Control and Cybernetics

vol. **22** (1993) No. 4

MCBARG system supporting multicriteria bargaining

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MCBARG is a computer based system supporting analysis and mediation process in bargaining, in case of multicriteria payoffs of players. An interactive procedure leading the players to a consensus is described. Main functions of the MCBARG system are presented.

Keywords: decision support systems, bargaining problem, multicriteria decision making.

1. Introduction

Many aspects of economic, environmental, or technological activity are influenced directly by bargaining between and among individuals, firms, and nations ("players"). In the bargaining problem, considered here, the bargaining conditions are determined entirely by the bounds of discussion, within which the final outcome is determined by the interaction of the players. Even in the case of one individual, firm or nation, there are many complex situations in which the decision maker needs help to learn about possible decision options and decision consequences. MCBARG is a computer based system which enables learning process of the players, and supports reaching the consensus outcome in the multicriteria bargaining problem. A theory of the multicriteria bargaining problem has been developed in Kruś, Bronisz and Lopuch 1990, Kruś and Bronisz 1993, Bronisz, Kruś and Wierzbicki 1988.

The multicriteria bargaining problem is a generalization of classical bargaining problem (see Nash 1950, Raiffa 1953, Kalai and Smorodinsky 1975, Roth 1979), under the assumption that there are not given explicitly utility functions of players. This generalization follows from the fact that an aggregation of participants' or players' objectives is often impossible because of various practical limitations of the utility theory.

In this paper an interactive negotiation procedure in multicriteria bargaining is described. Main function of the MCBARG system are presented. The multicriteria bargaining problem is illustrated by an example referring to acid rains problem.

2. Problem formulation

In most approaches (see Nash, 1950, Raiffa, 1953, Kalai and Smorodinsky, 1975, Roth, 1979), the bargaining problem has been considered in the case of unicriteria payoffs of players, i.e. when the preferences of particular players are expressed by utility functions. In many practical applications however, players trying to balance a number of objectives might have difficulties while constructing such utility functions. Moreover, the classic literature considers mostly axiomatic models of bargaining which yield one-shot solutions and do not result in procedures describing a process of reaching a binding agreement.

We consider n players, each with several objectives, so we deal with a multiobjective bargaining problem. In this problem, the players are faced with an agreement set of feasible outcomes. Any such outcome can be accepted as the result if it is specified by an unanimous agreement of all players. In the event that no unanimous agreement is reached, the status quo point is the result. If there are feasible outcomes which all participants prefer to the status quo point, then there is an incentive to reach an agreement. In most situations, players differ in their opinions which outcome is most preferable, hence there is a need for bargaining and negotiation.

Dealing with multiple payoffs, we do not assume that there exist explicitly given utility functions of the players. In this case the solution can be found in an interactive procedure. Such a procedure is considered here. The procedure starts from the status quo point and leads to a nondominated, individually rational solution belonging to the agreement set. During the interaction, players can express their preferences and can influence the course of the iterative process.

Let $N = \{1, 2, \ldots, n\}$ be the finite set of players, each player having m_i objectives. A multiobjective bargaining problem is defined as a pair (S, d), where an agreement set S is a subset of $\sum_{i=1}^{n} m_i$ – dimensional Euclidean space, called R^{NM}, and a disagreement point (status quo point) d belongs to S.

The bargaining problem has the following interpretation: every point $x \in R^{NM}$, $x = (x_1, x_2, \ldots, x_n)$, $x_i = (x_{i1}, x_{i2}, \ldots, x_{im_i})$, in the agreement set S represents payoffs for all the players that can be reached when they do cooperate with each other (x_{ij} denotes the payoff of the j-th objective for the i-th player). If the players do not cooperate, the disagreement point is the result.

Each objective can be maximized or minimized. For simplicity of problem formulation in Section 2 we assume that all objectives are maximized.

We employ a convention that for $x, y \in R^k$, $x \geq y$ implies $x_i \geq y_i$ for $i = 1, \ldots, k$, $x > y$ implies $x \geq y$, $x \neq y$, $x \gg y$ implies $x_i > y_i$ for $i = 1, \ldots, k$. We say that $x \in R^k$ is a weak Pareto optimal point in X if $x \in X$ and there is no $y \in X$ such that $y \gg x$; $x \in X$ is a strict Pareto optimal point in X if there is no $y \in X$ such that $y > x$.

We confine our consideration to the class of all multicriteria bargaining games (S, d) satisfying the following conditions:

(i) S is compact and there is $x \in S$ such that $x > d$,

(ii) S is comprehensive, i.e. for $x \in S$ if $d \leq y \leq x$ then $y \in S$.

(iii) For any $x \in S$, let $Q(S,x) = \{i : y \geq x,\ y_i > x_i \ \text{ for some } y \in S\}$. Then for any $x \in S$, there exists $y \in S$ such that $y \geq x,\ y_i > x_i$ for each $i \in Q(S,x)$.

Condition (i) states that the set S is closed, upper bounded and the problem is not degenerated. Condition (ii) says that objectives are disposable, i.e. that if the players can reach the outcome x then they can reach any outcome worse than x. $Q(S,x)$ is the set of all coordinates in R^{NM}, payoffs of whose members can be increased from x in S. Condition (iii) states that the set of Pareto optimal points in S contains no "holes". We do not assume convexity of S, however, any convex set satisfies Condition (iii).

The problem consists in supporting the players in reaching a nondominated solution, agreeable and close to their preferences.

Definitions:

A point $x^i \in S$ is defined as i-nondominated, $i \in N$, if there is no $y \in S$ such that $y_i > x^i_i$. A point $u \in R^{NM}$ is defined as a *utopia point relative to aspirations* (RA utopia point) if for each player $i \in N$, there is an i-nondominated point $x^i \in S$ such that $u_i = x^i_i$.

The i-nondominated point is an outcome which could be achieved by a rational player i if he would have full control of the moves of the other players. Let us observe that if in the unicriteria set there is only one i-nondominated point, then in the multicriteria case considered here there is a set of such points. Each player i, $i \in N$, is required then to investigate the set of i-nondominated points in S as m_i-dimensional multicriteria decision problem and then to select one i-nondominated point as his most preferable outcome.

The RA utopia point generated by the selected i-nondominated points, $i \in N$, carries information about the most preferable outcomes for all the players. The RA utopia point significantly differs from the ideal (utopia) point defined by the maximal values of all objectives in set S.

3. Interactive procedure

We are interested in a constructive procedure that is acceptable by all players, starts at the status quo point and leads to a strict Pareto optimal point in S. The procedure can be described as a sequence, $\{d^t\}_{t=0}^{k}$, of agreement points d^t such that $d^0 = d$, $d^t \in S$, $d^t \geq d^{t-1}$, for $t = 1, 2, \ldots$, d^k is a strict Pareto optimal point in S. (The assumption $d^t \geq d^{t-1}$ follows from the fact that no player will accept improvement of payoffs for other players at the cost of his concession.) At every round t, each player $i \in N$ specifies his preferable reference point $r^t_i \in R^{m_i}$, $r^t_i > d^t_i$ defining his improvement direction $\lambda^t_i \in R^{m_i}$, $\lambda^t_i = r^t_i - d^t_i$, and proposes his confidence coefficient $\alpha^t_i \in R$, $0 < \alpha^t_i \leq 1$. The improvement direction λ_i^t indicates the i-th players preferences over his objectives at round t. The confidence coefficient α_i^t reflects his ability at round t to describe preferences and to predict precisely all consequences and possible outcomes in S. (For more detailed justification, see Fandel, Wierzbicki, 1985, and Bronisz, Kruś, Wierzbicki, 1988).

The interactive procedure is defined by the process:

$$(*) \qquad \begin{aligned} &\{d^t\}_{t=0}^{\infty} \quad \text{such that} \quad d^0 = d, \\ &d^t = d^{t-1} + \varepsilon^t * \left[u(S, d^{t-1}, \lambda^t) - d^{t-1}\right] \qquad \text{for} \quad t = 1, 2, \dots, \end{aligned}$$

where

$\lambda^t \in R^{NM}$, $\lambda^t = (\lambda_1^t, \lambda_2^t, \dots, \lambda_n^t)$, is the improvement direction specified jointly by all the players,

$u(S, d^{t-1}, \lambda^t) \in R^{NM}$ is the utopia point relative to the direction λ^t at round t defined by

$u(S, d^{t-1}, \lambda^t) = \left(u_1(S, d^{t-1}, \lambda_1^t), u_2(S, d^{t-1}, \lambda_2^t), \dots, u_n(S, d^{t-1}, \lambda_n^t)\right)$,

$u_i(S, d^{t-1}, \lambda_i^t) = \max_{\geq} \left\{ x_i \in R^{m_i} : x \in S,\ x \geq d^{t-1},\ x_i = d_i^{t-1} + a\lambda_i^t \text{ for some } a \in R \right\}$,

$\varepsilon^t = \min(\alpha_1^t, \alpha_2^t, \dots, \alpha_n^t, \alpha_{\max}^t) \in R$, where $\alpha_{\max}^t$ is the maximal number α such that $d^{t-1} + \alpha\left[u(S, d^{t-1}, \lambda^t) - d^{t-1}\right]$ belongs to S.

The utopia point $u(S, d^{t-1}, \lambda^t)$ relative to the aspirations of the players (relative to direction λ^t) reflects the preferences of the particular players when the improvement direction λ^t is specified at round t. The individual outcome $u_i(S, d^{t-1}, \lambda_i^t)$ is the maximal payoff in S for the i-th player from d^{t-1} according to the improvement direction λ_i^t, while ε^t is the minimal confidence coefficient of the players at round t (we assume that no player can agree on a coefficient greater than his) such that a new calculated agreement point belongs to S. The preferred direction λ_i^t at round t is specified on the basis of interactive scanning of a number of solutions generated for assumed by players different reference points. The proposed approach is very closed to the achievement function concept (Wierzbicki, 1982) from the point of view of the user. Analogously, a special way of the parametric scalarization of the multiobjective problem is utilized to influence on the selection of solutions by changing reference points. To solve the problem, directional maximization is applied, using a bisection method. The scanning (called in the system the improvement directions testing) is performed independently by each of players. Given the information about the current status quo and ideal point the player proposes a number of reference points and confidence coefficient. For each reference point $r_i^t > d_i^t$ and confidence coefficient α_i^t given by the player at the round t, the system calculates:

RA-utopia:

$$u_i(S, d^{t-1}, \lambda_i^t) =$$
$$\max_{\geq} \Big\{ x_i \in R^{m_i} : x \in S,\ x \geq d^{t-1},\ x_i = d_i^{t-1} + a\lambda_i^t \quad \text{for some} \quad a \in R \Big\},$$

one-shot solution:

$$x^t = \max_{\geq} \Big\{ x \in S : x = d^{t-1} + a * \left[u(S, d^{t-1}, \lambda^t) - d^{t-1}\right] \quad \text{for some} \quad a \in R \Big\},$$

anticipated solution:

$$y^t = d^{t-1} + \varepsilon^t * \left[u(S, d^{t-1}, \lambda^t) - d^{t-1}\right]$$

maximal confidence coefficient:

$$\alpha_{\max}^t = \max_{\geq} \Big\{ a \in R : d^{t-1} + a * \left[u(S, d^{t-1}, \lambda^t) - d^{t-1}\right] \in S \quad \text{for some} \quad a \in R \Big\},$$

where $\lambda^t \in R^{NM}$, $\lambda^t = (\lambda_1^t, \lambda_2^t, \ldots, \lambda_n^t)$, $\lambda_j^t = \lambda_j^{t-1}$ for $j \neq i$, $\lambda_i^t = r_i^t - d_i^t$, $u(S, d^{t-1}, \lambda^t) \in R^{NM}$ is the utopia point relative to the direction λ^t.

Having the above information for a number of reference points, the player selects his preferred one. It defines the improvement direction of this player. Defined in this way improvement directions of all the players λ_i^t are used for calculation of the result d^t of the negotiation round.

The procedure is based on the following theoretical result (see: Kruś, Bronisz, 1993):

Theorem 1. For any multicriteria bargaining game (S, d) satisfying conditions (i), (ii) and (iii) and for any confidence coefficients α_i^t such that $0 < \varepsilon \leq \alpha_i^t \leq 1$, $t = 1, 2, \ldots, T$ there is a unique process d^t, $t = 0, 1, \ldots, T$, $T \leq \infty$, described by $(*)$ satisfying the following postulates:

P1. $d^0 = d$, $d^t \in S$ for $t = 1, 2, \ldots, T$,

P2. $d^t \geq d^{t-1}$ for $t = 1, 2, \ldots, T$,

P3. d^T ($= \lim_{t\to\infty} d^t$ if $T = \infty$) is a strict Pareto optimal point in S.

P4. *Principle of α-limited confidence*. Let $0 < \alpha_i^t \leq 1$ be a given confidence coefficient of the i-th player at round t. Then acceptable demands are limited by:

$$d^t - d^{t-1} \leq \alpha_{\min}^t \left[u(d^{t-1}) - d^{t-1}\right]$$

for $t = 1, \ldots, T$, where $\alpha_{\min}^t$ is a joint confidence coefficient at round t, $\alpha_{\min}^t = \min\{\alpha_1^t, \ldots, \alpha_n^t\}$, $u(d^{t-1})$ is the RA utopia point of the set $\{x \in S : x \geq d^{t-1}\}$ reflecting the preferences of the players.

P5. *Principle of recursive rationality*. Given d^t, at each round t, there is no such outcome $x \in S$, $x > d^t$, that x satisfies P4 (x substitutes d^t in P4).

P6. *Principle of proportional gains.* For each round t, $t = 1, \ldots, T$, there is a number $\beta > 0$ such that

$$d^t - d^{t-1} = \beta\big[u(d^{t-1}) - d^{t-1}\big].$$

The presented approach has been examined in a case of one-round process with confidence coefficients of the players equal to one. Corresponding one-shot solution has been characterized axiomatically (Kruś and Bronisz, 1993). It is easy to notice that in the unicriteria case, each game (S, d) has a unique utopia point which coincides with the ideal point and the one-shot solution coincides with the Raiffa solution (see Raiffa, 1953, Roth, 1979).

It is assumed that after testing of right amount of reference points each player selects his preferred direction. However, it may happen that a player has not sufficiently tested his set of nondominated points and selects a weak Pareto outcome as his preferred result. In such a case, even if all the players assume the confidence coefficients greater than the values of maximal confidence coefficients, the procedure should proceed in several iterations more, till it will reach strict Pareto solution in S. In the system this inconvenience is removed by application of an option of lexicographical improvement of weak Pareto solution to strict Pareto one without interaction of the players. The option is used only in the case when all the players assume their confidence coefficients greater then the values of maximal confidence coefficients. In such a case it is assumed that they are going to finish the interactive process. The lexicographical improvement proceeds in the following way:

Let us assume that in round t the obtained agreement point $d^t = d^{t-1} + \varepsilon * \big[u(S, d^{t-1}, \lambda^t) - d^{t-1}\big]$ is weak Pareto optimal. For a finite subsets of integer numbers I, J, let $e(I,J) = \big(e_1(I,J), \ldots, e_n(I,J)\big) \in R^{NM}$ be such that $e_{ij}(I,J) = \lambda^t$ for $i \in I$ and $j \in J$, otherwise $e_{ij}(I,J) = 0$.

Given $y \in S$ with $Q(S,y) \neq \emptyset$, define $x(S,y) \in S$ by

$$x(S,y) = \max\nolimits_{\geq} \Big\{ x \in S : x = y + a * e\big(Q(S,y)\big) \quad \text{for some} \quad a \in R \Big\}.$$

Intuitively, the vector $e\big(Q(S,y)\big)$ includes all the coordinates of vector λ^t, along with the solution can be improved. Otherwise, corresponding coordinate of the vector $e\big(Q(S,y)\big)$ is equal to 0. Then the lexicographical improvement can be defined by the sequence $\{x^j\}_{j=0}^{\infty}$ such that $x^0 = d^t$, and $x^j = x(S, x^{j-1})$ for $j = 1, 2, \ldots$. It can be shown that there is exactly one such sequence, moreover this sequence is finite.

The presented lexicographical improvement has been examined in a case of one-round process, i.e. when weak Pareto optimal solution is reached in the first round. It can be shown that in such a case the solution of the bargaining process can be described with the Rawlsian lexmin principle (Rawls, 1971). Moreover in the unicriteria case, the solution coincides with the Imai solution (Imai, 1983).

4. General function of MCBARG system

The MCBARG system is a decision support system designed to help in analysis of a decision situation and mediation in multicriteria bargaining problem in which a mathematical model of the problem can be formulated by a status-quo point and a system of inequalities describing agreement set in objective space of the players.

The system supports the following general functions:

1. The definition and edition of a model of the bargaining problem.
2. Interactive mediation.
3. Report of successive agreement outcomes.

The interactive mediation proceeds in a number of rounds and in each round the system supports the players in:

- Initial multiobjective analysis of the bargaining problem, resulting in an estimation of bounds on efficient outcomes and learning about the extreme and neutral outcomes.
- Unilateral, interactive analysis of the problem with stress on learning, organized through system response to user specified confidence coefficients and aspiration levels for objective outcomes. The systems responds with efficient (under the assumed confidence coefficient) objective outcomes.
- Calculation of the multilateral, cooperative solution of the round and reporting the results of the already performed rounds.

The system is self-explaining, it includes a set of information facilitating working with the system. User is provided with a set of menus which allow him to select a needed option easily.

The **MAIN** menu includes the following options: **INFORMATION, MODEL, OLD SESSION, NEGOTIATION**.

The **INFORMATION** option presents general information about the system.

The **MODEL** option enables definition and edition of a model of bargaining problem. The model includes: number of players and their names, criteria of each player, description of the criteria, their units, status of the criteria (to be maximized or minimized), status quo for each criterion, set of inequalities describing the agreement set.

The **OLD SESSION** option make it possible to load and view an old session. It is useful for analysis as well as for restarting from the previously performed and saved session.

The **NEGOTIATION** option activates the negotiation–mediation procedure.

The procedure consists of a number of rounds. Each round starts from the current status quo point (the first round starts from the initial status quo point). At each round the player specifies his confidence coefficient (i.e. defines part of the maximal improvement of the outcomes the counter players can obtain in the round) and indicates his preferred improvement direction. Usually, he needs to compare a number of possible outcome variants before making his decision. The MCBARG system helps him in an interactive scanning of outcome variants obtained for different reference points, confidence coefficients and assumed improvement directions of the other players.

The players get information about the range of possible outcomes and reasonable reference points. The system generates also some initial values serving the player introductory information. It is so called neutral outcome, a solution obtained by the system under the assumption that the improvement direction is defined according to the ideal point.

The scanning of the player outcomes is performed in the system through directional optimization and lexicographic improvement of the week Pareto outcomes. The system responds to the player with attainable, efficient (under the assumed confidence coefficient) outcomes that strictly correspond to the player-specified aspirations.

To finish this phase the player is required to select, according to his preferences, his reference point indicating his preferred improvement direction. The points selected independently by all the players are basis for a calculation of the result of the round. The result is calculated following the limited confidence principle (the minimal confidence coefficient is used for all players), trying to improve outcomes for all the players in the directions specified by their reference points. Thus, the system acts as a neutral mediator proposing a single-test provisional agreement improving the initial situation and forming a basis for the next round of negotiations.

The results are presented to the players in form of report, and the players can begin the next round assuming the obtained result as a new status quo point. The process terminates when the efficient, strict Pareto optimal solution in the agreement set is reached.

5. Example referring to acid rain problem

Let us consider two countries disputing programs reducing sulfur emissions. Each country is assumed to have an adopted plan for emission control and expects the emission level $\underline{E}_i$. However it is also assumed that the deposition levels resulting from the $\underline{E}_i, i = 1, 2$ are regarded as unacceptable, therefore an additional emission control program is requested and required for the program expenditures are discussed.

For each country $i = 1, 2$ there is a given cost function describing minimal required total cost C_i of reducing the total emission from the level $\underline{E}_i$ to the level E_i. The function is assumed to be decreasing and piece-wise linear.

The sulfur depositions in country i is described by the equation:

$$D_i = a_{i1}E_1 + a_{i2}E_2 + \underline{D}_i, i = 1, 2$$

where $a_{i,j}$ $(i, j = 1, 2)$ are parameters of the atmospheric transportation matrix (so called European Monitoring and Evaluation Programme matrix), $\underline{D}_i$ are depositions from emission of the other countries.

The model was inspired by the paper by Bergman L., H. Cesar, G. Klaassen (1990). The similar model, but with nonlinear cost functions has been considered with application of nonlinear multicriteria optimization package in paper by Stam, Cesar, Kuula (1989). In our paper a different approach based on the interactive bargaining is proposed.

The following two situations can be considered.

The first case deals with unilateral actions of the countries. In this case each country is assumed to enforce independently his additional program reducing sulfur emission. Paying $\overline{X}_i = \overline{C}_i$, it achieves the emission $\overline{E}_i$ and deposition $\overline{D}_i$ calculated with use of the cost function and the deposition equation (where $\overline{C}_i$ is cost of the program).

In the second case cooperation of the countries is assumed in a form of a bilateral agreement on a joint reducing program. In this case a joint fund is created, and the following equation is added to the model description:

$$X_1 + X_2 = C_1 + C_2$$

where X_i, $i = 1, 2$ are shares of the countries in the joint fund, C_i, $i = 1, 2$ are costs of reducing the emissions in particular countries. In this case a multicriteria optimization problem can be considered in which the expenditures X_1, X_2 and the depositions D_1, D_2 are minimized subject to the constraints described by the cost functions, the deposition equations and the equation defining the joint fund with respect to the variables: the costs C_1, C_2 and the emissions E_1, E_2. Solutions of the problem lay on a Pareto boundary of a simplex S in the four-dimensional objective space. Let S_+ be defined by

$$\begin{aligned} S_+ = \{ & (X_1, X_2, D_1, D_2) : \\ & (X_1, X_2, D_1, D_2) \leq (\overline{X}_1, \overline{X}_2, \overline{D}_1, \overline{D}_2), (X_1, X_2, D_1, D_2) \in S\} \end{aligned}$$

and called an agreement set. If S_+ is not empty, it describes benefits the countries can achieve as an effect of cooperation in comparison to the first case. This is an incentive to cooperation.

The bargaining problem consists of looking for an efficient solution in an agreement set, being subset of the simplex S, of all points dominating the point $d = (\overline{X}_1, \overline{X}_2, \overline{D}_1, \overline{D}_2)$, called further a status quo or a disagreement point. The solution should be selected according to the preferences of the countries considered further as players, and should assure the "fairness" rule. Roughly speaking, the problem consists in proper, agreeable to both the countries allocation of the benefits resulting from the cooperation. The MCBARG system supports analysis of the problem and selection of such a solution.

In the presented model of bargaining problem each of the two players (countries) has two objectives, namely: the expenditures, and the sulfur deposition, to be minimized. The agreement set is described by a set of linear inequalities obtained for assumed forms of the cost functions and given parameters in deposition equations. The parameters have been assumed in such a way that the first player represents developing country with highly polluting technologies and very limited funds for the reducing program, while the second player represents highly developed country, with advanced technologies.

Exemplary sessions have been performed with use of MCBARG system. Nondominated outcomes of the players in the agreement set have been found according to the players preferences with use of reference points. The results have shown that the cooperation can be beneficial for both the countries. In particular, it is reasonable for the highly developed country to locate some funds in the sulfur reducing program in the developing country.

6. Conclusions

The paper describes MCBARG system supporting the players in finding the agreeable, nondominated solution in multicriteria bargaining problem. The proposed interactive process implemented in the system consists in generation of a sequence of outcomes leading to a nondominatated solution. The process is based on a limited confidence principle, taken from practical observation, which says that the players have limited confidence in their ability to predict consequences and possible outcomes, hence each player tries to prevent other players from receiving disproportionally large gains. The generated outcomes are consistent with preferences of the players. The process assures some fairness rules and is resistant to the various manipulations of the players.

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