Control and Cybernetics

vol. **24** (1995) No. 2

m-D characteristic polynomial factorization using operators' method and state feedback control

by

N.E. Mastorakis, S.G. Tzafestas*, N.J. Theodorou

National Technical University of Athens
Electrical and Computer Engineering Department
Patission 42, Athens 10682, Greece.
* Zografou Campus, Athens 15773, Greece.

Abstract: This paper deals with the factorization of the characteristic polynomial of an m-D (m-dimensional) dynamic system through state feedback control. The factorization type of concern is that where one of the factors is at most (m-1)-dimensional, i.e. it contains at most m-1 variables. Under the assumption that such a factorization is not possible for the characteristic polynomial of the open-loop system, the paper derives necessary and sufficient conditions for the closed-loop polynomial to be factorizable. An illustrative example is provided.

1. Introduction

Much research effort has been devoted in recent years to problems involving multidimensional (m-D) signals and multidimensional (m-D) systems. An m-D signal is a function of more than one variable while an m-D system is an algorithm or a transformation that transforms an m-D input signal to an m-D output signal. If for a linear combination of two inputs, the same linear combination of their corresponding outputs is obtained in the output of the system, the system is said to be linear. If for each shifted m-D input signal a similarly shifted m-D output signal is obtained, the system is said to be shift invariant. Linear and shift invariant m-D systems, indicated by the symbol LSI, have recently attracted increasing attention. The reason is that many practical systems and applications lead to m-D models. Among these applications, we mention m-D digital filtering and image processing, biomedical and geophysical data processing, remote sensing, computer vision, underwater acoustics, moving-objects recognition, x-ray enhancement, digital memory modeling and distributed-parameter system analysis – Kaczorek (1985), Tzafestas (1986), Roesser (1975).

An m-D, LSI, SISO (single input, single output) discrete system, with input $u(n_1,\ldots,n_m)$, output $y(n_1,\ldots,n_m)$ and corresponding m-D z-transform $U(z_1,\ldots,z_m)$, $Y(z_1,\ldots,z_m)$ can be defined by its m-D transfer function (here $n_1,\ldots,n_m$ are positive integers):

$$G(z_1,\ldots,Z_m)=\frac{Y(z_1,\ldots,Z_m)}{U(z_1,\ldots,Z_m)}= \tag{1}$$
$$=\frac{\sum_{i_1=0}^{N_1}\cdots\sum_{i_m=0}^{N_m}Q_{i_1,\ldots,i_m}\cdot z_1^{i_1}\ldots z_m^{i_m}}{\sum_{i_1=0}^{N_1}\cdots\sum_{i_m=0}^{N_m}F_{i_1,\ldots,i_m}\cdot z_1^{i_1}\ldots z_m^{i_m}}$$

where $N_1,\ldots,N_m$ are positive integers, and $Q_{i_1,\ldots,i_m}\in\mathbf{R}$, $F_{i_1,\ldots,i_m}\in\mathbf{R}$.

The characteristic polynomial of the system is

$$\sum_{i_1=0}^{N_1}\cdots\sum_{i_m=0}^{N_m}F_{i_1,\ldots,i_m}\cdot z_1^{i_1}\ldots z_m^{i_m}$$

and is henceforth symbolized as $f_c(z_1,\ldots,z_m)$.

If an m-D polynomial can be written as a product of other lower degree polynomials, then it is said to be factorizable. Factorization of m-D polynomials is one of the primary processes in the field of m-D systems, since, in particular, it helps in performing simpler realizations, applying simpler stability tests, and designing simpler controllers – Theodorou, Tzafestas (1985); Mastorakis, Tzafestas, Theodorou (1990); Mastorakis, Theodorou (1992); Mastorakis, Theodorou, Tzafestas (1992); Mastorakis, Theodorou, Tzafestas (1994); Mastorakis, Theodorou, (1990). More specifically, if the numerator and denominator of the transfer function $G(z_1,\ldots,z_m)=g(z_1,\ldots,z_m)/f_c(z_1,\ldots,z_m)$ are factorized as:

$$\begin{aligned} g(z_1,\ldots,z_m) &= g_1(z_1,\ldots,z_m)\ldots g_N(z_1,\ldots,z_m)\\ f_c(z_1,\ldots,z_m) &= f_1(z_1,\ldots,z_m)\ldots f_N(z_1,\ldots,z_m)\end{aligned}$$

where the g_i's and f_i's are obviously simpler than g and f_c, respectively, one has to realize the simpler m-D transfer functions:

$$G_1(z_1,\ldots,z_m)=\frac{g_1(z_1,\ldots,z_m)}{f_1(z_1,\ldots,z_m)},\ldots,G_N(z_1,\ldots,z_m)=\frac{g_N(z_1,\ldots,z_m)}{f_N(z_1,\ldots,z_m)}$$

As the stability tests are in the form: "check if $f_c(z_1,\ldots,z_m)=0$ (in appropriate regions of $z_1,\ldots,z_m$)", it is important to factorize $f_c(z_1,\ldots,z_m)$ in factors $f_1(z_1,\ldots,z_m),\ldots,f_N(z_1,\ldots,z_m)$, because in this case the stability test is decomposed into simpler ones.

The factorization results of m-D polynomials are also useful in the theory of distributed-parameter systems (DPS), which are described by partial differential equations, since the characteristic polynomials of DPS are actually m-D polynomials.

Finally, some properties of m-D systems like controllability, observability, etc., are studied in a straightforward manner if $g(z_1, \ldots, z_m)$, $f_c(z_1, \ldots, z_m)$ are m-D factorizable polynomials.

It should be noted that, up to now, the general factorization problem, i.e. the factorization of any factorizable polynomial, has not yet been fully solved. For this reason, some more or less special types of m-D polynomial factorization have been studied – Mastorakis, Theodorou, Tzafestas (1992); Mastorakis, Theodorou, Tzafestas (1994); Mastorakis, Theodorou, (1990), Tzafestas, Theodorou (1985), Theodorou (1985).

In Theodorou, Tzafestas (1985), the factorization in monovariable $(1-D)$ factors i.e. $f(z_1, \ldots, z_m) = f_1(z_1) \ldots f_m(z_m)$, or in factors with no common variables, i.e. $f(z_1, \ldots, z_m) = f_1(\bar{z}_1) \ldots f_k(\bar{z}_k)$, where $\bar{z}_1, \ldots, \bar{z}_k$ are mutually disjoint groups of independent variables, has been completely solved.

In Mastorakis, Tzafestas, Theodorou (1990), the factorization is succeeded by considering the given polynomial as $(1-D)$ polynomial with respect to z_j and applying the well-known formulas from elementary algebra, in case that the given polynomial is of 2nd, 3rd, 4th degree in z_j. In Mastorakis, Theodorou (1992), factorization of the State-Space Model is presented. In Mastorakis, Theodorou, Tzafestas (1992), factorization of an m-D polynomial in linear factors i.e. $f(z_1, \ldots, z_m) = \prod_{i=1}^{N_1}(z_1 + a_{i,2}z_2 + \ldots + a_{i,m}z_m + c_i)$ has been studied. In Mastorakis, Theodorou, Tzafestas (1994), factorization of an m-D polynomial in general non-linear factors is presented. In Mastorakis, Theodorou (1990), the factorization of an m-D polynomial in factors where at least one factor contains no more than $m-1$ variables has been studied. The latter type of factorization is considered in the present work.

Each type of the above factorizations has some features and some advantages If a particular type of factorization is not obtained though it is desired, one should modify the system using feedback control.

Generally by applying feedback one attempts to make the system possess some particular features, e.g. stability (feedback stabilization), input-output decoupling (state feedback decoupling), pole-assignment, sensitivity reduction etc. The present technique of factorizing an m-D characteristic polynomial through application of state feedback belongs to this class of techniques. Actually, by the feedback the original (nonfactorizable in the given sense) system is transformed to a new system which is factorizable. The practical value of this has been discussed in many instances. See for example Tzafestas, Theodorou (1985); Theodorou, Tzafestas (1985).

The present paper is organized as follows: In Section 2, the m-D state space model is presented. In Section 3, the feedback-control law is presented as well as the relation between the original and the "closed-loop" characteristic polynomial. In Section 4, the factorization technique is developed.

2. The m-D state-space model

The canonical state-space model of the m-D transfer function (1) is:

$$\hat{x} = \mathcal{A} \cdot x + \mathcal{B} \cdot u \tag{2}$$

$$y = \mathcal{C} \cdot x + \mathcal{D} \cdot u \tag{3}$$

where:

$$x = \begin{bmatrix} x_1(n_1, \ldots, n_m) \\ \vdots \\ x_m(n_1, \ldots, n_m) \end{bmatrix}, \hat{x} = \begin{bmatrix} x_1(n_1+1, \ldots, n_m) \\ \vdots \\ x_m(n_1, \ldots, n_m+1) \end{bmatrix},$$

x is the state-space vector of $\tilde{N}$ dimension, $\tilde{N} = \tilde{N}_1 + \ldots + \tilde{N}_m$, N_i is the dimension of $x_i(n_1, \ldots, n_m)$, $i = 1, \ldots m$ ($\tilde{N}_i \geq N_i$, in general), $u = u(n_1, \ldots, n_m)$, $y = y(n_1, \ldots, n_m)$ are the scalar input and output respectively, and $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ are matrices of appropriate dimensions (the algorithmic way of their determination is given in Tzafestas, Theodorou, 1987).

It is well known that

$$f_C(z_1, \ldots, z_m) = \det(\mathcal{Z} - \mathcal{A}) = \sum_{i_1=0}^{N_1} \cdots \sum_{i_m=0}^{N_m} F_{i_1,\ldots,i_m} \cdot z_1^{i_1} \ldots z_m^{i_m} \tag{4}$$

where

$$\mathcal{Z} = \text{diag}\ (z_1 \mathbf{I}_{\tilde{N}_1}, \ldots, z_m \mathbf{I}_{\tilde{N}_m})$$

($\mathbf{I}_{\tilde{N}_i}$ is the unit (identity) $\tilde{N}_i \times \tilde{N}_i$ matrix)

Example. Consider the polynomial

$$f_C(z_1, z_2, z_3) = z_1^2 z_2 z_3 + 2z_1^2 z_3 - z_1^2 z_2 - z_1^2 + 3z_1 z_2 z_3 + 2z_1 z_3 + z_1 z_2 + z_3$$

and the transfer function:

$$G(z_1, z_2, z_3) = \frac{z_1^2 z_2 z_3}{f_C(z_1, z_2, z_3)}$$

which is of the form (1). Applying the analysis of Tzafestas, Theodorou (1987), the following state-space model can be found:

$$\begin{bmatrix} x_1(n_1+1,n_2,n_3) \\ \cdots \\ x_2(n_1,n_2+1,n_3) \\ \cdots \\ x_3(n_1,n_2,n_3+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \vdots & 0 & \vdots & 1 \\ 1 & 0 & 0 & 0 & \vdots & 0 & \vdots & 0 \\ -4 & -1 & -3 & -1 & \vdots & 1 & \vdots & 0 \\ 0 & 0 & 1 & 0 & \vdots & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & & \cdots \\ 6 & 2 & 4 & 2 & \vdots & -2 & \vdots & -1 \\ -4 & -1 & -3 & -1 & \vdots & 1 & \vdots & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} x_1(n_1,n_2,n_3) \\ \cdots \\ x_2(n_1,n_2,n_3) \\ \cdots \\ x_3(n_1,n_2,n_3) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \cdots \\ -2 \\ \cdots \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} -4 & -1 & -3 & -1 & \vdots & 1 & \vdots & 1 \end{bmatrix} \begin{bmatrix} x_1(n_1,n_2,n_3) \\ \cdots \\ x_2(n_1,n_2,n_3) \\ \cdots \\ x_3(n_1,n_2,n_3) \end{bmatrix} + \\ + \begin{bmatrix} 1 \end{bmatrix} \cdot u$$

3. State feedback control

An m-D SISO discrete system, defined by its transfer function (1), can be found under the feedback control law:

$$u = f^T \cdot x + \omega \tag{5}$$

where x is the state-space vector, f is a $\tilde{N}$-vector, and ω is the scalar input of the so called closed-loop system.

Using this feedback-control law, the characteristic polynomial f_C of the original system is changed into the closed-loop characteristic polynomial f_C^*. Let

$$f_C^*(z_1,\ldots,z_m) = \sum_{i_1=0}^{N_1} \cdots \sum_{i_m=0}^{N_m} \cdot F^*_{i_1,\ldots,i_m} \cdot z_1^{i_1} \ldots z_m^{i_m} \tag{6}$$

Then, the following equation holds for the relation of the closed-loop and the original polynomials, Theodorou, Tzafestas (1985):

$$f_C^*(z_1,\ldots,z_m) = f_C(z_1,\ldots,z_m) - f^T \cdot \Lambda(z_1,\ldots,z_m) \tag{7}$$

where

$$\Lambda(z_1,\ldots,z_m) = \text{adj}\,(\mathcal{Z}-\mathcal{A})\cdot\mathcal{B} = \sum_{i_1=0}^{N_1}\cdots\sum_{i_m=0}^{N_m} \lambda_{i_1,\ldots,i_m}\cdot z_1^{i_1}\ldots z_m^{i_m} \tag{8}$$

4. The factorization technique

The above state feedback property is useful for changing an m-D polynomial, which cannot be factorized, into a factorizable one. The factorization of concern is:

$$f(z_1,\ldots,z_m) = h(z_1,\ldots,z_{i-1},z_{i+1},\ldots,z_m)\cdot q(z_1,\ldots,z_m) \tag{9}$$

i.e. one factor of the polynomial does not include (at least) one variable. The necessary and sufficient condition for such a factorization is given in the Appendix (Mastorakis, Theodorou, 1985):

$$\frac{T\cdot f(z_1,\ldots,z_m)}{f(z_1,\ldots,z_m)} = \frac{T\cdot q(z_1,\ldots,z_m)}{q(z_1,\ldots,z_m)} \tag{10}$$

where $q(z_1,\ldots,z_m)$ is a polynomial factor of $f(z_1,\ldots,z_m)$ and T is a special linear operator, i.e.

$$T\cdot(c_1f_1+c_2f_2) = c_1T\cdot f_1 + c_2T\cdot f_2 \tag{11}$$

where c_1, c_2 are constants and f_1, f_2 are m-D polynomials, and T is "special with respect to z_i", because it acts only on the variable z_i. Also T is "special" because it can only assume the following four types of linear operators, Mastorakis, Theodorou (1990):

(i) Partial differentiation operator: $T = \frac{\partial}{\partial z_i}$

(ii) Integration operator with respect to z_i: $T = \int\cdots dz_i$

(iii) Displacement operator:

$$T\cdot f(z_1,\ldots,z_{i-1},z_i,z_{i+1},\ldots,z_m) = \\ = f(z_1,\ldots,z_{i-1},z_i+a,z_{i+1},\ldots,z_m)$$

(iv) Replacement operator:

$$T\cdot f(z_1,\ldots,z_{i-1},z_i,z_{i+1},\ldots,z_m) = \\ = f(z_1,\ldots,z_{i-1},r(z_1,\ldots,z_{i-1},,z_{i+1},\ldots,z_m),z_{i+1},\ldots,z_m)$$

Thus, a polynomial can be factorized as in (9), if and only if (10) holds.

If the characteristic polynomial of the system is not factorizable as in (9), the feedback control technique is applied. The necessary and sufficient condition for factorizability of the (unknown) closed-loop polynomial is

$$\frac{T \cdot f_C^*(z_1, \ldots, z_m)}{f_C^*(z_1, \ldots, z_m)} = \frac{T \cdot q(z_1, \ldots, z_m)}{q(z_1, \ldots, z_m)} \tag{12}$$

where $q(z_1, \ldots, z_m)$ is a desired factor of the closed-loop characteristic polynomial. Substituting (7) into (12), Eq. (13) is obtained:

$$q(z) \cdot T \cdot [f_C(z) - f^T \cdot \Lambda(z)] = [f_C(z) - f^T \cdot \Lambda(z)] \cdot T \cdot q(z) \tag{13}$$

where $f_C(z) = f_C(z_1, \ldots, z_m)$, $q(z) = q(z_1, \ldots, z_m)$ and $\Lambda(z) = \Lambda(z_1, \ldots, z_m)$, or after some algebraic manipulation:

$$f^T \cdot [\Lambda(z) T \cdot q - q \cdot T \cdot \Lambda(z)] = f_C(z) \cdot T \cdot q - q \cdot T \cdot f_C(z) \tag{14}$$

To find the unknown vector f, the coefficients of $z_1^{i_1} \cdots z_m^{i_m}$ on both sides of (14) are equated.

So, one finds a linear system of μ equations in $\tilde{N}$ unknowns ($\mu \geq \tilde{N}$). This linear system can be written in the compact form:

$$\mathcal{M} \cdot f = \mathcal{L} \tag{15}$$

Suppose that $\rho = \text{rank}\ (\mathcal{M})$. Then, there exists a $\mu \times \mu$ non-singular matrix $\mathcal{G}$, such that:

$$\mathcal{G} \cdot \mathcal{M} = \begin{bmatrix} \mathcal{M}_1 \\ \cdots \\ 0 \end{bmatrix} \quad \mathcal{G} \cdot \mathcal{L} = \begin{bmatrix} \mathcal{L}_1 \\ \cdots \\ \mathcal{L}_2 \end{bmatrix} \tag{16}$$

where $\mathcal{M}_1$ contains the p independent rows of $\mathcal{M}$. The algebraic system is then transformed to

$$\begin{bmatrix} \mathcal{M}_1 \\ \cdots \\ 0 \end{bmatrix} \cdot f = \begin{bmatrix} \mathcal{L}_1 \\ \cdots \\ \mathcal{L}_2 \end{bmatrix} \tag{17}$$

Hence:

$$\mathcal{M}_1 \cdot f = \mathcal{L}_1 \tag{18}$$

$$\mathcal{L}_2 = 0 \tag{19}$$

Equation (19) gives the $\mu - \rho$ necessary and sufficient conditions, under which (18), and hence the factorization problem has a solution. Equation (18)

is a linear system of ρ equations in $\tilde{N}$ unknowns and can be solved, having $\tilde{N}-\rho$ degrees of freedom. In the special case $\rho = \tilde{N}$, there exists a unique solution of the system, given by:

$$f = \mathcal{M}_1^{-1}\mathcal{L}_1 \tag{20}$$

So, the following theorem has been established:

THEOREM 4.1 *Given a non factorizable (in the sense of (9)), m-D characteristic polynomial $f_C(z_1,\ldots,z_m)$ of the form (4), the problem of choosing the state feedback gain-vector f, (obeying (5)), such that f_C is changed into a factorizable (in the sense of (9)) closed-loop polynomial f_C^*, with a desirable factor $q(z_1,\ldots,z_m)$, has a solution, if and only if, the compatibility $\mu-\rho$ conditions (19) are satisfied.*

If these conditions hold, the $\tilde{N}$-vector f is given by the solution of (18), which when $\rho = rank\ (\mathcal{M}) = \tilde{N}$ is uniquely determined by (20).

Example (continued). Consider the polynomial $f_C(z_1,z_2,z_3)$. Following the procedure presented in Mastorakis, Theodorou, Tzafestas (1990), one can see that this polynomial is not factorizable in the sense of (9). In order to change this polynomial into a factorizable closed-loop one, the feedback-control technique is applied. Therefore the respective state-space model is considered. Now, $\Lambda(z_1,\ldots,z_m)$ is found, where

$$\Lambda(z_1,\ldots,z_m) = \text{adj}\ (\mathcal{Z}-\mathcal{A})\cdot\mathcal{B} =$$

$$= \begin{bmatrix} 0\\1\\0\\-1\\0\\0 \end{bmatrix} z_2 + \begin{bmatrix} 1\\0\\-1\\0\\0\\0 \end{bmatrix} z_1z_2 + \begin{bmatrix} 0\\0\\1\\0\\0\\0 \end{bmatrix} z_1z_2z_3 + \begin{bmatrix} 0\\0\\0\\1\\0\\0 \end{bmatrix} z_2z_3 +$$

$$+ \begin{bmatrix} 0\\0\\0\\0\\-2\\0 \end{bmatrix} z_1^2z_3 + \begin{bmatrix} 0\\0\\0\\0\\-2\\0 \end{bmatrix} z_1z_2 + \begin{bmatrix} 0\\0\\0\\0\\1\\0 \end{bmatrix} z_1^2 + \begin{bmatrix} 0\\0\\0\\0\\0\\1 \end{bmatrix} z_1^2z_2$$

Let us choose the polynomial:

$$q = z_1 + z_2 + z_1z_2$$

as a desired factor of the unknown closed-loop polynomial. The operator $T = \frac{\partial}{\partial z_2}$ is considered. Introducing this operator in (14) one obtains:

$$f = [f_1, f_2, f_3, f_4, f_5, f_6]^T$$

Thus, equating the coefficients (of $z_1^{i_1} z_2^{i_2} z_3^{i_3}$) on both sides of (14), the next linear system results

$$\mathcal{M} \cdot f = \mathcal{L}.$$

where:

$$\mathcal{M} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 1 \\ 1 \\ 0 \\ \cdots \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In this case, $\mu = 14$, $\tilde{N} = 6$, $\rho = \text{rank}\, \mathcal{M} = 6$, $\mathcal{G} = \mathcal{I}$ (where $\mathcal{I}$ is the identity 14×14 matrix), and

$$\mathcal{M}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix}, \quad \mathcal{L}_1 = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathcal{L}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

One can see that the $\mu - \rho = 8$ compatibility conditions (19) are satisfied. Since these conditions hold and $\rho = \tilde{N}$, the solution is uniquely determined by (20). Therefore

$$f = [-2.5, 0, -1, 0, 0.5, 0.5]^T$$

The closed-loop characteristic polynomial can be found using the relation (7), i.e.

$$f_C^*(z_1, z_2, z_3) = z_2 z_3 - 0.5 z_1^2 - 0.5 z_1 z_2 + z_1^2 z_3 + z_1 z_3 - 0.5 z_1^2 z_2 + 2 z_1 z_2 z_3 + \\ + z_1^2 z_2 z_3$$

It is already known that $z_1 + z_2 + z_1 z_2$ is one of its factors. So, carrying out the division of $f_C^*(z_1, z_2, z_3)$ by $z_1 + z_2 + z_1 z_2$ one obtains the quotient: $-0.5z_1 + z_3 + z_1 z_3$

Thus,

$$f^*(z_1, z_2, z_3) = (z_1 + z_2 + z_1 z_2)(-0.5z_1 + z_3 + z_1 z_3)$$

5. Conclusion

In this paper, the problem of factorizing an m-D characteristic polynomial in the sense of (9) was solved through application of state feedback control. If the original polynomial is not factorizable in the sense of (9), it was found that the closed-loop polynomial is factorizable in this sense if the conditions (19) are satisfied. In this case the solution is provided by (18). In general, this feedback factorization technique may be used for m-D polynomial factorization in other senses too

References

KACZOREK T. (1985) Two-Dimensional Linear Systems, Springer-Verlag Berlin, Heidelberg.

MASTORAKIS N.E., THEODOROU N.J. (1992) State-Spacel Model Factorization in m-Dimensions. Application in Stability, *Found. Comp. Decision Science*, **17**.

MASTORAKIS N.E., THEODOROU N.J. (1990) Operators' Method for m-D Polynomials Factorization, *Found. Comp. Decision Science*, **15**, 2-3, pp. 159-172.

MASTORAKIS N.E., THEODOROU N.J., TZAFESTAS S.G. (1992) Multidimensional Polynomial Factorization in Linear m-D Factors, *Int. Journ. of Syst. Sci.*, **23**, ll, 1805-1824.

MASTORAKIS N.E., THEODOROU N.J., TZAFESTAS S.G. (1994) A General Factorization Method for Multivariable Polynomial, *Multidimensional Systems and Signal Processing*, **5**, pp. 151-178.

MASTORAKIS N.E., TZAFESTAS S.G., THEODOROU N.J. (1990) A Simple Multidimensional Polynomial Factorization method, *IMACS-IFAC Intern. Symp. on Math. and Intelligent Models in System Simulation*, Brussels, VII. B. 1-1.

ROESSER R.P. (1975) A Discrete State-Space Model for linear Image Processing, *IEEE Trans. Autom. Contr.*, **AC-20**, 1, pp. 1-10.

THEODOROU N.J., TZAFESTAS S.G. (1985) Characteristic Polynomial Assignment of 3-D Discrete Systems Using a Canonical State Space Model, *Int. Journ. Syst. Sci.*, **15**, 2, pp. 175-186.

THEODOROU N.J., TZAFESTAS S.G. (1985) Reducibility and factorizability of multivariable polynomials, *Control Theory and Advanc. Techn.*, **1**, pp. 25-46.

TZAFESTAS S.G. (1986) Multidimensional Systems, Techniques and Applications, Marcel Dekker, New York.

TZAFESTAS S.G., THEODOROU N.J. (1987) An Inductive approach to the State-Space Representation of Multidimensional Systems, *Control Theory and Advanced Technology*, **3**, pp. 293-322.

TZAFESTAS N.J., THEODOROU N.J. (1985) On the exact model matching of 3-D discrete systems using feedback control, *Canad. Elec. Eng. J.*, **10**, 1, pp. 22-31.

Appendix

The proof of (10) can be found in Mastorakis, Theodorou (1990). Since in this paper, $q(z_1, \ldots, z_m)$ is supposed to be a (polynomial) factor of $f(z_1, \ldots, z_m)$, (while in Mastorakis, Theodorou, 1990, this is not necessary), slight modifications in the proof are required.

THEOREM 5.1 *Let $q(z_1, \ldots, z_m)$ be a factor of $f(z_1, \ldots, z_m)$. The necessary and sufficient condition for a quotient $f(z_1, \ldots, z_m)/q(z_1, \ldots, z_m)$ not containing the variable z_i, is*

$$\frac{T \cdot f(z_1, \ldots, z_m)}{f(z_1, \ldots, z_m)} = \frac{T \cdot q(z_1, \ldots, z_m)}{q(z_1, \ldots, z_m)} \tag{21}$$

where T is a special linear operator with respect to z_i

Proof. Necessary: Suppose that

$$f(z_1, \ldots, z_m) = h(z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_m) \cdot q(z_1, \ldots, z_m) \tag{22}$$

and T is a special linear operator with respect to z_i, then

$$T \cdot f(z_1, \ldots, z_m) = h(z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_m) \cdot T \cdot q(z_1, \ldots, z_m) \tag{23}$$

The equations (22) and (23) yield (21).

Sufficient: The sufficiency is proved for each operator separately. Suppose that (21) holds.

(i) Let T be the partial differential operator with respect to z_i : $T = \frac{\partial}{\partial z_i}$ i.e. $T \cdot f = \partial f / \partial z_i = f'_{z_i}$

Equation (21) can be rewritten as $\frac{f_{z_i}{}'}{f} = \frac{q'_{z_i}}{q}$

or

$$f'_{z_i} \cdot q = f \cdot q'_{z_i}$$

or

$$\frac{f'_{z_i} \cdot q - f \cdot q'_{z_i}}{q^2} = 0$$

or

$$\left(\frac{f}{q}\right)'_{z_i} = 0 \tag{24}$$

Equation (24) shows that f/q is a function not containing z_i. Furthermore, since $q(z_1, \ldots, z_m)$ is a factor of $f(z_1, \ldots, z_m)$, f/q is a polynomial not containing z_i.

(ii) Let T be the integration operator with respect to z_i : $T = \int \cdots dz_i$ i.e. $F(z) = T \cdot f(z) = \int f(z) dz_i$
Then $f(z) = \frac{\partial}{\partial z_i}(T \cdot f(z)) = \frac{\partial}{\partial z_i} F(z) = F'_{z_i}(z)$. Suppose that (21) holds:

$$\frac{T \cdot f(z)}{f(z)} = \frac{T \cdot q(z)}{q(z)}$$

or

$$\frac{F}{f} = \frac{Q}{q} \tag{25}$$

or

$$\frac{f}{F} = \frac{q}{Q}$$

or

$$\frac{F'_{z_i}}{F} = \frac{Q'_{z_i}}{Q}$$

or

$$\frac{F'_{z_i} Q - F Q'_{z_i}}{Q^2} = 0$$

or

$$\left(\frac{F}{Q}\right)'_{z_i} = 0. \tag{26}$$

Equation (26) shows that F/Q is a function not containing z_i. From (25) we obtain that: f/q is also a function not containing z_i. Furthermore, since $q(z_1, \ldots, z_m)$ is a factor of $f(z_1, \ldots, z_m)$, f/q is a polynomial not containing z_i.

(iii) Let T be the displacement operator with respect to z_i:

$$T \cdot f(z_1, \ldots, z_{i-1}, z_i, z_{i+1}, \ldots, z_m) =$$
$$= f(z_1, \ldots, z_{i-1}, z_i + a, z_{i+1}, \ldots, z_m)$$

Then, if (21) holds, we have $\frac{T \cdot f}{T \cdot q} = \frac{f}{q}$, i.e.

$$\frac{f(z_1, \ldots, z_{i-1}, z_i + a, z_{i+1}, \ldots, z_m)}{q(z_1, \ldots, z_{i-1}, z_i + a, z_{i+1}, \ldots, z_m)} = \frac{f(z_1, \ldots, z_m)}{q(z_1, \ldots, z_m}$$

However $\frac{f}{q}$ is a rational function with respect to z_i.This function is not a periodic function (with respect to z_i) having period a. So $\frac{f}{q}$ does not contain z_i. Similarly, it is a polynomial not containing z_i.

(iv) Let T be the replacement operator with respect to z_i:

$$T \cdot f(z_1, \ldots, z_{i-1}, z_i, z_{i+1}, \ldots, z_m) =$$
$$= \quad f(z_1, \ldots, z_{i-1}, r(z_1, \ldots, z_{i-1}, , z_{i+1}, \ldots, z_m), z_{i+1}, \ldots, z_m)$$

Then, if $\frac{T \cdot f}{f} = \frac{T \cdot q}{q}$, we have $\frac{T \cdot f}{T \cdot q} = \frac{f}{q}$. Clearly $\frac{T \cdot f}{T \cdot q}$ does not contain z_i. Therefore $\frac{f}{q}$ does not contain z_i too. Similarly, f/q is a polynomial not containing z_i. ■

CONFERENCE ANNOUNCEMENT AND CALL FOR PAPERS

IEA $\frac{96}{}$ AIE

The Ninth International Conference on Industrial & Engineering Applications of Artificial Inteligence & Expert Systems

June 4-7, 1996, ARCOS Fukuoka, Tenjin Chuo-ku Fukuoka, Japan

Topics of interest include, but are not limited to:

Automated Problem Solving
CAD/CAM
Case-based Reasoning
Computer Vision
Connectionist Models
Dependability of AE/ES
Distributed AI Architectures
Expert Systems
Fuzzy Logic & Soft Computing
Genetic Algorithms
Heuristic Searching
Intelligent Computer Network
Intelligent Databases
Intelligent Interface
Intelligent Tutoring
KBS Methodologies
Knowledge Acquisition
Knowledge Representation
Machine Learning
Model-Based Reasoning
Natural Language Processing
Neural Networks
Planning & Scheduling
Practical Applications
Reasoning Under Uncertainty
Robotics
Sensor Fusion
Spatial & Temporal Reasoning
Speech Recognition
System Integration
Tools
Verification & Validation of KBS

Authors are invited to submit five copies of paperss, written in English, of up to 10 single-spaced pages, presenting the results of original research or innovative practical applications relevant to one or more of the listed areas of interest. Practical experiences with state-of-art AI methodologies are also acceptable when they reflect lessons of unique value to the conference attendees. Shorter works, up to 6 pagess, to be presented in 10 minutes, may be submitted as SHORT PAPERS representing work in progress or suggesting possible research directions. (Please indicate "short papers" in the submission letter in this case.) Submissions should be received by the Program Co-Chair Takushi Tanaka of the IEA/AIE-96 conference office by November 8, 1995. All papers should include a key word list. Notification of the review process will be made by January 22, 1996, and final copies of papers will be due for inclusion in the conference proceedings by February 20, 1996. References will be asked to nominate papers for a Best Paper Prize to be announced at the conference. All papers, but particularly those nominated for the Best Paper competition, will be automatically considered for a place in the Journal of Applied Intelligence.

Prof. Moonis Ali
General Chair of IEA/AIE-96
Department of Computer Science
Southwest Texas State University
San Marcos TX 78666-4616 USA
Phone: (+1) 512-245-3409
Fax: (+1) 512-245-8750
Email: ma04@swt.edu

Prof. Takushi Tanaka
IEA/AIE-96 Conference Office
Fukuoka Institute of Technology
Wajiro-Higashi, Higashi-ku
Fukuoka 811-02, Japan
Phone: (+81) 92-606-3131
Fax: (+81) 92-606-1342
Email: tanaka@fit.ac.jp

World Wide Web – http://www.fit.ac.jp/ieaaie.html
The procedings will be published and will be avaliable at the conference. Copies of the procedings of earlier conferences are avaliable from: *Gordon and Breach Science Publishers, Customer Service*, P.O. Box 786, Cooper Station, New York, NY 10276; Fax: (+!) 212-645-2459