidence.

Then, as proposed by Yager (1988),

$$w_k = \mu_Q(k) - \mu_Q(k-1), k = 1, ..., p; \mu_Q(0) = 0 \tag{9}$$

Just to give some examples of the w_i's associated with the particular quantifiers, notice that:

(1) if $w_p = 1$, and $w_i = 0$, $\forall i \neq p$, then this corresponds to $Q =$ "all";

(2) if $w_i = 1$ for $i = 1$, and $w_i = 0$, $\forall i \neq 1$, then this corresponds to $Q =$ "at least one".

The intermediate cases, which correspond to, e.g., *a half, most, much more than 75%, a few, almost all, ...* may be therefore obtained by a suitable choice of the w_i's between the above two extremes.

The OWA operators are therefore an interesting and promising class of aggregation operators, and their ability to formally express fuzzy linguistic quantifiers will be used here for deriving degrees of consensus under fuzzy majorities.

3. Degrees of consensus under fuzzy preferences and a fuzzy majority

To sketch the basic setting adopted in this work, suppose that we have a set of n options, $\mathcal{S} = \{s_1, \ldots, s_n\}$, and a set of m individuals, $\mathcal{I} = \{1, \ldots, m\}$. Each individual k provides his or her *(individual) fuzzy preference relation*, R_k, given by its membership function $\mu_{R_k} : \mathcal{S} \times \mathcal{S} \to [0, 1]$ which, if card S is small enough, may be represented by a matrix $[r_{ij}^k]$ such that $r_{ij}^k = \mu_{R_k}(s_i, s_j)$; $i, j = 1, \ldots, n$; $k = 1, ..., m$; $r_{ij}^k + r_{ij}^k = 1$.

The degree of consensus is now derived in three steps. First, for each pair of individuals we derive a degree of agreement as to their preferences between all the pair of options, next we aggregate these degrees to obtain a degree of agreement of each pair of individuals as to their preferences between $Q1$ (a fuzzy linguistic quantifier as, e.g., *most, almost all, much more than 50%, ldots*) pairs of options, and, finally, we aggregate these degrees to obtain a degree of agreement of $Q2$ (a fuzzy linguistic quantifier similar to $Q1$) pairs of individuals as to their preferences between $Q1$ pairs of options. This is meant to be the degree of consensus sought.

We start with the degree of (strict) agreement between individuals $k1$ and $k2$ as to their preferences between options s_i and s_j

$$v_{ij}(k1, k2) = \begin{cases} 1 & \text{if } r_{ij}^{k1} = r_{ij}^{k2} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where: $k1 = 1, \ldots, m-1$; $k2 = k1+1, \ldots, m$; $i = 1, ..., n-1$; and $j = i+1, \ldots, n$.

The degree of agreement between individuals $k1$ and $k2$ as to their preferences between $Q1$ pairs of options is

$$v_{Q1}(k1, k2) = \mathrm{OWA}_{Q1}(v_{ij}(k1, k2)) \tag{11}$$

where $\mathrm{OWA}_{Q1}(.)$ is the aggregation of $v_{ij}(k1, k2)$'s with respect to $Q1$ via the OWA operator as shown in Section 2.2.

In turn the degree of agreement of $Q2$ pairs of individuals as to their preferences between $Q1$ pairs of options, called the *degree of Q1/Q2 - consensus* is

$$\mathrm{con}(Q1, Q2) = \mathrm{OWA}_{Q2}(v_{Q1}(k1, k2))) \tag{12}$$

where $\mathrm{OWA}_{Q2}(.)$ is defined similarly as $\mathrm{OWA}_{Q1}(.)$.

Since the strict agreement (10) may be viewed too rigid, we can use the degree of *sufficient agreement* (at least to degree $\alpha \in [0,1]$) of individuals $k1$ and $k2$ as to their preferences between options s_i and s_j, as well as the the degree of strong agreement of individuals $k1$ and $k2$ as to their preferences between options s_i and s_j, obtaining the degree of $\alpha/Q1/Q2$ - consensus and $s/Q1/Q2$ - consensus, respectively (cf. Fedrizzi and Kacprzyk, 1988, Kacprzyk, 1987a, and Kacprzyk and Fedrizzi, 1986, 1988, 1989).

This constitutes our intended reformulation of the degrees of consensus proposed in our former works. by using the OWA operators to handle fuzzy linguistic quantifiers representing a fuzzy majority. An important issue, not yet dealt with here, is the addition of the importance of individuals and the relevance of options. This is a nontrivial problem which requires a deeper analysis, in particular in the context of the OWA operators. It is beyond the scope of this paper, and will be dealt with in another paper.

To illustrate the use of the OWA operators for the derivation of a degree of consensus, let us consider the same example that was used in our previous works (e.g., in Kacprzyk, Fedrizzi and Nurmi, 1992).

Example 2. Suppose that we have 3 individuals and 3 options.

The fuzzy linguistic quantifiers $Q1 = Q2 =$ "most" are given by (5), i.e.

$$\mu_{\text{"most"}}(x) = \begin{cases} 1 & \text{for } x \geq 0.8 \\ 2x - 0.6 & \text{for } 0.3 < x < 0.8 \\ 0 & \text{for } x \leq 0.3 \end{cases}$$

The fuzzy preference relations of the particular (three) individuals, $R^k = [r_{ij}^k]$, $i, j, k = 1, 2, 3$ are assumed to be:

$$R^1 = [r_{ij}^1] = \begin{bmatrix} 0.0 & 0.1 & 0.6 \\ 0.9 & 0.0 & 0.7 \\ 0.4 & 0.3 & 0.0 \end{bmatrix}$$

$$R^2 = [r_{ij}^2] = \begin{bmatrix} 0.0 & 0.1 & 0.7 \\ 0.9 & 0.0 & 0.7 \\ 0.3 & 0.3 & 0.0 \end{bmatrix}$$

$$R^3 = [r_{ij}^3] = \begin{bmatrix} 0.0 & 0.2 & 0.6 \\ 0.8 & 0.0 & 0.7 \\ 0.4 & 0.3 & 0.0 \end{bmatrix}$$

Now, if we follow steps (10) – (12), we obtain that $\mathrm{con}(most, most) \approx 0.4$. Notice that this value is not equal to that obtained by using the conventional Zadeh's calculus of linguistically quantified propositions (Kacprzyk, Fedrizzi and Nurmi, 1992a), which was approximately equal to 0.35, but it is more or less consistent. This is certainly a supporting argument for the use of the OWA operators. However, some deeper analysis of the relation between the w_k's and $\mu_Q(.)$ proposed as (9) should probably be performed which is, however, beyond the scope of this paper.

4. Concluding remarks

We have shown how to use the OWA operators to formally handle fuzzy linguistic quantifiers which are in turn a natural representation of a fuzzy majority. This has then been used to redefine degrees of consensus under fuzzy preferences and a fuzzy majority. The use of the OWA operators seems to help attain a favorable operationality maintaining the intuitive appeal of previously employed fuzzy-logic-based calculus of linguistically quantified propositions.

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