

**A methodology for optimizing managerial exploitation decisions in an industrial enterprise on the example of a power plant**

by

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**Summary.** The operational management and control process in a power plant distinguishes itself among other industrial processes by its complexity.

In order to optimize the managerial decisions in a power plant we generate a set of cooperating decision models, to be implemented in the form of software packages. Optimization becomes a "man-machine" (here: decision maker - computer) process. The paper presents synthetically the elements of more important decision models involved.

The set of relevant decisions is classified into decisions of local (optimized inside particular models) and global nature (optimized on the level of the whole system pertaining to the power plant, referred to nominally as the CEO level). The elements of a combined method of global optimization, based upon random search and experiment planning precepts, are presented.

The elements of the fuel selection problem, as an example of an important decision subprocess, are shown. The paper implies the feasibility and purposefulness of comprehensive modelling of the essential components of a complex industrial process, with the objective of optimizing all the relevant decisions.

## **1. Introduction**

Generation of electric and thermal energy in a power plant is an industrial process characterized by exceptional complexity.

A power plant is an element of the broader set of hierarchical systems, primarily of the energy system of a country. In itself, a power plant and its running

can also be represented as a hierarchical system. A number of levels can be distinguished within this system, such as the levels of units, general purpose subsystems, main facilities, installations, devices, subassemblies, and parts, as well as the functional subprocesses. A simplified scheme of structure of the process considered is shown in Fig.1.

The set of typical more important decision problems relevant for power plant running contains some 60 problems, decomposable into approximately 200 elementary typical optimized decision variables (decisions to be taken).

A *decision problem*  $P$ , referred to here, is understood as a *subset of mutually connected decision variables* ( $P \equiv \{Z\}$ ), usually jointly optimized in a single decision process. The actual decision set is further multiplied by appropriate assignment to particular units, devices and installations. Managerial decisions are often strongly mutually related.

## 2. Methodology of modelling and optimization of the power plant exploitation process

As mentioned at the beginning, the process of exploitation of a power plant is an element of a hierarchical system, here composed of two levels: the management of the electric power system (National Power Dispatching centre) - the power plant. Optimization in the two-level system (see Findeisen, 1974) is performed with iterative methods making use of coordination and decomposition procedures. For this purpose, within the entire set of decision problems relative to exploitation (running),  $\mathbf{P}$ ,  $\mathbf{P} = \{P\} = \{\{Z\}\}$ , a subset  $\mathbf{P}^S$ ,  $\mathbf{P}^S \subset \mathbf{P}$ , is distinguished of decisions optimized within on the upper level (the National Power Dispatching). Optimization of these decisions is not the subject of the present paper. The remaining decision problems form the set  $\mathbf{P}^V$ , optimized on the level of the power plant ( $\mathbf{P} = \mathbf{P}^S \cup \mathbf{P}^V$ ). Within the set  $\mathbf{P}^V$  a subset is yet distinguished,  $\mathbf{P}^W$ , of problems optimized within the individual decision models (local optimisation) and the subset  $\mathbf{P}^N$  of problems optimized on the superior level of the process (global optimization),  $\mathbf{P}^V = \mathbf{P}^W \cup \mathbf{P}^N$ . Complexity of the overall process makes it impossible to encompass with just one decision model all the exploitation decisions. A set of cooperating computerized decision models is generated. Optimization becomes a "man-machine" ("decision maker - computer") process, in which the superior level ("decision maker") carries out the process of global optimization, controlling the trajectory of computations passing through particular decision models ("computer").

The elements mentioned above form together the notion of the managerial decision system. It is represented by the following ordered 5-tuple:

$$S = (\mathbf{P}, \mathbf{K}, \mathbf{A}, \mathbf{M}, \mathbf{D}) \quad (1)$$

where:

$S$  power plant management and control decision system,

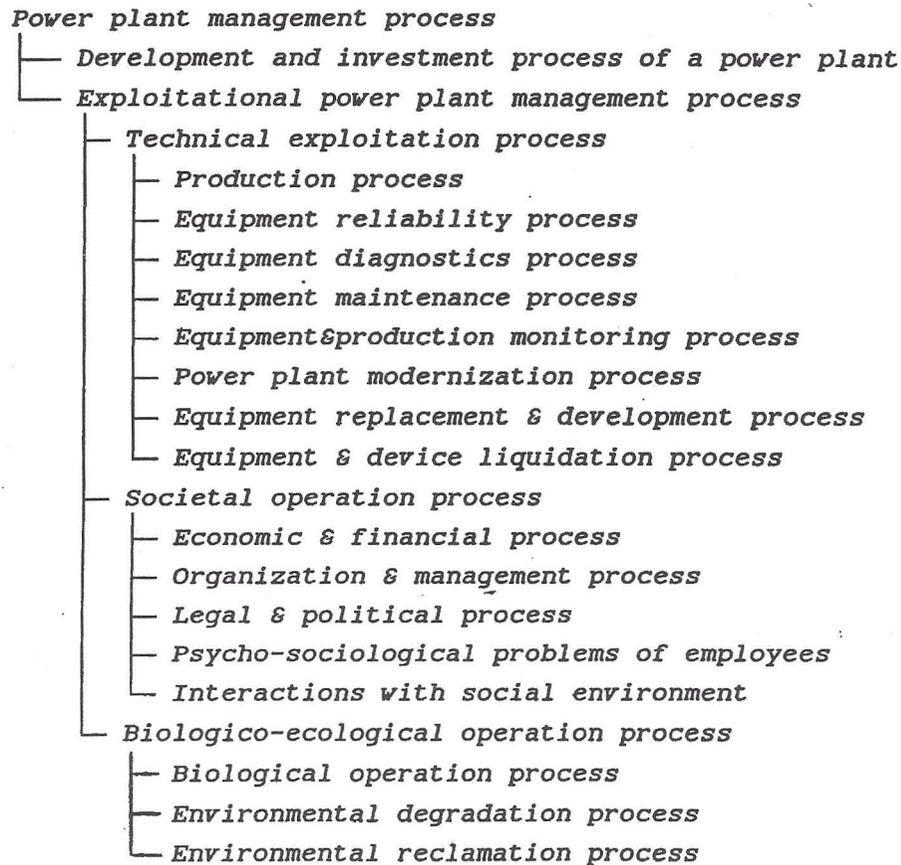


Figure 1. Simplified view of the hierarchical structure of the power plant operation processes.

- P** set of power plant decision problems,  
**K** objective function of the global optimization process,  
**A** set of methods of global optimization,  
**M** set of computerized decision models,  
**D** set of decision makers (members of management staff).

Then, the  $m$ -th decision model,  $M_m^d$ , is an ordered 6-tuple:

$$M_m^d = (D_m^d, \mathbf{P}_m^d, \mathbf{V}_m, \mathbf{R}_m, \mathbf{O}_m, \mathbf{A}_m), \quad m = 1, \dots, M, \quad M_m^d \in \mathbf{M}, \quad (2)$$

where:

- $m$  index of computerized decision models,  
 $M$  number of computerized decision models,  
 $M_m^d$   $m$ -th decision model of power plant operation,  
 $D_m^d$  domain of  $m$ -th decision model,  
 $\mathbf{P}_m^d$  set of decision problems of the  $m$ -th model,  
 $\mathbf{V}_m$  set of parameters (not subject to decision) of the  $m$ -th decision model,  
 $\mathbf{R}_m$  set of relations of the model,  
 $\mathbf{O}_m$  objective function (or a set of objective functions) of local optimization for the model,  
 $\mathbf{A}_m$  set of methods of local optimization for the model.

Let us note at this point that the notation here introduced and referred to thereafter uses the superscripts as the parts of variable or parameter names, with the lower superscripts playing the role of proper indices.

Previous studies conducted by the present author, Brzozowski (1995), have led to determination of the following structure of the set of computerized decision models of particular processes, this structure fulfilling definite additional conditions, given in the reference quoted:

- $M_1^d$  simulation model of the economic-financial system of the power plant,  
 $M_2^d$  decision model of operation of power generation units,  
 $M_3^d$  simulation model of reliability of equipment and maintenance planning of power plant units,  
 $M_4^d$  decision model of unit overhaul execution,  
 $M_5^d$  to  $M_8^d$  as  $M_2^d$  to  $M_4^d$ , but for the general purpose systems and the technical infrastructure of the power plant,  
 $M_9^d$  decision model of organization and management system of the power plant,  
 $M_{10}^d$  decision model accounting for the psycho-sociological problems of employees.

Thus,  $M = 10$ .

Theoretically speaking, the operation process should be optimized, as a dynamic process, in a continuous (or quasi-continuous) manner. In practice, however, in view of the requirements of the management process, several independent decision processes are distinguished, implemented for selected, relatively independent subsets of decision variables. These processes would have a discrete, repetitive and multistage nature, with a definite temporal cycle of decision making,  $T^y$ , and a definite time horizon of accounting for consequences of

these decisions,  $T^h$ . The space of decisions for a given distinct  $k$ -th global optimization process, after the initial procedures of problem and decision variable selection and filtration have been carried out, is then defined by the set

$$\{Z_{pze}^n\}, \text{ with } p = 1, \dots, \Xi_k, z = 1, \dots, Z_p, \quad (3)$$

$$e = 1, \dots, E_k, Z_k = E_k \sum_{p=1}^{\Xi_k} Z_p$$

where:

$k$  index of decision process,  $k = 1, \dots, K$ ,

$K$  number of distinguished decision processes,

$p$  index of decision problem,

$\Xi_k$  number of decision problems in  $k$ -th decision process,

$z$  index of decision variable,

$Z_p$  number of decision variables of  $p$ -th problem,

$e$  index of stage of decision process,

$E_k$  number of stages of  $k$ -th decision process,

$Z_{pze}^n$   $z$ -th decision variable of  $p$ -th decision problem, globally optimized in  $e$ -th stage of  $k$ -th decision process,

$Z_k$  joint number of decision variables in the  $k$ -th decision process.

The set of decision variables  $\{Z_{pze}^n\}$ , described above, is equivalent to the sum of the selected – according to a definite decomposition criterion – decision problems ( $\{Z_{pze}^n\} = \{P_p\}$ ,  $p = 1, \dots, \Xi_k$ ,  $P_p \in \mathbf{P}^V$ ,  $P_p \in \mathbf{P}^n$ , since, conform to the adopted categorisation, a decision problem is equivalent to a set of decision variables). The variables constitute, respectively, the elements of the selected  $M_k$  computerized decision models ( $\{Z_{pze}^n\} \subset \{\mathbf{P}_m^d\}$ ,  $m = 1, \dots, M_k$ ), which have to be made use of in the optimization calculations of a given  $k$ -th decision process. The objective function of each  $k$ -th process,  $k = 1, \dots, K$ , is taken as the maximum of the expected value of the benefit function – i.e. the average discounted profit to be divided within the power plant, calculated over the period of functioning:

$$\max_{\{Z_{pze}^n\}} \bar{F} \quad (4)$$

subject to constraints:  $\Theta_r = 0$ ;  $\{T\} \gg \{W\}$  where:

$$\bar{F} = \frac{\sum_{i=1}^I F_i^f}{I}; F_i^f = \sum_{r=0}^R [F_{ir}^a (1 + \alpha)^{-r}] \quad (5)$$

for  $p = 1, \dots, P_k$ ,  $z = 1, \dots, Z_p$ ,  $e = 1, \dots, E_k$ ,  $i = 1, \dots, I$ , and the meanings of particular notions are as follows:

$\bar{F}$  average discounted profit to be divided within the power plant,

- $\Theta_r$  cumulative financial deficit of the power plant in the  $r$ -th year,  
 $\{T\} \gg \{W\}$  relation of fulfilment of the requirements concerning security (especially in nuclear plants), environment and technical operation by the technological parameters,  
 $i$  index of a random run (repetition) of the optimization calculations,  
 $I$  total number of the above mentioned runs,  
 $F_i^f$  discounted profit calculated during one ( $i$ -th) calculation run,  
 $F_{ir}^a$  annual profit for the  $r$ -th consecutive year, calculated in the  $i$ -th run; this quantity is determined for the  $k$ -th decision process, and therefore is calculated with  $M_k$  decision models; it belongs to the economico-financial system of the power plant and is defined within the model  $M_1^d$ ,  
 $\alpha$  discount rate,  
 $r$  year index,  
 $R$  number of years of power plant exploitation taken in the time horizon  $T^h$  of optimization of the  $k$ -th decision process (relative to accounting for the consequences of a decision).

The possibility of adopting the above single economic objective function results from the repetitiveness of the operational decisions, and from the nature of the process of power plant exploitation. This process is a well structured one in the sense proposed by Simon (1977), stochastic (i.e. described with the identifiable probability distribution), but still rationally conducted, without a very significant element of uncertainty and risk (constancy of distribution characteristics).

### 3. A synthetic view of models for separate decision processes

#### 3.1. Model of the economico-financial system of the power plant

**Symbol:**  $M_1^d$ .

**Model type:** decision-oriented, computerized, probabilistic, analytical (in terms of the form of relations), dynamic, simulation (in terms of solution methods).

**Simulation type:** discrete, with changeable step, event-oriented.

**Simulation language:** original pseudo-language developed by the author, based upon one of the high level computer languages.

**Implementation:** microcomputer package MDE1.

More important inputs/outputs of the model, in the setting proposed in Gutenbaum (1987):

$$M_1^d : \{T^{in}, E^{in}\} \Rightarrow \{\forall(r \in [R', R'']), (F_r^a), T^{out}, E^{out}\} \quad (6)$$

where:

- $T^{in}$  set of aggregated technological input parameters of the power plant: parameters of the unit load schedule, overhaul plan parameters, characteristics of fuel use etc. – interactions with models  $M_{2,3,4,5}^d$ ,
- $E^{in}$  set of economico-financial input parameters, such as prices, interest rates etc.,
- $F_r^a$  as before, profit to be internally divided within the power plant, calculated here for purposes of further use in global optimization, see (5),
- $R', R''$  initial and terminal years of the optimized operation period in a given decision process,
- $T^{out}$  set of technological output parameters of the power plant, such as production, capacity, output power etc.,
- $E^{out}$  set of economico-financial output parameters, including write-offs to particular funds, accounts and funds, credits, debts, as well as calculation of production costs etc.

Locally optimized decision variables of the model: parameters of payment and receipt operations and of credit economy.

Local objective function:

$$\min_{\{O\}} Q_r \quad (7)$$

subject to constraints as in (4),

where:

$Q_r$  sum of extraordinary losses of the power plant in the  $r$ -th year,

$\{O\}$  set of financial operations of payment/receipt relative to the current account of the power plant,

Every criterion of local optimization, relative to particular models, cooperates with the objective function of the global process optimization, formula (4).

Local optimization method: procedure consisting in realization of the disbursement/receipt operations relative to the current account of the power plant in the sequence of priorities assigned to individual partial operations, so as to minimize accruing interest on credit (components of extraordinary losses).

Local optimization is carried out within particular models each time in the consecutive runs of global optimization calculations (see sections 4 and 5).

Decision variables of the globally optimized model: the remaining decision variables of the economico-financial model.

Relations in the model: The model contains a submodel of clearings, costs and sales.

According to the simulative character of the model, relations accounted for represent the events of economic type (resulting from financial balances and the like), of financial type (payments, account and fund changes, credit taking etc.), as well as events of technological kind (repairs, fuel supplies, supplies of spare

parts and equipment etc.), the latter usually entailing the events of the two first kinds.

The model was elaborated in the versions corresponding to various structural solutions in the electric power sector: the currently existing structure, and the hypothetical corporate structure, as well as the free market of wholesale electric power economy.

Identification of the model: the set of parameters of the economic o-financial system (like tax exemptions, inflation rate, etc.) in the second and the later future years of the period considered in power plant optimization requires estimation. Appropriate forecasting methods are used for this purpose.

The analysis of errors in model parameters: the mathematical analysis of the error in the value of objective function in dependence of the errors in parameters directly based upon the algorithm of the model would be very difficult in view of high degree of complexity of the algorithms applied. The present author made use, however, of an originally developed method based upon simulation of the statistical distribution of the errors in parameters.

Analysis of sensitivity of the model parameters (influence of parameters on optimum solutions): initially, the maximum set of input variables of the model was adopted; potential corrections of the model (elimination of less important variables) might not be performed immediately on the basis of the study of sensitivity, but rather on the basis of experiences from a long period (of, for instance, 10 years) of functioning of the system.

The simulation model of the economico-financial system of the power plant was already – separately – described in a number of publications, see, e.g., Brzozowski (1987).

### 3.2. Decision model of power plant productive operation

**Model symbol:**  $M_2^d$ .

**Model type:** decision-oriented, computerized, deterministic, analytic, static.

More important input/output variables of the model:

$$M_2^d : \{P^{ss}, T^{in}, U^{in}\} \Rightarrow \{T^{out\alpha}, T^{outu}\} \quad (8)$$

where:

$P^{ss}$  subset of the set of decision problems, relative to productive operation, optimized at the level of National Power Dispatching ( $P^{ss} \subset P^s$ , i.e. coordination decision variables set at the level mentioned),

$T^{in}$  set of detailed technological input parameters of the power plant (including, for instance, fuel parameters, as well as the levels of power generation, like temperature of live steam etc.),

$U^{in}$  set of parameters describing the degress of typical failures of equipment – interaction with models  $M_3^d, M_4^d, M_5^d$ ,

$T^{outa}$  set of aggregate technological output parameters of the power plant (i.e. parameters of the unit loadings, fuel use characteristics etc.) – interaction with model  $M_1^d$ ,

$T^{outu}$  set of detailed technological output parameters of the power plant, decisive for intensity of the failure process (parameters of loadings of particular installations and devices etc.) – interaction with models  $M_3^d$ ,  $M_4^d$  and  $M_5^d$ .

Locally optimized decision variables of the model: none (all the variables of the model are globally optimized).

Model relations: the model contains submodels for particular main equipment items of the power generation unit: boiler, turbine, generator, individual auxiliary installations, as well as water-steam cycle. Relations of the model constitute the subsets (limited to the domain of exploitation processes) of relations concerning construction and design calculations for heat and electricity devices of the power plant unit.

Identification of the model: some of the technological parameters of power plant unit require appropriate estimation.

Analysis of sensitivity and errors in model parameters: same as for  $M_1^d$ .

### 3.3. Decision model of unit reliability, diagnostics and maintenance planning

**Model symbol:**  $M_3^d$ .

**Model type:** decision oriented, computerized, probabilistic, analytically-heuristic, dynamic, simulation.

**Simulation type and language:** as for  $M_1^d$ .

**Implementation of the model:** auxiliary computer model TEST was developed meant for meta-simulation of the respective process (with purpose of verification of the methodology adopted in the model from the point of view of the crucial problem of time of computer calculations); the corresponding microcomputer package for the model is still under development.

More important input/output variables of the model:

$$M_3^d : \{P^s, T^{in}, U^{in}\} \Rightarrow \{T^{out}, U^{out}\} \quad (9)$$

where:

$P^s$  complete set of coordination variables, i.e. the ones concerning productive functioning and repairs, optimized at the level of the National Power Dispatching, i.e. the coordination decision variables, set at the level mentioned,

$T^{in}$  set of detailed input technological parameters of the power plant, decisive for intensification of the failure processes (parameters of the medium, loads on particular installations and devices etc.) – including, for instance, interaction with the model  $M_2^d$ ;

- $U^{in}$  set of parameters describing the degrees of typical equipment failures (defined by the systems of diagnostics, including also expert systems);
- $T^{out}$  set of aggregate output technological parameters of the power plant (parameters describing planned, optimum schedule of the envisaged overhauls and repairs of the unit within the optimized period of power plant operation, as well as the forecasted temporal distribution of forced stops and power failures due to breakdowns) – interaction with model  $M_1^d$ ;
- $U^{out}$  set of output parameters describing the degrees of typical failures of equipment – interaction with model  $M_2^d$ .

Variables of the model optimized locally: parameters of the cycle of planned overhauls of the unit and of the individual installations of a unit. Individual decisions refer to putting the units to planned overhauls, as well as on the scope and degree of the planned overhaul.

Local optimization criterion:

$$\max_{\{Z_b^r\}} F_b^t, b \in \{1, \dots, B\}, F_b^t = \sum_{r=0}^R (F_r^a (1 + \alpha)^{-r}) \quad (10)$$

subject to the same constraints as in (4) before,  
where:

- $\{Z_b^r\}$  set of locally optimized decision variables of the process of overhaul planning;
- $b$  unit index;
- $B$  number of units;
- $F_b^t$  discounted profit to be internally divided within the power plant, defined from a single simulation run relative to the repairs of the  $b$ -th unit, for one variant of the overhaul plans (overhaul translocation),
- $F_r^a$  profit magnitude as  $F_{ir}^a$  in formula (5), but for the run as above, quantity determined by the model  $M_1^d$  (in realization of the local optimization procedure, the  $M_3^d$  model cooperates with  $M_1^d$ );
- $R, \tau, \alpha$  as in formula (5).

Method (procedure) of local optimization: the procedure consists in heuristic translocation of overhauls of maximum scope, so as to achieve an improvement of the objective function (10).

Decision variables of the model optimized globally: the remaining decision variables of the overhaul planning process.

Relations of the model: the structure of the model contains the subsets of relations referring to modelling of: typical equipment failures, reliability structure of the power plant, as well as maintenance systems of a unit and of particular installations of a unit, and then the subsets of relations referring to simulation of the failure and repair processes, and finally – optimization of the unit overhaul life and putting in operation. A new type of maintenance system was defined: the system of forecasted overhauls.

Identification of the model: it is necessary to identify the parameters of some typical equipment failures (like types and parameters of distributions of time of non-failure operation, limit values of failures etc.; a number of these parameters have already been determined in the studies conducted by now). It is also necessary to develop diagnostic expert systems, cooperating with the model here considered.

Analysis of errors and sensitivity of the model parameters: as for  $M_1^d$ .

### 3.4. Decision model of unit repair execution

**Model symbol:**  $M_6^d$ .

**Model type:** decision-oriented, computerized, deterministic (quasi-probabilistic) analytico-heuristic, network, dynamic.

**Implementation of the model:** microcomputer package PERT'88 (implementing the optimization procedures A and B, see further on).

More important input/output variables of the model:

$$M_6^d : \{X^{in}, G^r, N^{in}\} \Rightarrow \{T^0, L^0, H_b^0, Y^{out}\} \quad (11)$$

where:

$X^{in}$  set of input parameters characterizing the scope of given planned and/or damage-removing overhauls of a unit (values of Boolean variables  $\Gamma_{bu}^n$ ,  $u \in \{1, \dots, U_b\}$ , corresponding to the fact of repair of the given  $u$ -th failure at the given overhaul);

$G^r$  set of parameters characterizing the structure of the overhaul network;

$N^{in}$  set of parameters describing the magnitude and structure (organization of work teams) of availability of means (labour and transport) over time, in carrying out an overhaul ( $\Phi_x = f'(t)$ ,  $\Phi_x^{\max}$ ,  $x \in \{1, \dots, X\}$ ), or the shape of dependence of the three characteristic times of the PERT method on the requirement for means ( $(T_c^p, T_c^o, T_c^n) = f''(\Psi_{cx'})$ ), depending upon the selection of the optimization procedure, A, B or C;

$t$  time measured in elementary units, usually hours;

$c$  index of operation in a project;

$C$  number of operations in a project;

$x$  index of kinds of means in a project;

$x'$  value of  $x$  above for the labour force of general specialization;

$\Phi_x$  availability of the  $x$ -th means in the project;

$\Phi_x^{\max}$  maximum availability of the  $x$ -th means in a project;

$T^0$  output parameter defining the minimum time necessary for realization of an overhaul, depending upon the procedure applied,  $T^{0A}$ ,  $T^{0B}$ ,  $T^{0C}$ ;

$L^0$  set of output parameters defining the minimum requirements on labour force, ( $\Psi_{x'}^{0B}$ ,  $\Psi_{x'}^{0C}$ , depending upon the nature of the procedure applied, as above);

$H_b^0$  optimal schedule of a given overhaul of the  $b$ -th unit (set of Boolean variables  $H_c(t)$ ,  $c \in \{1, \dots, C\}$ , corresponding to the fact of realization of the  $c$ -th operation in the  $t$ -th time instant);

$Y^{out}$  set of output parameters for the optimum realization of the overhaul, characterizing the distribution of demand for means over time.

Locally optimized decision variables of the model: all the decision variables of the process (in case there exists the hierarchy of the objectives: minimum project time – minimum demand for means).

Criteria of local optimization:

Procedure A:

$$\text{objective function: } \min_{H_b^r} T^p, \quad (12)$$

subject to constraints

$$\Psi_x^{\max A} \leq \Phi_x^n, \Psi_{cx} = \Psi_{cx}^n; x = 1, \dots, X, c = 1, \dots, C \quad (13)$$

$$\text{as in the formula (4)} \quad (14)$$

Procedure B:

$$\text{objective function 1: } \min_{H_b^r} T^p \quad (15)$$

$$\text{objective function 2: } \min_{\{H_b^r, \{\Psi_{cx}\}\}} (\Psi_x^p | T^p = T^{0B}); x = 1, \dots, X \quad (16)$$

subject to constraints

$$\Psi_x^{\max B} \leq \Phi_x^n, \Psi_{cx} = \Psi_{cx}^n; x = 1, \dots, X, c = 1, \dots, C \quad (17)$$

$$\Psi_x^{0B}(t) = \text{const}, \text{ for } t \leq T^{0B}, x = 1, \dots, X \quad (18)$$

$$\text{as in the formula (4)} \quad (19)$$

Procedure C:

$$\text{objective function 1: } \min_{H_b^r} T^p \quad (20)$$

$$\text{objective function 2: } \min_{\{H_b^r, \{\Psi_{cx}\}\}} (\Psi_x^p | T^p = T^{0C}) \quad (21)$$

subject to constraints:

$$\Psi_x^{\max C} \leq \Phi_x^{\max}, \Psi_{cx}^{\min} \leq \Psi_{cx} \leq \Psi_{cx}^{\max}; x = 1, \dots, X, c = 1, \dots, C \quad (22)$$

$$\Psi_x^{0C}(t) = \text{const}, \text{ for } t \leq T^{0C}, x = 1, \dots, X \quad (23)$$

$$\text{as in the formula (4)} \quad (24)$$

where:

$H_b^T$  set of all the feasible realizations of a given overhaul of the  $b$ -th unit (all the 2-element variations with repetitions of the Boolean variables  $H_c(t)$ ,  $c \in \{1, \dots, C\}$ ,  $t \in \{1, \dots, T^{\max}\}$ );

$T^P$  project (overhaul) realization time;

$\Psi_x^P$  demand for the  $x$ -th means in the (overhaul) project;

$\min \Psi_x^P | T^P = T^{0B}$ ,  $\min \Psi_x^P | T^P = T^{0C}$  criterion of the minimum of demand for means under the condition of attainment of minimum time of project realization in the implementation of, respectively, procedures B and C;

$\Psi_x^{\max A}$ ,  $\Psi_x^{\max B}$ ,  $\Psi_x^{\max C}$  maximum demands for the  $x$ -th means in the project, when various procedures of optimization are applied;

$\Phi_x^n$  normative availability of the  $x$ -th means in the project;

$T^{0A}$ ,  $T^{0B}$ ,  $T^{0C}$  optimum (minimum) time of realization of the project when various optimization procedures are applied;

$\Psi_{cx}$ ,  $\Psi_{cx}^n$ ,  $\Psi_{cx}^{\min}$ ,  $\Psi_{cx}^{\max}$  demand for the  $x$ -th means in the  $c$ -th operation (variable, normative, minimum and maximum), as an argument of a function: time demand;

$t$  time (time instants).

Methods (procedures) of local optimization: the model encompasses three hierarchical analytico-heuristic procedures, referred to as A, B and C. These procedures perform minimization of the time of realization of the overhaul project, as well as minimization of the resource demand and use.

In modelling of maintenance execution process the PERT method (Programme Evaluation and Review Technique) was used, though network optimization was carried out with an original own computerized algorithm. This algorithm accounts for a number of specific properties and requirements of maintenance economy in the power plants.

Relations in the model: within the structure of the model the subsets of relations are distinguished, connected with the previously described optimization procedures A, B and C.

Identification of model parameters: the structure of the repair network requires identification, together with the parameters of particular repair operations for every atypical repair (for the cases of typical repairs these parameters had been defined in the studies already carried out).

Analysis of undefiniteness and sensitivity of the model parameters: as for  $M_1^d$ .

#### 4. Methods of global optimization of the process

The selection of the global optimization method depends upon the complexity and nature of a given decision process. In the case of decision processes of low complexity (low number of optimized decision variables and/or low admissible ranges of variability of these variables), a simple method of calculation and comparison of all the combinatorially defined decision variants is preferred. On the other hand, in the case of more complex processes the methods developed or applied within the domain of operations research are referred to, such as mathematical and heuristic programming, as well as random search for optimum, or the methods of experiment planning. In this section some more important features are presented of the combination of two methods known from literature, proposed by the present author. These two were: the method of random search for extremum of a multivariate function, based upon the work of Rastrigin school from the Latvian Academy of Sciences (Kalinnikov, Lifshits, Malts, 1969, and Rastrigin, 1975), and the method from the domain of theory of experiment planning, with main reference being Polański (1984).

The initial point of the method is the space of decisions defined by the formula (3). The Monte Carlo mechanism using a standard pseudo-random number generator is used to obtain  $L'$  initial points of search, indexed with  $l$ , i.e.  $l \in \{1, \dots, L'\}$ . This set is complemented with  $L'' (\geq 1)$  points selected on the basis of a heuristic scheme as candidates for the hypothetical global optimum of the objective function, so that we finally have  $L = L' + L''$  points. Now, from each  $l$ -th initial point,  $l \in \{1, \dots, L\}$ , an independent realization of the procedure of search for the extremum of the objective function is carried out.

Around the initial point the decision variables are normalized in accordance with the formula

$$\hat{Z}_{pze}^n = \frac{Z_{pze}^n - Z_{pzel}^n}{\Delta Z_{pze}^n} \quad (25)$$

where:

$Z_{pzel}^n$  value of the decision variable in the  $l$ -th initial point of search;  
 $\hat{Z}_{pze}^n$  decision variable normalized into the interval  $[-1, 1]$ , around the initial point of search, i.e. in the interval  $[Z_{pzel}^n - \Delta Z_{pze}^n, Z_{pzel}^n + \Delta Z_{pze}^n]$ ;  
 $\Delta Z_{pze}^n$  adopted step of the increment of the decision variable value.

In the close vicinity of each  $l$ -th initial point the objective function  $\bar{F}$  is being approximated with the linear polynomial  $F^{apr}$ , for which the gradient, taking the form given below, is being calculated,

$$\hat{g} = \text{grad} \hat{F}^{apr} = \sum_{p=1}^{\Xi_k} \sum_{z=1}^{Z_p} \sum_{e=1}^{E_k} \frac{\partial \hat{F}^{apr}}{\partial \hat{Z}_{pze}^n} z_{pze}^n \quad (26)$$

where:

$$\hat{F}^{appr} = \hat{\omega}_o + \sum_{p=1}^{\Xi_k} \sum_{z=1}^{Z_p} \sum_{e=1}^{E_k} \hat{\omega}_{pze} \hat{Z}_{pze}^n \quad (27)$$

and:

$\mathbf{z}_{pze}^n$  the orthonormal vector of the decision variable,

$\omega$  coefficients of the approximating function,

$\hat{\cdot}$  symbol of the normalized quantities.

The direction of the gradient of approximating function constitutes itself the approximation of the direction of steepest change of the objective function. The gradient vector is determined with the use of the experiment planning methods, using the fractional plan PS/DS-P: $2^{\gamma-\delta}$ , being simultaneously a rotatable and an orthogonal plan, Polański (1984).

The scope of variability of the substitute index of the decision variables, here introduced,  $q \in \{1, \dots, \gamma\}$ ,  $\gamma = Z_k$ , is divided into two subintervals:  $q = q'$ ,  $q' \in \{1, \dots, \gamma - \delta\}$  and  $q = q''$ ,  $q'' \in \{\gamma - \delta + 1, \dots, \gamma\}$ . For the decision variables of the index values  $q''$  the generating relations are defined, being the functions of the decision variables of the index value  $q'$ :

$$\hat{Z}_{q''}^n = \prod_{q'=1}^{q'=\gamma-\delta} \beta_{q'} \quad (28)$$

where  $\beta_{q'} = |\frac{1}{\hat{Z}_{q'}}|$ ;  $q' = 1, \dots, \gamma - \delta$ ;  $q'' = \gamma - \delta + 1, \dots, \gamma$ , with

$\beta_{q'}$  being an auxiliary variable,

$\gamma$  number of settings of the complete plan, and

$\delta$  quantity by which the number of the settings of the fractional plan is decreased.

The auxiliary variables  $\beta_{q'}$  are selected in such a manner as to make the products  $\prod_{q'=1}^{q'=\gamma-\delta} \beta_{q'}$  constitute chosen, but in each case different, combinations

$$C_{\gamma-\delta}^2, C_{\gamma-\delta}^3, \dots, \text{ or } C_{\gamma-\delta}^{\gamma-\delta}, \quad (29)$$

of the variables  $\hat{Z}_{q''}^n$ ,  $q' = 1, \dots, \gamma - \delta$ , with  $C_n^m$  denoting corresponding the mathematical construct of combinations.

The generating relations are the basis for reduction of the complete plan PS/DK- $2^\gamma$  to the fractional one, PS/DS-P: $2^{\gamma-\delta}$ . In the thus defined plan there are  $J = 2^{\gamma-\delta}$  calculations of the objective function value, leading to determination of consecutive values  $\bar{F}_j$ , with the index  $j = 1, \dots, J$ ,  $J = 2^{\gamma-\delta}$ . In these calculations the values of the decision variables of indices  $q'$ ,  $q' = 1, \dots, \gamma - \delta$ , form  $2^{\gamma-\delta}$  - i.e. all the possible  $-\gamma - \delta$ -element variations with repetitions from

the two-element set of normalized values  $\{-1, +1\}$ , while the values of decision variables of indices  $q''$  are defined from the generating relations (28) and (29).

Now, the coefficients of the normalized approximating function are being determined according to the relations

$$\hat{\omega}_0 = \frac{1}{J} \sum_{j=1}^J \bar{F}_j; \hat{\omega}_q = \frac{1}{J} \sum_{j=1}^J \bar{F}_j \hat{Z}_q^n; q = 1, \dots, \gamma; J = 2^{\gamma-\delta} \quad (30)$$

where

$j$  index of realizations of the experiment plan,

$J$  number of realizations of the plan.

The calculated coefficients determine the vector of normalized gradient of the approximate to the objective function, i.e.

$$\text{grad } \hat{F}^{appr} = [\hat{\omega}_1, \hat{\omega}_2, \dots, \hat{\omega}_q, \dots, \hat{\omega}_\gamma] \quad (31)$$

Along this gradient direction  $\Lambda$  working steps are performed. According to the classical method of statistical gradient only one step ( $\Lambda = 1$ ) is performed, after which the gradient calculations of the objective function approximation is repeated each time. According to the steepest descent techniques the number of steps is bigger than 1 ( $\Lambda > 1$ ), but after each step the sign of the objective function change is checked. The decrease of the objective function value means that a step was a "failure" and then a return to the last "successful" step is performed, where a new gradient of the approximate objective function is calculated. Calculation of the objective function value takes place at every step conform to the formula (5), along the trajectory of calculations passing through selected decision models (an example of such a trajectory for calculation of fuel selection is presented further on).

The value of the decision variable in the  $\lambda$ -th step, following the  $\sigma$ -th ( $\sigma \in \{1, \dots, \Sigma_l\}$ ) procedure of determination of the gradient of approximate objective function,  $Z_{pze\lambda}^n$ , is

$$Z_{pze\lambda}^n = Z_{pze\sigma}^n + \epsilon \lambda \hat{\omega}_{pze} \Delta Z_{pze}^n, \quad (32)$$

for  $p = 1, \dots, \Xi_k$ ,  $z = 1, \dots, Z_p$ ,  $e = 1, \dots, E_k$ ,  $\lambda = 1, \dots, \Lambda_\sigma$ ,  $\sigma = 1, \dots, \Sigma_l$ ;  
where:

$Z_{pze\sigma}^n$  value of the decision variable in the  $\sigma$ -th point of determination of the gradient of objective function approximate;

$\Delta Z_{pze}^n$  assumed increment of the decision variable;

$\sigma$  index of realization of the procedures of determination of the approximate to the objective function;

$\Sigma_l$  the number of calculations of the above procedures along the trajectory of extremum search from a single  $l$ -th initial search point;

$\lambda$  working step index;

$\Lambda_\sigma$  number of steps carried out after the  $\sigma$ -th point of determination of the gradient of the approximate to the objective function;  
 $\epsilon$  assumed multiplicity of the step length in comparison with the increments of variable values;  
 $\hat{\omega}_{pze}$  normed coefficient of the objective function approximate.

The approach forwarded by the Rastrigin school, (Kalinnikov, Lifshits, Malts, 1969) differentiates the coefficient  $\epsilon$  of the multiplicity of step length depending upon the number of "failure" steps taken previously over the trajectory of search from a given initial search point.

If the number of successful steps taken from the last point at which the gradient of the objective function approximate has been determined,  $\Lambda_{\Sigma_l}$ , is equal zero, then it is assumed that the local extremum point has been found (not to be confounded with the notions of local and global optimization, introduced at the beginning of the paper) of the objective function  $\bar{F}_l^*$  from the  $l$ -th initial point of search. After the procedure is repeated for all the  $L$  initial points of search the global maximum of the objective function is determined as:

$$\bar{F}^0 = \max_{l \in \mathcal{L}} \bar{F}_l, \text{ for } \mathcal{L} = \{1, \dots, L\} \quad (33)$$

where:

$\bar{F}^0$  global maximum of the objective function;

$\bar{F}_l^*$  local maximum of the objective function, found on the trajectory of search starting from the  $l$ -th initial point of search.

The set of values  $\{Z_{pze}^n\}$ ,  $p \in \{1, \dots, \Xi_k\}$ ,  $z = \{1, \dots, Z_p\}$ ,  $e = \{1, \dots, E_k\}$ , corresponding to the point of the global maximum found as above constitutes the optimum solution to the  $k$ -th decision process.

Among other methods of global optimization the most perhaps known is the general technique of Gauss-Seidel. This method of finding extrema of multivariate functions is applied in these decision processes in which there are no constraints or they have been appropriately transformed. An example of such a process can be, in particular, given definite simplifying assumptions, provided by the fuel selection problem in a power plant.

## 5. An instance of the global decision optimization process of the power plant operation

We have chosen the fuel selection process in a power plant as an example of the global optimization process. Fuel costs account for the highest share in own costs of power plant, and that is why fuel selection is decisive for the power plant economy. Simultaneously, there is a very important influence exerted by the fuel (and especially by its quality) on the damage rate and durability of the equipment, and on the efficiency of a unit. Optimization of fuel selection must therefore be connected with modelling of failure processes and with optimization of the planned overhaul life of the power generation units, as well as with

the process of execution of planned and emergency repairs. All of the above mentioned problems constitute a given  $k$ -th (here, nominally,  $k = 1$ -st) decision process in the power plant.

Decisions concerning fuel selection should be optimized (verified) within the power plant not less frequently than every  $T^y = 2$  years with the time horizon of the envisaged consequences of these decisions of the order of some  $T^h = 10$  years. This amounts to an  $E_1 = 5$ -stage decision process.

If in the subsequent stages we do not envisage essential changes in the structure of fuel prices, in the fuel characteristics, in supply conditions (e.g. quantity limitations), in transport pricing structure, nor finally in the characteristics of the power plant that could influence the efficiency of power generation units, then the process can be simplified to a single-stage one (in the sense of the structure of decision space; physically, in reality, verification of decisions should still be performed in appropriate stages).

The set of decision variables of the process includes:

- type of fuel purchased (fuel type is described by the vector composed of: calorific value, price of purchase, transport cost, supplier and/or producer, chemical composition and physical characteristics (with special emphasis on sulphur content, water, ash content, granulation, grindability, temperature of ash softening etc.); a fuel type may also be constituted by a definite mixture of (simple) fuels of known characteristics;
- parameters of the planned unit overhaul life (number of overhaul degrees, types of overhaul degrees, intervals between repairs, overhaul duration);
- optimum schedule of repairs (sequence of operations, system of their divisibility, time of operation execution, allocation of resources to operations);
- system of repair execution (described by the vector composed of numbers of persons in teams, worker skills, assignment of teams to repair tasks or operations).

All the other variables are treated in the given decision process as parameters (i.e. not as decision variables). The subset of globally optimized variables encompasses exclusively the type of fuel purchased. The remaining decision variables can be optimized locally within respective decision models.

The process here considered makes use of decision models denoted  $M_1^d, M_2^d, M_3^d, M_4^d$  (see section 2).

We could use, as the global optimization method, in the case of a simplified single-stage decision process, the procedure of enumeration and comparison of all the decision variants.

In the case of a multi-stage process, when the number of stages is  $E_k = 5$  and  $Z_{pze}^n = 1, \dots, 10$  ( $k = 1, p = 1, z = 1, e \in \{1, \dots, 5\}$ , 10 being the maximum practicable number of fuels considered, mixtures of fuels of various structures taken into account, after the variable scope has been subject to filtering), the number of decision variants is  $10^5$ , which already makes it impossible to enumerate and compare all of them (calculations relative to a single variant require

each time passing through the trajectory described further on, steps 2 to 6).

It is possible, on the other hand, to apply the method of combined random search of extremum of a multivariate function and experiment planning, outlined before (it is then advised to order the feasible realizations of the fuel type vector according to the increasing equivalent fuel price, i.e. taking into account purchasing price of fuel and calorific value), though, in view of the relative independence of selection of the types of fuel in consecutive stages, as well as lack of constraint function set on the choice of types, there exists, exceptionally, the possibility of application to this particular case of the classical Gauss-Seidel algorithm.

The trajectory of optimization calculations is composed of the following steps:

1. The process is controlled by the algorithm of global optimization, defining every time the values of decision variables before particular computation runs, depending upon the adopted method of optimization. In the case of the Gauss-Seidel algorithm only one decision variable is changing its value with the given definite, locally optimal values of the remaining variables.
2. Given the values of decision variables, determined in step 1 above, the realization of repetitions of the random calculation runs is taking place. The necessary number of these repetitions should result from the analysis of variance of the objective function in implementation of a given decision process.
3. For a given set of values of the decision variables and a given random repetition, a realization of the decision model  $M_3^d$  takes place, this model being related to questions of reliability, diagnostics and planning of the unit overhauls, for the operation horizon equal  $T^h$ , and for all the units of the power plant in a sequence.  
During calculations the model cooperates with  $M_2^d$ , the decision model of power plant operation, and with  $M_4^d$ , the decision model of overhaul execution. Inside the models  $M_3^d$  and  $M_4^d$  a local optimization of the decision variables concerning planning and execution of repairs is carried out.
4. After the totality of operation and maintenance calculations for a given set of values of decision variables, given random repetition and all the units, has been performed, the decision model  $M_1^d$ , related to the economic-financial system of the power plant, sets in, and is realized for the operation period of  $T^h$  and for the whole of the power plant, in a single run. Resulting from calculations there is the value  $F_i^f$  of discounted profit to be shared/divided within the power plant, as a single value of the random variable.
5. Another random repetition is initiated and the steps 3 and 4 are repeated. When calculations for the last envisaged random repetition have already been performed, then the procedure passes over to step 6.

6. Objective function value is calculated, namely the expected value  $F$  of the discounted profit to be divided within the power plant, based upon the values obtained in particular random repetitions (see formula (5)).
7. Return to step 1 takes place, i.e. to the procedure of global optimization, with the aim of comparing the obtained values of the objective function with the ones from other calculation runs and with the aim of selection of some other set of decision variable values.

When the algorithm finishes the search process, the last maximum value obtained of the objective function, the one determined in the recent iteration, will constitute the global optimum of the given process (with Gauss-Seidel algorithm having been applied) and will thereby determine the sought optimal values of the globally optimized variables (here: fuel types in subsequent stages of the process). Simultaneously, the optimum values of the local decision variables (here: overhaul parameters), defined inside the models  $M_3^d$  and  $M_4^d$  in the same iteration, will constitute their optimum values sought.

## 6. The status of implementation of the methodology of modelling and optimization in industrial practice

Models  $M_1^d$  and  $M_4^d$  are already transformed into software in the form of computer packages (MDE1 and PERT'88) and as such implemented in one of large Polish power plants. They were used for a number of useful optimization calculations. This gave rise to conclusions indicating the potentially high economic utility of full implementation of the methodology here described.

A number of test programs have been developed as well, meant for verification of the algorithms of the remaining models. Work is underway on software development for the particularly important model  $M_3^d$ , relative to reliability of equipment and overhaul planning, as well as on the expert systems for technical diagnostics, which will cooperate with the model mentioned.

The precondition for the success of the process of implementation of the methodology described is the parallel development of modern computer management systems that would secure an adequate source of information for optimization calculations.

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