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Survival game: n-person bargaining for controlling coalition[1]

by

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A dynamic process for determining the coalition formation and payoff distribution for sidepayment games is presented. The novelty of the approach is in a particular process of bids and counter-bids which induces the crucial players to join the survival coalition. Besides, an algorithm for calculating solutions and an application to the forecasting of the new government coalition are given once the first electoral results are known.

Keywords: bargaining games, dynamic games, government coalition, political forecast, shareholding.

1. Introduction

Coalitions of control are formed in various types of real situations: from political ones (for example, in government) and economic ones (for example, the control of shares) to the most complex international relations. The mechanism governing formation of the controlling coalition cannot easily be explained even by those involved in it. In fact it takes place through complex dealings based not only on power relations, but also on human relations, likes and dislikes, external influences, nuances, skills and psyhological factors. It is, however, worth trying to find a description which "captures" the most characteristic aspects of this process, so as to provide a reference point for individual players and onlookers.

In the field of political science and game theory many descriptions of bargaining between n persons are already known. The principal approaches may

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be divided into three classes: power indices (for further information see Gambarelli, 1983;1991;1993; Gambarelli, Owen, 1994; Owen 1982), bargaining sets, which were introduced by Aumann and Maschler in 1964 (for those details most inherent to this paper see Bennett, 1987; Bennet, Houba, 1992; Kalai, Maschler, Owen, 1975) and combinatory and recursive models (see Bennett, 1987; Kalai, Maschler, Owen, 1975; Myerson, 1977; Rapoport, Kahan, 1974; Stearns 1968). Mention must be made also of the modifications to the indices of Shapley-Shubik (1954) and Banzhaf-Coleman (1965) carried out in (Owen, 1977;1981; Carreras, Owen, 1988). These modifications have the advantage of being more suitable for real situations, allowing, for example, the formation of winning sub-coalitions. The first two types of approaches are generally of more a regulating nature rather than a descriptive one. The combinatory and recursive models have the disadvantage of linking the solution to a strict behavioural pattern. They have, however, the advantage of being particularly suitable for computing, and thus, permit an analysis of stability through repeated simulations. This paper may be included in the last category.

2. Description of the model

There are several potential coalitions. Each of them will obtain certain earnings only if the coalition is actually formed, that is, if all its components definitely decide to join, thus abandoning all other potential coalitions. Single players, who on their own would have no earnings, must decide which side to be on. Let us suppose that initially a certain agreement is being negotiated for the formation of a particular coalition. The members excluded from this coalition then make a bid to the players of the coalition being formed so as to induce them to abadon it in their favour. The sequence of bids and counter-bids could theoretically go on for ever, but actually at a certain point it does stop, since (even though the time variable does not affect the model) it is in the interest of each player to find solution and to be paid on this basis. Such a solution must be as stable as possible, that is, unassailable by the player excluded. This solution must minimize the greatest regret of each member of the coalition; indeed it is sufficient that a single member leaves for the coalition to break down. Thus, at each stage of the negotiation, each player is putting forward his demands to every potential coalition as follows. These demands should not displease the other potential allies too much. In any case the distribution he proposes should not displease any one member more than the others (in other words, it has to be acceptable by all members to the same degree).

If the dynamic model of negotiation converges, then each of the winning players knows exactly what is the best bid to make to the winning coalition and the game ends with the actual payoff distribution.

It must be pointed out that the model may as well not converge. This fact may seem negative, but it is actually correct. Indeed it sometimes happens in real situations that a solution cannot be reached (for example, in politics, when it is necessary to resort to elections held earlier than originally planned). A model which provided a solution for any type of game would therefore not give a reliable description of reality.

3. The static game

3.1. Notations

Let $N = \{1, \dots, n\}$ be the set of players, and v be a non-negative real function ("potential value") defined on subsets of N ("potential coalitions") which is null for all subsets consisting of less than two elements. The set of parts of N each having more than one element can be arranged lexicographically from $(1, 2)$ to N, and can then be identified by the elements of the ordered set $\mathcal{G} = \{1, \dots, m\}$ where $m = 2^n - n - 1$. For all $s \in \mathcal{G}$, we call v_s the potential value of the s-th coalition.

We now define the following game in normal form. For all i from 1 to n, for all s and $r \in \mathcal{G}$, let $x_i = \{x_{i1}, \dots, x_{im}\}$ be a real vector satisfying:

$$\begin{cases} 0 < x_{is} \leq v_s & \text{and} \\ x_{ir} = 0 & \text{if the } i\text{-th player does not belong to the } r\text{-th coalition.} \end{cases}$$

Such a vector is the strategy of the i-th player, and can be interpreted as the payment demanded by its player from the players of s-th coalition, in order to take part in the game. The payments p_i are so defined for all $i \in N$:

$$p_i = \begin{cases} x_{is} & \text{if } x_{js} > x_{jr} \text{ and } \sum_i x_{is} = v_s \\ & \text{for all } j \text{belonging to the } s\text{-th coalition} \\ & \text{and for all } r \in \mathcal{G} \text{ different than } s; \\ 0 & \text{elsewhere} \end{cases}$$

This is justified by the following. A player receives the payment requested from the s-th coalition only if this coalition "survives", that is, if its members do actually unite (which occurs if all the members of the s-th coalition are paid more highly by the latter than by all the others). All the other players have no earnings.

3.2. Properties of the solutions

We can note that all the strategies of the members of the surviving coalitions are Pareto-optimal and are Nash equilibria satisfying individual and group rationality (in fact, for all $p_i > 0$, it is $x_{is} > 0$, and $x_{js} > x_{jr}$ for all $r \in \mathcal{G}$). We call them "survival startegies" and we call X their set. There can be several surviving coalitions in any one game, but in that case they are disjoint.

4. An example

Let $\mathcal{G}$ be a group of coalitions such that the intersection of any two of them is non-empty. Let v be positive for all the coalitions in $\mathcal{G}$, null for all the others and let there be a single s with a maximum potential value v_s. Then the s-th coalition is the only surviving one and solutions are all vectors $x \in X$ so that $x_{js} > v_r$ for all j belonging to the s-th and r-th coalitions and for all $r \in \mathcal{G}$ different that s.

5. The dynamic game

5.1. General description

In this section we present a dynamic model which provides a solution (where a solution actually exists) for the games lacking in static solution. The process begins by assigning a payoff distribution to each coalition. Given these payoffs, for each coalition a threat point is calculated by using, as each player's component, the maximum payoff the player can obtain, given the initial distribution. For a given solution this threat point may or may not be a feasible payoff distribution. The new payoff distribution in the coalition is found by increasing (wherever feasible, otherwise by decreasing) each player's payoff from the threat point by equal amounts to "reach the boundary". A rearranging process is carried out if the decrease goes against individual rationality. Given this new set of payoff distributions, the process is repeated. A solution is a fixed point of this bargaining process.

5.2. Notations

For each stage t, we call $d^t(S)$ the payment demanded by the i-th player from coalition S; $b_i^t(S)$ the "best last bid" which S could propose to i according to the survival scheme (where $b_i^t(S) = d_i^t(S) = 0$ if $i \notin S$). We call D^t the vector having as columns (in lexicographic order of cardinality and components) the attributions $d^t(S)$. We define the winning payment demanded in stage t as $d^t(S)$ (and its respective coalition S) so that, for every $i \in S$ and $R \in \mathcal{G}$ with $R \neq S$:

$$d_i^t(S) > d_i^t(R)$$

5.3. Detailed description

5.3.1. Start

Having established at the beginning that $t = 0$, we start with a generic allocation D^*, satisfying group rationality ($\sum d^t(S) = v(S)$ for all S) and individual rationality ($d_i^t(S) \geq v(i)$ for all $i \in S$).

5.3.2. Determination of the best last bid

In the following stage every coalition, in order to assure its own members, has to offer each one the maximum among the previous offers of the other coalitions. Such a bid is then:

$$b_i^t(S) = \max_{R \neq S} d_i^t(R)$$

The earnings of every coalition S in stage t are then

$$e^t(S) = v(S) - \sum_{j \in S} b_j^t(S)$$

5.3.3. Determination of Payments Demanded

The following elaborations are carried out for each coalition.

5.3.4. Winning Demands

If the winner can satisfy all demands, if i.e., $e^t(S) \geq 0$, then the "winning payment demanded" is determinded by sharing the earnings equally among the members of S, in order to minimize the maximum complaint and the consequent risk of losing a component. Then we obtain:

$$d_i^{t+1}(S) = b_i^t(S) + \frac{e^t(S)}{|S|}$$

where $|S|$ is the number of members of S.

5.3.5. Losing Demands

If $e^t(S) < 0$ then the boundary is reached using a formula approaching the above. In fact, the latter expression must be modified when some $d^{t+1}(S)$ are negative, thus violating individual rationality. Here is the formula for losing demands:

$$d_i^{t+1}(S) = \max\left(0, b_i^t(S) + \frac{e^t(S)}{|S|} + \bar{k}\right)$$

where $\bar{k}$ is the maximum among all k so that

$$\sum_{j \in S} \max\left(0, b_i^t(S) + \frac{e^t(S)}{|S|} + \bar{k}\right) = v(S)$$

The existence and uniqueness of $\bar{k}$ emerges from the strict monotonicity (and consequent existence and monotonicity of the inverse) of the above function of k, in the interval

$$\left[\min_{j\in S}\left(-b_i^t(S)-\frac{e^t(S)}{|S|}\right),\max_{j\in S}\left(-b_i^t(S)-\frac{e^t(S)}{|S|}\right)\right]$$

A bisection method suffices for the computation of the solution.

5.3.6. Go back

having carried out the calculation of bids in stage $t+1$ for all coalitions, we return to the determination of offers until conditions of interruption are met.

5.3.7. Conditions of interruption

The interruption is carried out when a fixed point x_{is} in the mapping $D^t \rightarrow D^{t+1}$ is achieved, or when oscillating behaviour or conditions of non-convergence satisfying a suitable metric are identified. if, for instance, we set

$$\mathrm{dist}(D^t, D^u) = \max_{i,S} |d_i^t(S) - d_i^u(S)| \text{ (or a standard norm } l_1 \text{ or } l_\infty)$$

a good indicator may be the behaviour of

$$\mathrm{dist}(D^t, D^{t+1}) \text{ for } t \rightarrow +\infty :$$

5.3.8. General flow

The flow of the algorithm can be summarized as follows:

(begin)

- generation of D^*
- repeat until interruption conditions:
  - computation of the bid
  - for all $S \in \mathcal{G}$:
    - computation of payments demanded:
      winning (for feasible bid) or
      losing (for non-feasible bid)
  - end for
- end repeat

(end)

5.3.9. Technical details

So as to avoid cycling in the computation of $b_i(S)$, it is necessary to add to the maximum among the previous suppliers of the other members, a positive incentive ϵ : $b_i^t(S) = \max d_i^t(R) + \epsilon$.

As already stated in section 3.2, the eventual subadditive coalitions are not surviving ones. It is easy to verify that they do not affect the development of the algorithm after the second step. Therefore such coalitions may be eliminated at the first stage.

5.4. An example

Let $v(1,3) > v(1,2) > 0$ and $v(S)$ be equal 0 for all other coalitions. We have:

$$D^0 = \begin{array}{ccc} & (1,2) & (1,3) \\ 1 & x & y \\ 2 & v(1,2)-x & - \\ 3 & - & v(1,3)-y \end{array}$$

$(0 \leq x \leq v(1,2);\ 0 \leq y \leq v(1,3))$. It is easy to verify that matrix D^t converges to

$$\begin{bmatrix} v(1,2) & v(1,2)+\frac{v(1,3)-v(1,2)}{2} \\ 0 & - \\ - & \frac{v(1,3)-v(1,2)}{2} \end{bmatrix}$$

independly of the starting conditions, with winning coalition $(1,3)$ and share

$$\begin{array}{ll} x_{1(1,3)} = v(1,2)+\frac{v(1,3)-v(1,2)}{2} & \text{for player 1} \\ x_{3(1,3)} = \frac{v(1,3)-v(1,2)}{2} & \text{for player 3} \end{array}$$

6. Applications

6.1. Determination of government coalition

Let us consider the damage occuring to each party by participating in every coalition. A party has a stronger bias for forming coalitions with politically close parties, with respect to a suitable distance (see Bennett, 1987; Mercik, Kołodziejczyk, 1986). Otherwise it requires the highest compensation possible to balance the bigger problems that entry into a difficult coalition would imply: for instance discontinuation of aid from allies, deviation from the electorate indications and consequently a loss of votes in subsequent elections, risk of a "putsch", numerically weak majority, an excessive shift to right or left in leading the country, etc. We must also allow for the benefits deriving from coalition (prestige, satisfying political policy, etc.).

The "global damage" of each coalition is the sum of the percentage dameges of the single members; earnings $v(S)$ of every coalition S are then the complement of the global damage to 100%. Of course, each "damage for coalition" suffered will be added, for each member, to the final distribution.

A coalition will be rejected:

(a) if gain is negative

(b) if it is not a majority.

Let us point out as regards this subject, that a coalition can be a majority nominally, but a minority in practice, when some of the members of a party boycott it (a case of "franc tireurs"). Then, an index of interior cohesion for each party is established beforehand, with respect to all possible coalitions. Such an index will serve to correct the party's number of votes, and then, globally, the coalition's number.

6.2. Case Discussion

In past Italian elections, Table 1 was prepared from interviews with members of Parliament and political scientists. An aid in setting out data comes from Bennett's (1987) method. For data synthesis, medians have been used. One difficulty in quantifying damage has been overcome by considering that only data necessary for the elaboration were the fifteen numbers of the last column, i.e. the characteristic function. As a simplifying point of reference it was requested that totals be integers from 1 to 100. Then, the eleboration was made for different vectors in a neighborhood of the input vector, in order to estimate the stability of the solution, referring to variations of input evaluations.

The result was that final sharing depended on the starting bid D^0; moreover, for all D^0, the five-party (pentaparty) coalition (DC, PSI, PLI, PSDI, PRI) won. Table 2 shows the results of the elaboration based on the input of Table 1, where D^0 corresponds to an equal sharing of the bid. It is verified that the relative deviations in column II of Table 2 were limited, for all D^0, within the band of 2%.

As regards stability (i.e. when the model was applied in a neighborhood of the data of Table 1), the following results were obtained. Pentaparty losses in favor of the three-party coalition (DC-PSI-PRI) if, other gains remaining unvaried, the gain of (DC, PSI, PRI) was estimated at 74. For a pentaparty loss in favor of the three-party coalition (DC-PSI-PSDI), an increase in the gain of the latter from 55 to 63 is needed, other gains of Table 1 remaining unvaried.

An interesting result concerns the role of the Communists in the historical compromise (DC-PCI) and in the left alternative (PCI-PSI-PRI-PSDI-PR-ECOLOGISTS). First, as we can observe in Table 1 the former coalition wins 3/2 over the latter one. Such an evaluation can be explained by the role of the PSI in the left alternative, compared to the role of the DC in the historic compromise. The DC is in a better position, because it has to share its gain only with the Communist Party (PCI), while the PSI has to satisfy many key-parties.

Then the model suggests that, against the PSI's threat of forming the left alternative, the DC's defence is the counterthreat of a historical compromise. We have proof if we revise the program with the same data, but lower the gain of the historic compromise from 22 to 15: in such a case pentaparty remains the winner, but with greater gains for the PSI. In any case, both coalitions remain improbable, though, as we can see, their hypothetic possibility influences the power-sharing process.

Column IV of Table 2 has been obtained from an evaluation of the present distribution of power in the pentaparty, in order to compare the results of the model with an estimate of the real situation. The weighting factors, which led to such estimation, are quoted in Table 3. Of course, any breakdown in the government in favor of other coalitions does not necessarily bring about the three-party coalitions mentioned above. This may occur since the breakdown in a government if often caused by substantial changes in likes and dislikes, and thus in input data (Table 1).

In particular, the government composition described here, is the one following past Italian elections. That government subsequntly fell because of the Christian Democrat "*franc-tireurs*". A reorganization of the government followed with the same distribution of ministries and under-secretaries, but with a total increase of five under-secretaries. Of these, one more of the Premiership was given to the DC and of the other four, two more were given to the DC (preasumably to satisfy the *franc-tireurs*), and one more each to the PSI and the PRI. In practice, however, the situation has hardly changed at all: in Tables 2 and 3 the actual power has become 52.2, 29.9, 8.4, 4.8, 4.8 and the maximum deviation (col. IV of Table 2) has become +3.5% for the DC, but it is counterbalanced by greater "coalition damage" due to the *franc-tireurs*.

7. Users

A user of this model could generally be a decision-maker in change of the formation of the controlling coalition or simply interested in a forecast to use as a starting point for future decisions.

Besides political applications, the model can describe economic problems: subcontracts, parallel bargaining and share-control (see e.g. Gambarelli, 1982; 1990; Gambarelli, Hołubiec, 1989; Gambarelli, Hołubiec, Kacprzyk, 1988; Gambarelli, Owen, 1994; Gambarelli, Szegö, 1982).

As regards political applications, this model could be used by a party in order to have a reference point for the "minimal" level of requests in terms of power percentage: are they too low? and how far is it convenient to go, before breaking up the coalition? Other uses could concern regional and local committees; in such a context determination of evaluational parameters is certainly easier. More generally, the model could be employed by a nation interested in a forecast of the political configuration of another nation, in real time, as soon as the first electoral projections are notified. It should be noted that all data (except the number of seats) can be set before: i.e. damage and indices of internal cohesion of each party referring to every possible coalition. In any case, a last minute retouch of data will be necessary, for potential electoral swings that would considerably change the estimation of damage.

8. Reliability

When usnig mathematical models, it is clear that the user should be quite aware that the validity of the results is a function of the truth of the hypotheses and rules of inference on which the model is based. In particular, with this model the following must be kept in mind.

8.1. As regards input

It is necessary to consider the extreme difficulty which occurs in evaluating situations which, by their very nature, are vague and sometimes deliberately confused. Especially in politics, where the predominant strategy is often that of "non-clarity", writing down numbers and placing any weight on them seems difficult. Such a task may be promoted by various dominating techniques, in particular, by starting with relations of order and refining calculations in subsequent checks. It is also essential to get several different opinions from people whose political beliefs are different. A good data synthesis also seems to require the use of "medians" in order to avoid the influence of any exaggerated opinions. More trustworthy calculations are possible for local or municipal bodies as well as economic applications. Then it must be kept in mind that only data which concern the model are "total damages" of potential superadditive coalitions, which are usually few. It is threfore possible to test the validity of the solution by applying the algorithnm several times with different variations of such values.

8.2. As regard the model

A doubt could arise about whether the results supplied by the model actually coincide with the calculation of previously existing damage, that is, that the model does not bring to light new facts, as output basically expresses the evaluations of input. An answer is that the algorithm points out the influence of the players which could belong to different coalitions. Such influences are not always obvious, particularly in the case of complex models. The algorithm brings, in any case, definite advantages in terms of simulation, as it allows for testing of the stability of a coalition regarding the various estimates of damage, and for singling out coalitions which should be monitored when variations in the damage to a party were brought to a breaking point. This allows a study of the strategy of "approach" and "threat" that each party could adopt towards its allies and non-allies, in order to increase its own power, without risking the approach of a breaking point.

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Damage from coalitions

Coalitions	DC	PCI	PSI	PRI	PSDI	PLI	PR	ECOLG	Totals	Winings
DC, PSI, PRI, PSDI, PLI (=pentapar.)	7	-	11	2	2	1	-	-	23	77
DC, PSI, PRI, PLI	8	-	12	6	-	3	-	-	29	71
DC, PSI, PRI	13	-	15	6	-	-	-	-	34	66
DC, PSI, PSDI, PLI	13	-	15	-	4	3	-	-	35	65
DC, PSI, PSDI	18	-	20	-	7	-	-	-	45	55
DC, PSI, PLI, ECOLG	20	-	20	-	-	7	-	3	50	50
DC, PSI, PLI	22	-	24	-	-	6	-	-	52	48
DC, PCI, PSI	25	5	27	-	-	-	-	-	57	43
DC, PSI, PR	33	-	32	-	-	-	7	-	72	28
DC, PSI, ECOLG	33	-	32	-	-	-	-	7	72	28
DC, PCI	70	8	-	-	-	-	-	-	78	22
DC, PSI (=histo. comp.)	36	-	37	-	-	-	-	-	73	27
PCI, PSI, PRI, PSDI, PR, ECOLG (=left alt.)	-	5	30	25	15	-	6	4	85	15
PCI, PSI, PRI, PSDI, PR	-	6	32	25	17	-	7	-	87	13
PCI, PSI, PSDI, ECOLG	-	5	30	30	29	-	-	6	91	9

Table 1. Input data. The only numbers of interests for the model are those of the last column.

	I Coalition Damage (according to tab. 1)	II Earnings (according to the algorithm)	III Foreseen Power (=col. I+II)	IV Actual Power (according to tab. 3)	V Deviations (=col. IV-III)
DC	7	41.7	48.7	51.8	+3.1
PSI	11	19.6	30.6	30.2	-0.4
PRI	2	8.3	10.3	8.2	-2.1
PSDI	2	3.1	5.1	4.9	-0.2
PLI	1	4.4	5.4	4.9	-0.5
Totals	23	77.0	100.0	100.0	0.0

Table 2. Distribution of power within the pentaparty. Column II shows the output of the model.

Ministry	DC	PSI	PRI	PSDI	PLI	TOTALS
Presidency	11	-	-	-	-	11
Vice-presidency	-	7	-	-	-	7
Foreign	5	-	-	-	-	5
Interior	5	-	-	-	-	5
Treasury	-	5	-	-	-	5
Justice	-	4	-	-	-	4
Finance	4	-	-	-	-	4
Defence	-	-	-	-	4	4
Balance	4	-	-	-	-	4
Post and Telecommunication	-	-	4	-	-	4
Industry	-	-	3	-	-	3
Labour	-	3	-	-	-	3
South	3	-	-	-	-	3
Education	2	-	-	-	-	2
Civil Protection	2	-	-	-	-	2
Environment	-	2	-	-	-	2
Work	-	-	-	2	-	2
Research	-	2	-	-	-	2
State Participation	2	-	-	-	-	2
Transport	2	-	-	-	-	2
Agriculture	2	-	-	-	-	2
Health	2	-	-	-	-	2
Foreign Trade	-	2	-	-	-	2
Tourism	-	2	-	-	-	2
Merchant Navy	1	-	-	-	-	1
Parliamentary Relations	1	-	-	-	-	1
Communitary Politics	-	-	-	1	-	1
Cultural Goods	-	-	-	1	-	1
Reg. Aff. & Ist. Ref.	-	-	1	-	-	1
Public Functions	1	-	-	-	-	1
Special Affairs	1	-	-	-	-	1
Urban Areas	-	1	-	-	-	1
Under Secretariats						
– of council presid.	1	-	-	-	-	1
– other (half weight)	14.5	9	2	2	2	29.5
Points of Power	63.5	37	10	6	6	122.5
Power Percentage	51.8	30.2	8.2	4.9	4.9	100

Table 3. Computation of the present sharing out of power