Computational methods for robust stability analysis of a class of quasi-polynomials[1]

by

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The paper proposes two computational methods (the so-called testing function methods) for robust D-stability analysis of quasi-polynomials of the retarded type with linearly dependent coefficient perturbations. The first method (based on a necessary and sufficient condition) requires a priori knowledge of a particular set of quasi-polynomials, named vertex quasi-polynomials. The second method, which is based on a sufficient and on a necessary conditions, does not require knowledge of the set of such quasi-polynomials and is simpler to apply. Moreover, on the basis of this method, the largest D-stability domain in the space of perturbed parameters for which the proposed sufficient condition is satisfied can be easily computed.

1. Introduction

Consider a linear time-delay system of the retarded type whose characteristic quasi-polynomial depends linearly on m uncertain physical parameters. The characteristic quasi-polynomial of such a system can be expressed in terms of deviations q_k $(k = 1, 2, \ldots, m)$ of uncertain parameters from their nominal values, as follows

$$w(s, q) = w_0(s) + q_1 w_1(s) + q_2 w_2(s) + \ldots + q_m w_m(s), \; q \in Q \subset R^M, \quad (1)$$

where quasi-polynomials $w_k(s)$, $k = 0, 1, \ldots, m$, are of the form

$$w_k(s) = a_{n0}^k s^n + \sum_{i=0}^{n-1} s^i \sum_{j=0}^{M} a_{ij}^k \exp(-s h_j). \quad (2)$$

We assume that all the coefficients a_{ij}^k are real with $a_{n0}^0 = 0$ and $0 = h_0 < h_1 < \ldots < h_M < \infty$. The quasi-polynomial $w_0(s) = w(s,0)$ is the nominal quasi-polynomial corresponding to nominal values of uncertain parameters; $q = [q_1, q_2, \ldots, q_m]^T$ is the vector of deviations of uncertain parameters and

$$Q = \{q \in R^m : q_k \in [b_k, c_k], \ b_k \leq 0, \ c_k \geq 0, \ k = 1, 2, \ldots, m\}. \tag{3}$$

Let assume that the coefficient of s^n in $w(s,q)$ is strictly positive for all $q \in Q$.

As q varies over the prescribed bounding set Q, we obtain the family of quasi-polynomials

$$\mathcal{W} = \{w(s,q) : q \in Q\}. \tag{4}$$

The family (4) can be expressed as the convex hull of finitely many generating quasi-polynomials $p_1(s), p_2(s), \ldots, p_K(s)$, i.e.

$$\mathcal{W} = \mathrm{conv}\{p_1(s), p_2(s), \ldots, p_K(s)\}, \tag{5}$$

where

$$p_i(s) = w(s, q^i), \ i = 1, 2, \ldots, K, \ K \leq 2^m, \tag{6}$$

and $q^i = [q_1^i, q_2^i, \ldots, q_m^i]^T$ with

$$q_k^i = b_k \ \text{or} \ q_k^i = c_k, \ k = 1, 2, \ldots, m. \tag{7}$$

Let D be the union of a finite number N of compact connected sub-regions D_i in the complex plane with $D_i \cap D_j = \emptyset$, $i = j$. Obviously D lies in the open left-half plane. Moreover, according to Fu, Olbrot, Polis (1989), we assume that the region D satisfies the requirement that for any point $x \in D^c$ (the complement of D) there exists a continuous path in D^c connecting x to some point y with an arbitrarily large absolute value and with the real part larger than some prescribed number. Examples of such regions are given in Fu, Olbrot, Polis (1989).

The quasi-polynomial of the retarded type, of the form (2), has an infinite number of roots located in the half plane $Re[s] < \alpha$, where α is a finite real number (Bellman, Cooke 1963).

Suppose that the nominal quasi-polynomial $w_0(s)$ has n_i roots inside the separate sub-region D_i, $i = 1, 2, \ldots, N$, where n_i is finite for D_i lying in the strip $\alpha_1 < Re[s] < \alpha_2$ (α_1 and α_2 are finite nonpositive real numbers) and infinite for D_i with an unbounded negative real part, lying in the half plane $Re[s] < \alpha < 0$

DEFINITION 1.1 *The perturbed quasi-polynomial $w(s,q)$ of the form (1) is called D-stable if for all $q \in Q$ it has in the separate sub-region D_i ($i = 1, 2, \ldots, N$) the same number of roots (finite or infinite) as the nominal quasi-polynomial $w_0(s) = w(s,0)$.*

The D-stability problem of the polytopic family $\mathcal{W}$ of quasi-polynomials can be solved by applying the "Edge Theorem" given in Bartlett, Hollot, Lin (1988) and Fu, Barmish (1989) for the polytopic family of polynomials and extended by Fu, Olbrot, Polis (1989) for the D-stability of the polytope $\mathcal{W}$ of quasi-polynomials. According to this theorem, $\mathcal{W}$ is D-stable if and only if all the exposed edges of $\mathcal{W}$ are D-stable. The exposed edges of the polytope (5) are of the form

$$p_{rk}(s, \lambda) = (1 - \lambda)p_r(s) + \lambda p_k(s), \ \lambda \in [0, 1], \tag{8}$$

where $p_r(s)$ and $p_k(s)$ are generating quasi-polynomials (vertices) of $\mathcal{W}$, i.e. $p_r(s) = w(s, q^r)$ and $p_k(s) = w(s, q^k)$. Note that not all pairwise combinations of the vertices of $\mathcal{W}$ are necessarily the exposed edges.

The D-stability problem of the edge polynomial (8) has been considered by many authors. They gave various methods for special cases of interest (see Ackermann, Barmish 1988; Białas 1985 and Zeheb 1989, for example). These methods are not, however, useful in the D-stability analysis of the edge quasi-polynomial since the quasi-polynomial usually has an infinite number of roots.

An approach to checking the D-stability of the edge quasi-polynomial (8) was given in the paper of Fu, Olbrot, Polis (1989). From this paper and from Definition 1.1 we have the following lemma.

Let $f_i(y)$, $y \in Y_i$, denote a parametric description of the boundary of the sub-region D_i, $i = 1, 2, \ldots, N$.

LEMMA 1.1 *The edge quasi-polynomial (8) is D-stable if and only if for all separate sub-regions D_i $(i = 1, 2, \ldots, N)$ the following conditions hold*

(i) *the quasi-polynomial $p_r(s)$ is D-stable, i.e. the plot of $p_r(f_i(y))/w_0(f_i(y))$, $y \in Y_i$, does not encircle the origin of the complex plane,*

(ii) *the plot of $p_k(f_i(y))/p_r(f_i(y))$, $y \in Y_i$, does not cross the nonpositive part of the real axis.*

The bounding set Q of the form (3) is a hyperrectangle in R^m. It has 2^m vertices and $m2^{m-1}$ exposed edges. Hence, the polytope (5) has 2^m vertices and $m2^{m-1}$ exposed edges in general. From this it follows that the Edge Theorem requires, in general, D-stability checks (on the basis of Lemma 1.1) of $m2^{m-1}$ edge quasi-polynomials of the form (8). This is a difficult problem when the number m of uncertain physical parameters is large. If, for example, $m = 8$ then the number of edge quasi-polynomials, the D-stability of which must be checked, is 1024. To avoid this difficulty in the case of the polytopic family $\mathcal{W}$ of polynomials, in recent years various methods have been presented for D-stability checking which do not require separate calculations for each exposed edge of $\mathcal{W}$ (see Barmish 1989; Barmish, Tempo 1991; Busłowicz 1993, 1994; and Cavallo, Celentano, De Maria 1991, for example).

The method given in Barmish (1989) was generalized by Barmish, Shi (1989) to the considered class of quasi-polynomials. But this method, in which the so-called testing function is constructed on the basis of two boundary sweeping functions, is not simple to apply.

In this paper we give two computational methods for robust D-stability analysis of the perturbed quasi-polynomial (1) with the additional assumption on the set D introduced in Fu, Olbrot, Polis (1989).

The first method, based on a necessary and sufficient condition, requires a priori knowledge of the set of 2^m generating quasi-polynomials (6) of the polytopic family $\mathcal{W}$. In the case when $\mathcal{W}$ is a polytope of polynomials, this method extends (with some modifications) the main result of Cavallo, Celentano, De Maria (1991) into the class of non-monic polynomials with linearly dependent coefficient perturbations.

The second method is a generalization of the main result of Busłowicz (1994) to the class of the perturbed quasi-polynomials of the form (1). This method, which is based on a necessary and on a sufficient conditions, does not require a priori knowledge of the generating quasi-polynomials (6). Moreover, on applying the second method, we can easily find the largest D-stability domain in the space of perturbed parameters for which the proposed sufficient condition is satisfied.

2. The main results

Consider the separate sub-region D_i whose boundary has a parametric description $f_i(y)$, $y \in Y_i$.

LEMMA 2.1 *The perturbed quasi-polynomial (1) and the nominal quasi-polynomial $w_0(s)$ have the same number of roots in the sub-region D_i if and only if the polytope (convex polygon)*

$$\mathcal{P}_i(y) = conv\{\tilde{p}_1(f_i(y)), \tilde{p}_2(f_i(y)), \ldots, \tilde{p}_K(f_i(y))\}, \tag{9}$$

where

$$\tilde{p}_k(f_i(y)) = p_k(f_i(y))/w_0(f_i(y)), \ k = 1, 2, \ldots, K, \tag{10}$$

does not include the origin of the complex plane for all $y \in Y_i$.

Proof. From Barmish, Shi (1989) it follows that quasi-polynomial $w(s, q)$ of the form (1) has in the region D_i the same number of roots as nominal quasi-polynomial $w_0(s)$ if and only if

$$w(f_i(y), q) = 0 \text{ for all } y \in Y_i \text{ and all } q \in Q.$$

The above condition is equivalent to

$$\tilde{w}(f_i(y), q) = 0 \text{ for all } y \in Y_i \text{ and all } q \in Q,$$

where

$$\tilde{w}(f_i(y), q) = w(f_i(y), q)/w_0(f_i(y)). \tag{11}$$

From (11) and (1) for $s = f_i(y)$ we have

$$\tilde{w}(f_i(y), q) = 1 + q_1 \tilde{w}_1(f_i(y)) + \ldots + q_m \tilde{w}_m(f_i(y)), \tag{12}$$

where

$$\tilde{w}_k(f_i(y)) = w_k(f_i(y))/w_0(f_i(y)), \ k = 1, 2, \ldots, m. \tag{13}$$

For any fixed $q \in Q$ and any fixed $y \in Y_i$ the point $\tilde{w}(f_i(y), q)$ lies in the convex polygon (9). Hence, $\tilde{w}(f_i(y), q) = 0$ for all $y \in Y_i$ and all $q \in Q$ if and only if the polygon (9) does not include the origin of the complex plane for all $y \in Y_i$. This completes the proof. ∎

From (12) and the definition (3) of the bounding set Q it follows that the point $(1, 0)$ of the complex plane lies in the polytope (9) for any $y \in Y_i$. This means that a part of the polygon (9) always lies in the right-half plane. Hence, the convex polygon (9) does not include the origin of the complex plane if and only if all its exposed edges do not cross the nonpositive part of the real axis.

For simplicity we may consider the set of all edges of the convex polygon (9) instead of the set of all exposed edges.

Let $[\tilde{p}_r(f_i(y)), \tilde{p}_k(f_i(y))]$ denote an edge of $\mathcal{P}_i(y)$ (line segment in the complex plane with endpoints $\tilde{p}_r(f_i(y))$ and $\tilde{p}_k(f_i(y))$). This edge does not cross the nonpositive part of the real axis for all $y \in Y_i$ if and only if

$$|\arg(\tilde{p}_r(f_i(y))) - \arg(\tilde{p}_k(f_i(y)))| < \pi, \ \forall y \in Y_i, \tag{14}$$

where $\arg(s) \in [-\pi, \pi)$ denotes the main argument of the complex number s.

From the above considerations we have the following lemma.

LEMMA 2.2 *The perturbed quasi-polynomial (1) has in the sub-region D_i the same number of roots as the nominal quasi-polynomial $w_0(s)$ if and only if the condition (14) holds for all $r, k = 1, 2, \ldots, K, \ r > k$.*

We now introduce the testing function $F_i(y)$, associated with the separate sub-region D_i, defined by

$$F_i(y) = \pi - \phi_i(y), \tag{15}$$

where for all fixed $y \in Y_i$

$$\phi_i(y) = \max\{|\arg(\tilde{p}_r(f_i(y))) - \arg(\tilde{p}_k(f_i(y)))| : r, k = 1, \ldots, K; r > k\}. \tag{16}$$

It is easy to see that the condition (14) is satisfied if and only if

$$F_i(y) > 0, \ \forall y \in Y_i. \tag{17}$$

By considering all sub-regions D_i $(i = 1, 2, \ldots, N)$ of the region D we have the following theorem

THEOREM 2.1 *The perturbed quasi-polynomial $w(s,q)$ of the form (1) is D-stable if and only if the condition (17) holds for all separate sub-regions D_i, $i = 1, 2, \ldots, N$.*

In the case of the polytopic family $\mathcal{W}$ of polynomials, Theorem 2.1 extends (with some modifications) the main result of Cavallo, Celentano, De Maria (1991) into the class of non-monic polynomials with linearly dependent coefficient perturbations.

Computing the testing function $F_i(y)$ from the formulae (15) and (16) requires knowledge of the generating quasi-polynomials (6) of the polytopic family $\mathcal{W}$. Knowledge of the exposed edges of $\mathcal{W}$ is not required. Moreover, testing the condition (17) requires analysis of only one plot of $F_i(y)$, whereas testing the condition (ii) of Lemma 1.1 requires analysis of $m2^{m-1}$ plots of the function $p_k(f_i(y))/p_r(f_i(y))$, $y \in Y_i$, corresponding to all the separate exposed edges of $\mathcal{W}$.

Now we give a simple computational method for D-stability analysis, which does not require a priori knowledge of the generating quasi-polynomials of the polytopic family $\mathcal{W}$.

It is easy to see that the convex polygon (9) does not include the origin of the complex plane for all $y \in Y_i$ if it lies entirely in the open right-half plane, i.e. if

$$B_i(y) > 0, \ \forall y \in Y_i, \tag{18}$$

where

$$B_i(y) = \min\{Re[\tilde{p}_1(f_i(y))], \ldots, Re[\tilde{p}_K(f_i(y))]\}, \tag{19}$$

or, equivalently,

$$B_i(y) = \min\{Re[\tilde{w}(f_i(y), q)] : \ q \in Q\}, \tag{20}$$

where $\tilde{w}(f_i(y), q)$ has the form (12).

Hence, the D-stability of the perturbed quasi-polynomial (1) can be checked by using the method given in Busłowicz (1994) for the polynomials with linearly dependent coefficient perturbations.

Now we give a short description of this method with extension into the considered class of perturbed quasi-polynomials.

The method of Busłowicz (1994) is based on the analysis, for all separate sub-regions D_i, of the complex testing function $T_i(y)$, where

$$Re[T_i(y)] = B_i(y) \tag{21}$$

and $B_i(y)$ is defined by (20).

The complex testing function $T_i(y)$, associated with the separate sub-region D_i, can be computed for all fixed $y \in Y_i$ from the formula

$$T_i(y) = 1 + t_{1i}(y) + t_{2i}(y) + \ldots + t_{mi}(y), \tag{22}$$

where

$$t_{ki}(y) = \begin{cases} b_k \tilde{w}_k(f_i(y)), & \text{if} \quad Re[\tilde{w}_k(f_i(y))] \geq 0, \\ c_k \tilde{w}_k(f_i(y)), & \text{if} \quad Re[\tilde{w}_k(f_i(y))] < 0 \end{cases} \tag{23}$$

Note that the complex testing function $T_i(y)$ is generated using only $(m+1)$ quasi-polynomials $w_k(s)$, $k = 0, 1, \ldots, m$, of the perturbed quasi-polynomial (1), whereas generation of the real testing function $F_i(y)$ requires a priori knowledge of the generating quasi-polynomials (6) of the polytopic family $\mathcal{W}$.

The plot of the complex testing function $T_i(y)$ is piecewise continuous and smooth and it is composed of the pieces of the plots of $\tilde{p}_k(f_i(y)) = p_k(f_i(y))/w_0(f_i(y))$, where $p_k(s)$ is the generating quasi-polynomial of the form (6).

By generalization of the main result of Busłowicz (1994) to the class of considered perturbed quasi-polynomials we have the following theorem.

THEOREM 2.2

1. If
$$B_i(y) = Re[T_i(y)] > 0, \ \forall y \in Y_i, \tag{24}$$
   for all separate sub-regions D_i, $i = 1, 2, \ldots, N$, then the perturbed quasi-polynomial (1) is D-stable,
2. if at least one plot of $T_i(y)$, $i = 1, 2, \ldots, N$, crosses the nonpositive part of the real axis, then the perturbed quasi-polynomial (1) with bounding set Q of the form (3) is not D-stable,
3. the largest bounding set $\hat{Q}$, for which the condition (24) holds for all sub-regions D_i, $i = 1, 2, \ldots, N$, has the form
$$\hat{Q} = \{q \in R^m : q_k \in (\delta b_k, \delta c_k), \ b_k \leq 0, c_k \geq 0, \ k = 1, 2, \ldots, m\}, \tag{25}$$
   where
$$\delta = \min\{\delta_1, \delta_2, \ldots, \delta_N\}, \tag{26}$$
$$\delta_i = 1/(1 - \alpha_i), \ i = 1, 2, \ldots, N, \tag{27}$$
$$\alpha_i = \min\{B_i(y) = Re[T_i(y)] : y \in Y_i\}, \tag{28}$$
   where $T_i(y)$ is computed from (22) and (23) with the bounding set Q of the form (3).

Theorem 2.2 gives the computational method for the D-stability analysis of the perturbed quasi-polynomial (1). This method, which is essentially based on the sufficient condition (24), may be too conservative in some cases. However, the method remains attractive due to its simplicity.

3.   Illustrative example

In this section we apply the proposed methods to the D-stability analysis of a
real life control system with delay (feedback control of a wind tunnel), provided
in Fu, Olbrot, Polis (1989). The characteristic quasi-polynomial of this system
has the form

$$
\begin{aligned}
w(s,\tau,k) \;=\; & \tau s^3 + (6\tau + 1)s^2 \\
& + (13.75\tau + 6 + 1.82\tau e^{-0.165s} + 0.42\tau e^{-0.33s})s \\
& + 13.75 + 1.82 e^{-0.165s} + (0.42 - 1305k)e^{-0.33s},
\end{aligned}
\tag{29}
$$

where $\tau \in [0.739, 2.58]$, $k \in [-0.0144, -0.0029]$, with the nominal values $\tau_0 =
1.964$ and $k_0 = -0.0117$.

All roots of the nominal quasi-polynomial $w(s, \tau_0, k_0)$ have a real part less
than -1 (Fu, Olbrot, Polis 1989).

We need to check if all the roots of the perturbed quasi-polynomial (30) have
real parts less than -1, or equivalently, if this quasi-polynomial is D-stable,
where

$$
D = \{s : \; Re[s] < -1\}.
\tag{30}
$$

The parametric description of the boundary of D has the form $f(y) = -1 +
jy$, $y \in Y = R$. Because the coefficients of the quasi-polynomial (30) are real
and D is symmetric with respect to the real axis, we can restrict Y to the finite
interval $[0, y_f]$.

Quasi-polynomial (30) can be rewritten in the form

$$
w(s, q) = w_0(s) + q_1 w_1(s) + q_2 w_2(s), \; q \in Q,
\tag{31}
$$

where

$$
\begin{aligned}
w_0(s) \;=\; & 1.964 s^3 + 12.784 s^2 + (33.005 + 3.57448 e^{-0.165s} \\
& + 0.82488 e^{-0.33s})s + 13.75 + 1.82 e^{-0.165s} + 15.6885 e^{-0.33s}, \\
w_1(s) \;=\; & s^3 + 6s^2 + (13.75 + 1.82 e^{-0.165s} + 0.42 e^{-0.33s})s, \\
w_2(s) \;=\; & -1305 e^{-0.33s}
\end{aligned}
$$

and

$$
Q = \{q \in R^2 : \; q_1 \in [-1.225, 0.616], \; q_2 \in [-0.0027, 0.0088]\}.
\tag{32}
$$

Parameters q_1 and q_2 represent deviations of uncertain parameters τ and k from
their nominal values, respectively.

On computing from (22) and (23) the complex testing function $T(y)$, $y \in
Y = [0, 5]$, we find that the plot of this function crosses the negative part of the
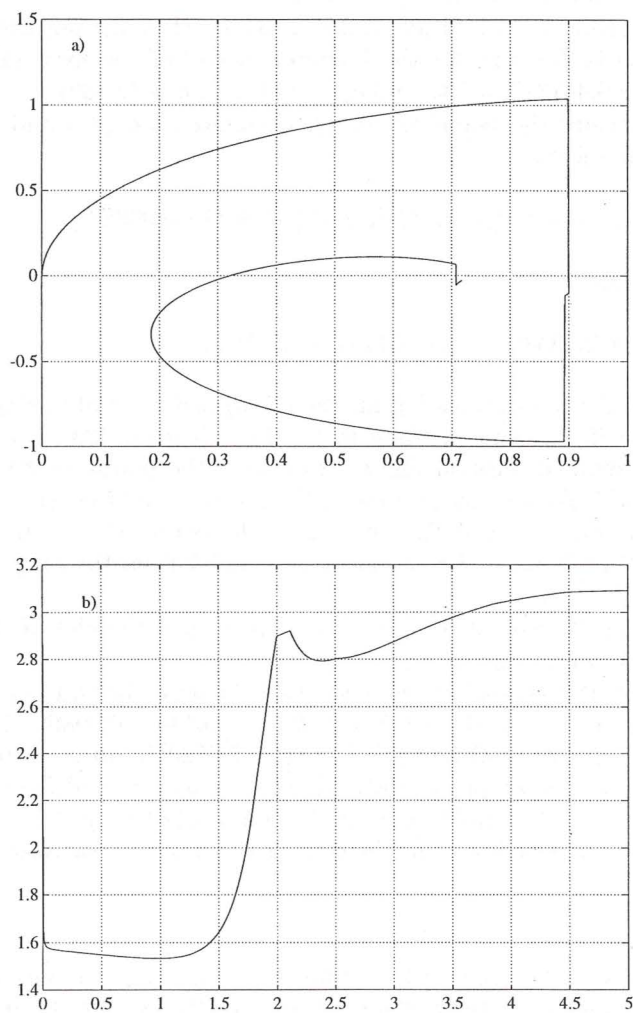real axis, namely $T(0) = \alpha + j0$, where $\alpha = \min\{Re[T(y)]: y \in Y\} = Re[T(0)] =$

Figure 1. Testing functions for quasi-polynomial (31) with bounding set (33):
a) complex testing function $T(y)$; b) real testing function $F(y)$.

-1.2659. Hence, from condition 2 of Theorem 2.2 it follows that the perturbed quasi-polynomial (31) with bounding set Q of the form (32) is not D-stable. The same result was obtained in Fu, Olbrot, Polis (1989) from the analysis, according to condition (ii) of Lemma 1.1, of four plots of $p_k(f(y))/p_r(f(y))$, corresponding to all the exposed edges of $\mathcal{W}$.

From the condition 3 of Theorem 2.2 it follows that the perturbed quasi-polynomial (31) is D-stable for the bounding set $\hat{Q}$ of the form (25), where $\delta = 1/(1-\alpha) = 0.441317$ and b_k and c_k $(k = 1, 2)$ are as in (32).

We now consider the bounding set Q, obtained from (32) and (25) with $\delta = 0.4413$, of the form

$$Q = \{q : \ q_1 \in [-0.5406, 0.2718], \ q_2 \in [-0.00119, 0.00388]\}, \tag{33}$$

which corresponds to

$$\tau \in [1.4234, 2.2358], \ k \in [-0.01289, -0.00782]. \tag{34}$$

The plots of the complex testing function $T(y)$ and the real testing function $F(y)$, $y \in Y = [0, 5]$, corresponding to the bounding set (33), are shown in Figure 1. The straight lines on Figure 1a) denote the jumps of the plot $T(y)$ from the plot $\tilde{p}_k(f(y))$ onto the plot $\tilde{p}_i(f(y))$. Because $\min\{Re[T(y)]: \ y \in Y\} = 0.0007$ from the condition 1 of Theorem 2.2 it follows that the perturbed quasi-polynomial (30) is D-stable for all values of τ and k from the ranges given in (34).

The above result also follows from Theorem 2.1 and the plot of the testing function $F(y)$ shown in Figure 1b).

In Fu, Olbrot, Polis (1989) it was shown that the perturbed quasi-polynomial (30) is D-stable for $\tau \in [1.571, 2.357]$ and $k \in [-0.0144, -0.0088]$ which corresponds to $q_1 \in [-0.393, 0.393]$ and $q_2 \in [-0.0027, 0.0029]$ in the quasi-polynomial (31). In such a case, on computing the testing functions $T(y)$ and $F(y)$ we obtain $\min\{Re[T(y)]: \ y \in Y\} = Re[T(0)] = 0.039$ and $\min\{F(y): \ y \in Y\} = 1.5322$. This means that the conditions of Theorems 2.1 and 2.2 are satisfied.

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context-free languages has been developed.

T. Onisawa, University of Tsukuba, Japan;
J. Kacprzyk, Polish Academy of Sciences, Warsaw,
Poland (Eds.)

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1995. X, 376 pp. 106 figs., 48 tabs.
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sibility theory in reliability and safety analysis. It pre-
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of uncertainty in these problems.

P. Bosc, IRISA/ENSSAT, Lannion, France;
J. Kacprzyk, Polish Academy of Sciences, Warsaw,
Poland (Eds.)

Fuzziness in Database Management Systems

1995. VIII, 433 pp. 100 figs., 32 tabs.
(Studies in Fuzziness, Vol. 5)
Hardcover DM 168,-   ISBN 3-7908-0858-X

This book provides a comprehensive treatise on the
use of fuzzy sets, fuzzy logic, and possibility theory
for dealing with imprecise and vague information in
database management systems. Theoretical works
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approach to database security is considered.

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