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The limit of using probability of a fuzzy event in a fuzzy decision problem

by

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The object of our study is construction of a decision rule in a fuzzy environment. From this view point, fuzzy-Bayes decision rules have been proposed to cope with a fuzzy state of nature named fuzzy event. These decision rules are based on the probability of fuzzy events defined by Zaheh. Because this decision rule is based on the probability of a fuzzy event, there one supposition i.e. orthogonal condition with regard to the membership function of a fuzzy event. In this paper we show the limit of the introduction the probability of fuzzy events to a decision rule on fuzzy events. On the other hand, the framework from the domain of possibility theory has been constructed. Furthermore, a probability-possibility transformation has been studied. Without the constraint on membership functions of fuzzy events, first, we propose to extend the above decision rule. Second, in the decision problem that is not tackled by the extended decision rule, we propose that we had better use the decision rule based on the possibility measure of fuzzy events rather than the probability of fuzzy events.

1. Introduction

In a fuzzy environment, a decision rule on a fuzzy state of nature named fuzzy event, on the basis of the probability of fuzzy events, Zadeh (1968), has been proposed, Okuda, Tanaka, Asai (1978). Because this decision rule is based on the probability of a fuzzy event, there is supposition on membership functions of fuzzy events that the membership functions of fuzzy event are orthogonal (the sum of membership functions is one). The main object of our study is to extend this decision rule without the orthogonality condition by introducing the concept of indifferent events and the reserved judgement to the above decision rule. When the sum of membership functions is less or equal one, this extension is very handily applied. But in the other case, this extension cannot be applied.

This is the limit of the application of probability of a fuzzy event to a fuzzy decision problem. On the other hand, the possibility measure of a fuzzy event was defined by Zadeh (1977). And the framework of the possibility distributions was develompment in Dubois, Prade (1988); Tanaka, Ishibuchi (1992a;b). Furhermore, the decision rule based on the possibility measure of a fuzzy event has been proposed in Uemura, Sakawa (1993); Uemura (1993a;b), Uemura (1994). In the decision problem that is not approached with the extended decision rule, we propose that we had better use the decision rule based on the possibility measure of fuzzy events rather than the probability of fuzzy events.

2. A decision rule based on the probability of a fuzzy event

2.1. A decision rule

Okuda, Tanaka and Asai (1978) considered a decision rule based on a fuzzy state of nature named fuzzy event. Now, let us express the Bayes decision rule as $\langle S, A, \pi \rangle$. S is a state of nature, $A(A_1, \ldots, A_m)$ is a set of actions, π is the prior distribution on S. On the other hand, we express a decision rule for a fuzzy event as $\langle F, A, L, \pi \rangle$. $F(F_1, \ldots, F_n)$ is a set of fuzzy events on a state of nature. And $L(A_i, F_k)$ is a fuzzy loss between a fuzzy event and an action. Here, the setting rule for $L(A_i, F_k)$ has been constructed on the basis of the concept of the representation value of the fuzzy loss function, Uemura (1993a;b).

The probability of a fuzzy event $P(F_k)$ is defined, Zadeh (1968), as follows:

$$P(F_K) = \int \mu_{F_k}(s)\pi(s)ds \tag{1}$$

Here $\mu_{F_k}(s)$ is the membership function of a fuzzy event.

The fuzzy expected loss $E_1(A_i)$ is obtained as:

$$E_1(A_i) = \sum_k L(A_i, F_k)P(F_k) \tag{2}$$

We take the optimal decision A^*, with minimum value of $E_1(A_i)$ for all i. This is the concept of a decision rule on a fuzzy event.

Note that there is one supposition named orthogonal condition with regard to $\mu_{F_k}(s)$ as follows:

$$\sum_k \mu_{F_k}(s) = 1 \text{ for all } s. \tag{3}$$

2.2. The indifferent event and the reserved judgement

We hereunder consider a more fuzzy situation, eliminating the orthogonal condition as follows:

$$\sum_k \mu_{F_k}(s) \leq 1 \text{ for all } s. \tag{4}$$

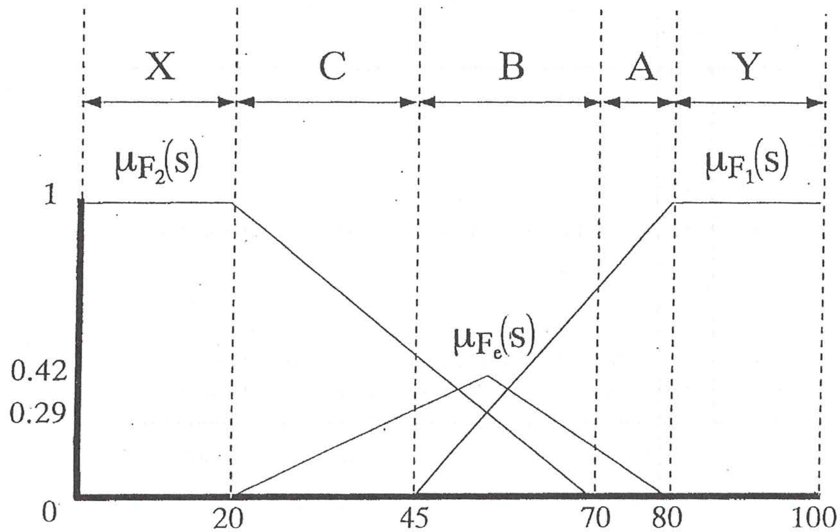

Figure 1. Indifferent event

Now it is natural that we consider the indifferent event for the situation with less information on fuzzy events. Let us exemplify by considering the results of the examination. The state of nature varies in the interval $[0, 100]$. We have two fuzzy events $F_1 =$ *good* and $F_2 =$ *bad*. $\mu_{F_1}(s)$ is expressed as the membership function of F_1, and, $\mu_{F_2}(s)$ is one of F_2. These membership functions are shown in Fig. 1.

In this example, $\sum_k \mu_{F_k}(s) \leq 1$ for every s. In this case the membership function of the indifferent event F_e is calculated as follows:

$$\mu_{F_e}(s) = 1 - \sum_k \mu_{F_k}(s) \tag{5}$$

We have the meaning of the indifferent event F_e for every zone in Fig. 1. In the zones x and y, the value of $\mu_{F_e}(s)$ is zero. This means that the situation can be separated completely into good and bad. Zone A is a conditional indifference zone in which we cannot judge whether or not it is bad though we understand it is not good. Zone B is a indifferent zone in which we can judge that it is neither good nor bad. Zone C is the conditional indifference zone in which we cannot judge it is good though we understand it is not bad.

In this example, $P(F_1) + P(F_2) + P(F_e) = 1$. If $P(F_e)$ is very big, it is dangerous that we make a decision in only a strict set of decisions, because we regard $P(F_e)$ as the index of the decrease of information on fuzzy environment.

Because of the decrease of information on fuzzy environment, a decision-making by sole addition of the indifferent event to a set of fuzzy events is very

		A_1	$\cdots$	A_i	$\cdots$	A_m	A_r
$P(F_1)$	F_1	$L(A_1, F_1)$	$\cdots$	$L(A_i, F_1)$	$\cdots$	$L(A_m, F_1)$	$L(A_r, F_1)$
.	.	.		.		.	.
.	.	.		.		.	.
.	.	.		.		.	.
$P(F_k)$	F_k	$L(A_1, F_k)$	$\cdots$	$L(A_i, F_k)$	$\cdots$	$L(A_m, F_k)$	$L(A_r, F_k)$
.	.	.		.		.	.
.	.	.		.		.	.
.	.	.		.		.	.
$P(F_n)$	F_n	$L(A_1, F_n)$	$\cdots$	$L(A_i, F_n)$	$\cdots$	$L(A_m, F_n)$	$L(A_r, F_n)$
$P(F_e)$	F_e	$L(A_1, F_e)$	$\cdots$	$L(A_i, F_e)$	$\cdots$	$L(A_m, F_e)$	$L(A_r, F_e)$
		$E_1(A_1)$	$\cdots$	$E_1(A_i)$	$\cdots$	$E_1(A_m)$	$E_1(A_r)$

Figure 2. The extended decision table

dangerous. To avoid this danger, we consider that we add the reserved judgement A_r to the set of actions. The fuzzy loss between the reserved judgement and a fuzzy event is set:

$$L(A_r, F_k) = \frac{\max_i L(A_i, F_k) + \min_i L(A_i, F_k)}{2} \tag{6}$$

We obtain the decision table as Fig. 2:

Here, $L(A_i, F_e)$ is set just like $L(A_i, F_k)$, on the basis of the concept of the representation value of the fuzzy loss function, Uemura (1993a;b).

Like in 2.1, we make a decision in the extended set of both fuzzy events and actions, on the basis of the ordering of the fuzzy expected loss.

However, in the other case, i.e. when $\sum_k \mu_{F_k}(s) > 1$, this extended rule is not applied. Using a transformation for membership functions, for example, the formal transformation, the subjectivity of a decision maker with respect to fuzzy events is ignored. Therefore, this decision rule is applied only in the case of $\sum_k \mu_{F_k}(s) \leq 1$. This is the limit of using the probability of a fuzzy event in a fuzzy decision problem.

3. A decision rule based on the possibility measures of fuzzy events

The framework of the possibility theory has been developed in Dubois, Prade (1988); Tanaka Ishibuchi (1992a;b). And the possibility measure of a fuzzy event $M(F_k)$ is defined by Zadeh (1977) as follows:

$$M(F_k) = \max\min(\mu_{F_k}(s), \prod(s)) \tag{7}$$

Here $\prod(s)$ is a possibility distribution of a state of nature. And the identification of $\prod(s)$ from data has been constructed by Tanaka, Ishibuchi (1992a;b), furthermore the transformation of a possibility distribution from a probability distribution has been proposed by Tomas (1992). In view of this, as well as a prior distribution of a state of nature, we can obtain a possibility distribution of a state of nature from data or a probability-possibility transformation.

We obtain a fuzzy expected loss $E_2(A_i)$ as follows:

$$E_2(A_i) = \sum_k L(A_i, F_k) M(F_k) \tag{8}$$

Like in section 1, we take the optimal decision A^*, with minimum value of $E_2(A_i)$ for all i.

In this decision rule, we are not constrained by the orthognal condition of the membership function of fuzzy events.

Therefore, we consider that we had better the possibility measure of a fuzzy event rather than the probability of a fuzzy event. However, if $\sum_k \mu_{F_k}(s) \leq 1$, we had better use the extended decision rule based on the probability of a fuzzy event of section 2, because we do not need to transfer a possibility distribution of a state of nature from a prior distribution. If $\sum_k \mu_{F_k}(s) \geq 1$, we cannot use the extended decision rule based on the probability of a fuzzy event of section 2. Therefore, we must use a decision rule based on the possibility measure of a fuzzy event in this case.

4. Conclusion

In this paper, we showed the limit of using the probability of a fuzzy event in a fuzzy decision problem. And we released the orthogonal condition on the membership functions by introducing indifferent events to fuzzy events and the reserved judgement. Furthermore, we could obtain the meaning of the indifferent events in every zone of the support. But the method could not pass this limit. Furthermore, if $\sum_k \mu_{F_k}(s) \leq 1$, we had better use an extended decision rule based on the probability of a fuzzy event, because we can save labour of transformation of a possibility distribution of a state of nature from a prior distribution. Otherwise, we must use a decision rule based on the possibility measure of a fuzzy event.

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