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Multidimensional scaling for comparison of dendrograms

by

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In the introductory part a survey is given on classification methods, proximity measures, representation and interpretation of results obtained by clustering and/or multidimensional scaling. In the subsequent part comparison of dendrograms is discussed. Data were collected by a questionnaire and hierarchical clustering was done to explore the value structure of selected Hungarian families. Multidimensional scaling is offered to check the results.

1. Introduction

It is required of a scientific classification that identification of classes be based on objective criteria. Determination of classes is the result of a learning process two main types of which are distinguished.

In the first one a pre-defined decision criterion is given and classification is based on this information (learning with teacher or by supervised training).

In the second one classes are separated exclusively on the basis of the sample. Pattern recognition or learning process is done without additional information (without teacher or by unsupervised training).

2. Cluster analysis

Methods of cluster analysis are of the second kind. The basic model of cluster analysis does not define types before selecting sample elements and class limits are not determined beforehand.

The variety of cluster models can be looked at from several viewpoints. In particular, to meet practical purposes overlapping classification and disjunct classification can be used, depending upon the case on hand.

Two major well known groups of approaches are used for classification in multivariate statistics: the hierarchic and the non-hierarchic ones.

The non-hierarchic model divides the sample into the predefined number of k classes. The hierarchical classification can be either divisive or agglomerative. The latter aggregates all the elements into one single class in a step by step aggregation procedure.

The first step of any statistical analysis is data collection. According to J. Kruskal (1977), there are three types of data used in clustering. The first type is multivariate data, the second one is proximity data and the third one is clustering data.

Multivariate data can be measured on ordinal or other available higher scales. Taking the multivariate data matrix as our starting-point we first measure proximities among data items.

The notion of proximity measure, as used here, refers either to similarity or dissimilarity. An $(n \times n)$ or $(p \times p)$ proximity matrix derived from the original data matrix will be the output of this stage of analysis. Proximity data can be given originally or they can be obtained, like outlined from earlier results.

The third type of data are clustering data which means sorting data. If many subjects are asked to sort or cluster variables or observations which are very similar in some way the result will be subjective clustering.

The clustering-algorithm takes (preliminary) step of processing proximity data from multivariate data and then proceeds the conversion of proximities into clustering data. Many authors refer to the first stage as prior to clustering and not an organic part of it.

The selection of measures of closeness (similarity) or distance (dissimilarity) will highly influence the partition of the sample. Importance of this intermediate stage is beyond any doubt, but it is out of the scope of this paper.

Conversion of proximities into clustering data is referred to as aggregation of observations. Using an agglomerative procedure, objects are combined in the first step and the resulting classes are joined in subsequent steps. Joining can be done using several cluster techniques based on the estimation of distances between groups.

As it was mentioned earlier, input of these techniques can be constituted by the similarity or distance measures. Different measures can result in different solutions for the same clustering method. In some cases there are well known formulations: the median, centroid and Ward's methods use squared Euclidean distances. Other methods, such as single and complete linkage and the average distances between and within clusters can be used both with similarity or distance measures.

How can we select a method which serves our purposes best? Or should we use only one cluster technique? There are certain rules of thumb for selection, but it is better to follow more sophisticated and reliable ways of analysis.

Thus, for instance, with the Euclidean distance measure we can theoretically use all of the seven techniques made available in the SPSS. The output can be seen on a special tree diagram (the so called dendrogram). These dendrograms can show different pictures using different methods.

The statistical problem is that of deciding which of the different cluster partitions presented by algorithms are real? Following Hartigan's definition (J.A. Hartigan, 1977): "A data cluster is real if it corresponds to one of the population clusters". This problem is stated as an asymptotic distribution theory problem, because finite and exact theory is almost out of question.

In many applications, large amount of multivariate data are collected in order to explore the underlying structure. The researcher wishes the data to show some partition and it is not so important to get the exact number of clusters.

If a statistician has some initial intuition or some results of yet another study on the same set of data, he might like to confirm the pre-existing classification in an efficient way.

There are a number of procedures suggested in the literature for evaluating the adequacy of cluster analysis. But, on one hand, no generally accepted statistical method has been proposed for computing the goodness-of-fit for classification. On the other hand the computer packages do not contain any measure of fit indices of partition hierarchy.

From an inferential point of view, the techniques of cluster analysis are perceived as even more than descriptive tools of multivariate statistics. A number of branches of statistics are relevant to clustering: discriminant analysis, princial component analysis and factor analysis, analysis of variance, contingency tables, regression and multidimensional scaling can be used in evaluating clusters.

3. Multidimensional scaling

Clustering and multidimensional scaling (MDS) are both methods for preliminary analysis of data. To some extent, they compete with each other. Sometimes they are complementary and then they can also be used together. In theory, however, they are competitors. If one model fits better, the other model must fit worse. In practice, though, a positive relationship is observed: when one model fits better the other one fits better, too.

The fact that both cluster analysis and MDS may fit well does not mean that they focus on the same kind of information about the data analysed. The MDS is a procedure for interpreting the matrix of proximities as distances between points in a low-dimensional Euclidean space. The proximities, as mentioned before, are ususally similarities or dissimilarities among objects.

Multidimensional scaling converts the proximity data into multivariate data, constituting thereby an opposite of the preliminary step of clustering. In the second stage the MDS algorithm forms distances from this multivariate data matrix and provides a spatial representation of objects on the basis of the so called secondary proximities or derived proximities. Each object can thereby be represented by a point in space. The accuracy of the representation is what is sought in the overall procedure.

The latter means that it is required the difference between input proximities and spatial distances be as small as possible. The measure of fit, the stress index is expressed through the square root of the proportion of the total sums of squares of error ($E = T - D^2$) to the transformed data (T). D^2 indicastes the squared distances.

$$S - stress = \left(\frac{\|T - D^2\|}{\|T\|}\right)^{1/2}$$

The MDS algorithms optimize stress in the given dimensionality of final representation.

The main objective is twofold: to maximize the accuracy and decrease the dimensionality of representation at the same time. The final result is based on a compromise, the goodness of fit being measured in a pre-determined dimensionality of representation.

Statistical softwares offer several types of multidimensional scaling procedure. Classical MDS is the simplest kind of scaling. CMDS can be used for only a single matrix of dissimilarities. Metric CMDS uses linear transformation of the dissimilarities while nonmetric CMDS gives monotonic transformation of dissimilarities. The output contains, among others, a scatterplot of linear or nonlinear fit, stimulus coordinates, a plot of objects on map and the value of the Kruskal stress index.

To interpret the MDS map the location of stimulus (position of variables or observations given by coordinates) has to be investigated. A statistician looks for virtually distinguishable groups of points and tries to explain the dimensions of the plot. In this stage of the analysis it is necessary to apply other multivariate statistical methods. The situation is quite similar to the interpretation of factors and clusters.

Our practical conclusion is the following:

Although the procedures of clustering and scaling are different, the results are pretty much in line with each other. The output of clustering helps to interpret the results of scaling and vice versa, so that it is reasonable to use these two analyses together.

4. Comparing life styles of families

We have faced the problem of interpretation of clusters in several socio-economic and sociological analyses. Using the hierarchical cluster analysis procedures, commonly accepted and offered in terms of available software, it is easy to compute results with different methods for the same problem but to form groups and synthetize the results can cause difficulties.

In this part of the paper, joint application of clustering and scaling will be discussed in a case study.

It has been established that the number of telephones is in close connection with the general economic development of a country. The number of telephones is in tight positive correlation with the GDP per person. This is why a questionnaire was designed to explore the demand for telephones in Hungary. The questionnaire contained several questions concernig both the standard living and the life style of 2000 Hungarian families.

Using telephones is a part of every day life in developed countries both in business and in private life. In Easter and Central Europe per capita number of telephones is lower than in the Western countries. This difference is combined with dissimilar business communication practices with a greater emphasis on written communication in Easter and Central Europe.

The value system of Hungarian families seems to be different from that of families in the developed countries. The life style differences can be seen in the tightness of family ties. Frequency of telephone calls and the importance for people to be accessible and to access others are asked for in the above mentioned questionnaire. One of the main purposes was to make clear the price and cost level of supply of the Hungarian Telephone Company and to determine the elasticity of the private and office demand. It can be stated that there are very limited number of phones in Hungary. In spite of this fact the usual variables as income and price are not significant in the econometric models. Finally the experts looked for explanation using questionnaire.

As many as 72 questions were asked and investigated in order to analyse the opinions of people. One of them is the following:

"Is it important for your family ...?"

1.	to have enough money to buy everything (LUXURY)	[12.5]
2	to save money for the future (FUTURE)	[71.1]
3	to reach an acceptable living standard (LIFE)	[89.0]
4	to survive (SURVIVE)	[70.9]
5	to follow God (GOD)	[47.9]
6	to have several friends, acquantances (CONNECT)	[42.6]
7	to have an adventurous life (ADVENTURE)	[8.8]
8	to live in peace (PEACE)	[96.7]
9	to keep the family together (FAMILY)	[95.4]
10.	to learn, to improve yourself (LEARN)	[73.1]
11.	to have tasks and to fulfill them honestly (TASKS)	[92.2]

The answer could be: "important" (corresponding to attribution of value=1) or "not impotant" (=0), so it is measured on a dichotomous scale. [Percentages of positive answers (of 1's) are given in brackets.]

The dichotomous values allow for the measurement of the differences between items using binary squared Euclidean distance.

$$d_{ik}^2 = \sum_{j=1}^{p} (x_{ij} - x_{kj})^2, \text{ where } x_{ij} \text{ and } x_{kj} \text{ can be 0 or 1.}$$

The 11×11 ordered distance matrix is the input of the hierarchical clustering procedure.

5. Value structure on dendrograms

Let us now turn to the analysis of the data obtained. These data can be interpreted in terms of the most important values guiding families questioned. In order to explore the value structure of Hungarian families hierarchical cluster analysis was first conducted. Agglomeration schedule using seven clustering techniques was repeated. Due to missing data, some cases were excluded from computations. Generally, 1600 families filled out the questionnaire in its entirety.

At first sight, it appears that hierarchical clustering trees are very similar (Figures 1 through 7). Values inedxed 8-9-11 and 3 form a stable group on each dendrogram. Rescaled distances are less than 5.

Values no. 1 and 7 coalesce at the rescaled distance level between 2 and 12. The smallest distance level can be seen on the dendrogram obtained when using Ward method, the highest one is the result of the Median method.

Starting from the top of the tree one outlier can be separated. Value no. 5 joins to the other ones at the distance level between 20 and 25. The only exception appears in the Ward method which gives quite a different picture. Value no. 5 joins to value no. 4 at the distance level 6 and none of the values is an outlier.

After having a general look at the trees one cannot be sure of the number of clusters. A bigger and a smaller group plus one outlier can be seen on dendrograms using Average linkage between groups and within group. Two groups and two outliers are shown on the trees of Single linkage and Centroid methods. Two groups and three outliers are separated on the Median method's tree. Complete linkage and Ward methods show values nearer to each other and they form only two clusters. At this point, further statistical analysis was necessary to help the decision-maker to having a clear picture.

Nonhierarchic clustering was run with several cluster numbers to identify groups of people (heads of families) who are accepting similar values mentioned in the questionnaire. Finally six groups formed of 1609 heads of families were separated and described.

The K-means procedure used starts with analysis of variance with the purpose of finding variables with a high value of F-test. All of the 11 items proved to be distinguishable variables at each level of significance.

Three smaller and three bigger groups are formed.

The greatest group contains 756 heads of families (47%) whose answers are the following:

The values of the final cluster centers are in brackets. These numbers show the group mean of each standard variable. The mean of the total sample is zero. The higher is the value the most characteristic is the variable in this group.

6. it is important to have friends (CONNECT) (0.89)
10. it is important to learn (LEARN) (0.42)
2. it is important to save for the future (FUTURE) (0.34)

The other 6 cluster centers are above zero as well but they are too small.

Only two items are not so important, 4 (to SURVIVE -0.26) and 5 (to follow GOD -0.14).

The second group contains 698 families from the sample (43%). For them

6. it is less important to have friends (CONNECT) (-0.81)
4. it is important to survive (SURVIVE) (0.31)
11. it is important to fulfill tasks (TASKS) (0.29)
9. family is important (FAMILY) (0.22)

Cluster centers are positive at two other items and have negative values at 5 answers.

The third group contains 108 heads of families (7%), who are in a very bad position. They do not think about anything, the lowest values are:

11. TASKS (-2.85)
9. FAMILY (-1.46)
10. LEARN (-1.25)
2. FUTURE (-0.87)

The only positive value is 8 (PEACE 0.18).

There are three small groups with 20 - 15 - 12 heads of families in each. They have some common characteristics, for instance 8 (PEACE) has no importance for them (-5.45), they do not want 11 (TASKS -2.82; -2.13) and 10 (LEARN -1.65;-0.97). On the other hand they are different, for instance 20 persons think of 9 (FAMILY) as important (0.22) and the others have an opposite opinion (-4.56). These groups are too small to warrant more explanation.

Table of Final Cluster Centers

Value	Cluster 1	2	3	4	5	6
Luxury	-.3782	-.2104	.2490	.0245	-.2484	-.3782
Future	-.5765	-.8747	.3363	-.5397	-.1533	-1.5692
Life	-1.2467	-1.0691	.3048	-.5010	-.0745	-2.8447
Survive	-.1293	-.0111	-.2585	-1.1194	.3067	.0908
God	-.4583	-.0136	-.1380	-.5584	.1710	.0420
Connect	-.7593	-.7106	.8942	-.5908	-.8169	-.6919
Adventure	.0422	-.1473	.1514	-.3106	-.1438	-.0166
Peace	-5.4490	.1834	.1834	-5.4490	.1834	-5.4490
Family	.2189	-1.4645	.1683	-4.5656	.2189	-4.5656
Learn	-.9720	-1.2518	.4184	-1.6485	-.1722	-1.6485
Tasks	-2.1335	-2.8517	.2758	-2.4443	.2906	-2.8172

Number of cases in each cluster

Cluster	Cases
1	20
2	108
3	756
4	15
5	698
6	12
Total	1609

Comparing the hierarchic tree representation and the nonhierarchic cluster centers the result are in accordance with each other. The dendrogram using Ward method shows the same value structure as the cluster centers of the biggest group. This does not mean perfect fit because of differences of the approach. Dendrograms give two-dimensional spatial output while K-means clustering produces $k = 6$ cluster centers in 11 dimensions.

At this stage of the analysis it becomes worth finding another spatial representation of values in order to compare dendrograms and select the most reliable one.

Multidimensional scaling seems to be the proper tool of such comparison. Classical MDS uses only one proximity matrix as input data and after transforming the dissimilarities the spatial configuration is given as output.

Using the model options two and three dimensional solutions are computed. The Kruskal Stress index is equal to 0.05641 for the configuration derived in 2 dimensions, so goodness of fit is quite perfect.

Looking at the map (Figure 8) we see that the first dimension shows great differences. Values no. 3-8-9 and 11 are on the left and values no. 1 (LUXURY) and 7 (ADVENTURE) are on the right end of the axis. These two groups are separated on the dendrograms as well.

Second dimension helps to understand the dendrograms. The greatest difference can be seen between value 6 (FRIENDS) and 5 (GOD). Value 2 (FUTURE) and 10 (LEARN) are relatively close to each other on the two-dimensional map and they are not too far from the bigger group. The fourth value (SURVIVE) is on the opposite side.

By drawing lines between points according to the dendrograms it is easy to check the reliability of the classification. The lines do not cross each other and this can be a proof of the stable structure. The lengths of the lines are expected to be similar to the rescaled distances.

In our case, there is only one question. Value no. 2 and 10 are very close on the MDS map and they do not join on the trees with a low rescaled distance. The only exception is the dendrogram obtained using Ward method. Nonhierarchic cluster centers of these two values seem to be close as well. What could be the reason for the differences? How can we explain different results?

The MDS model option allows us to produce the two dimensional configurations. The goodness-of-fit measure indicates, however,the necessity of a higher dimensional solution. Dendrograms have to be two-dimensional trees. In hierarchic clustering there is no measure of fit. How can we avoid misclassification of sample?

By finding differences in classification seen throught dendrograms and MDS maps we can conclude that it is obviously necessary to repeat the multidimensional scaling procedure in higher dimension.

By increasing the number of dimensions we will improve the goodness-of-fit. In our case the three dimensional configuration (Figure 9) gives the value 0.01326 for the stress index, which means a truly perfect fit.

In the third dimension, it is clear that the location of values 2 and 10 are different. They are opposites. The stimulus coordinate of FUTURE is -0.6127 and the same value for LEARN is 0.6642. This is the second greatest difference in the third dimension. This distance between the values of -0,6 and 0,6 is shown on all of the six dendrograms.

Excepting the importance of this difference we tried to find group of people who evaluate the values 2 and 10 quite opposite way. The value structure of groups of people, as expressed through the results of the nonhierarchic clustering, was expected to explain the axes of MDS map. The importance of values measured by cluster centers in each group can be in line with the coordinates along particular dimensions.

Cluster centers of the largest group (Cluster 3) get along well the value structure presented along the second dimension. Members of Cluster 3 give positive evaluation to values 2 (FUTURE) and 10 (LEARN). Cluster centers of the second largest group (Cluster 5) follow each other as the values are located on the horizontal axis. Both values 2 and 10 are less important for this group, so they have negative cluster centers. They are on the right side of the first dimension. Importance is decreasing from left to right. Value coordinates on the third axis of the MDS map are quite different comparing them with the cluster centers of the four other groups.

We have not found the explanation that we looked for. None of the six groups differentiate these two values (Future and Learn). Could it be a consequence of the different distance concept used in agglomerative cluster techniques? Or are only answers of a couple of heads of families responsible for separating values 2 and 10? Separation of outliers within the value structure and further investigation of subsamples can be suggested. There is no reliable answer to the original question. The table is open for further research.

References

Anderson, T.W., (1958) *An Introduction to Multivariate Statistical Analysis*, John Wiley and Sons, New York.

Cooley, W.W. and Lohnes, P.R., (1971) *Multivariate Data Analysis*, John Wiley and Sons, New York.

Coombs, C.H. and Kao, R.C., (1960) On a Connection Between Factor Analysis and Multidimensional Unfolding, *Psychometrika*, **25**, 3.

COX, D.R., (1977) *The Analysis of Binary Data*, Chapman and Hall, London.

DILLON, W.R. and GOLDSTEIN, M., (1984) *Multivariate Data Analysis*, John Wiley and Sons, New York.

FÜSTÖS, L. and KOVÁCS, E. (1989) *Computer Oriented Statistical Analysis* (in Hungarian), University Press, Budapest.

FÜSTÖS, L., MESZÉNA, GY., MOSOLYGÓ-SIMON, N., (1981) Cluster Analysis, *Acta Economica*, **26** (3-4), Publishing House of the Hungarian Academy of Sciences.

GANESALINGAM, S., (1989) Classification and Mixture Approaches to Clustering via Maximum Likelihood, *Appl. Statistics*, **38**, 3.

HUBERT, L.J. and BAKER, F.B., (1977) An Empirical Comparison of Baseline Models for Goodness-of-Fit in r-Diameter Hierarchical Clustering *Classification and Clustering*, Edited by J.V. Ryzin, Academic Press, New York.

KRISHNAIAH, P. and RAO, C.R., (1982) *Handbook of Statistics Vol.2 Classification, Pattern Recognition and Reduction of Dimensionality*, North-Holland.

KRUSKAL, J., (1977) The Relationship between Multidimensional Scaling and Clustering, *Classification and Clustering*, Edited by J.V. Ryzin, Academic Press, New York.

KRUSKAL, J.B. and WISH, M., (1978) *Multidimensional Scaling*, Sage Univ. Paper Series, Quantitative Applications in the Social Sciences, No.07-011. Beverly Hills, London.

SHEPARD, R.N., (1962) The Analysis of Proximities: Multidimensional Scaling with an Unknown Distance Function *Psychometrika*, **27**, 2.

SOKAL, R.R., (1977) Clustering and Classification: Background and Current Directions *Classification and Clustering*, Edited by J.V. Ryzin, Academic Press, New York.

TOU, J.T. and GONZALES, R.C., (1974) *Pattern Recognition Principles*, Addison-Wesley Publishing Company, Reading.

YOUNG, T.Y. and CALVERT, T.W., (1974) *Classification, Estimation and Pattern Recognition*, Elsevier, New York.

Stage	Clusters Combined Cluster 1	Cluster 2	Coefficient	Stage Cluster 1st Appears Cluster 1	Cluster 2	Next Stage
1	8	9	66.000000	0	0	2
2	8	11	112.000000	1	0	3
3	3	8	178.000000	0	2	5
4	1	7	245.000000	0	0	8
5	3	10	381.500000	3	0	6
6	2	3	435.200012	0	5	7
7	2	4	563.000000	6	0	10
8	1	6	630.500000	4	0	9
9	1	5	790.333313	8	0	10
10	1	2	1019.178589	9	7	0

Rescaled Distance Cluster Combine

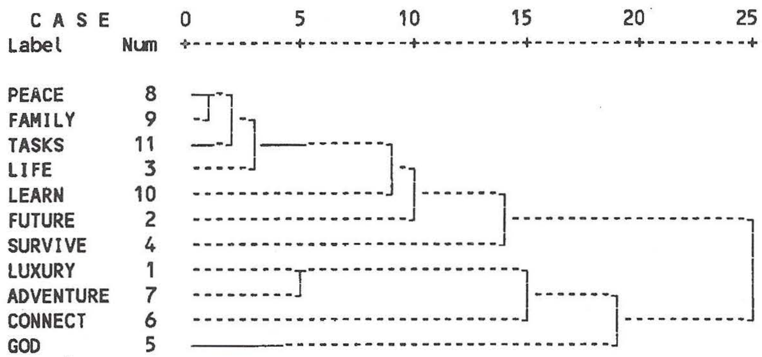

Figure 1. Agglomeration Schedule using Average Linkage (Between Groups)

Stage	Clusters Combined Cluster 1	Cluster 2	Coefficient	Stage Cluster 1st Appears Cluster 1	Cluster 2	Next Stage
1	8	9	66.000000	0	0	2
2	8	11	96.666664	1	0	3
3	3	8	137.333328	0	2	4
4	3	10	235.000000	3	0	6
5	1	7	245.000000	0	0	10
6	2	3	301.733337	0	4	7
7	2	4	376.380951	6	0	8
8	2	6	484.107147	7	0	9
9	2	5	557.611084	8	0	10
10	1	2	733.054565	5	9	0

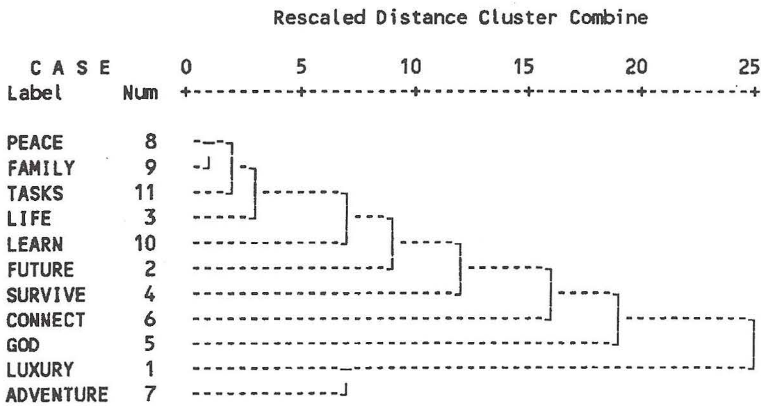

Figure 2. Agglomeration Schedule using Average Linkage (Within Groups)

Stage	Clusters Combined Cluster 1	Cluster 2	Coefficient	Stage Cluster 1st Appears Cluster 1	Cluster 2	Next Stage
1	8	9	66.000000	0	0	2
2	8	11	107.000000	1	0	3
3	3	8	173.000000	0	2	5
4	1	7	245.000000	0	0	8
5	3	10	352.000000	3	0	6
6	2	3	408.000000	0	5	7
7	2	4	472.000000	6	0	9
8	1	6	616.000000	4	0	9
9	1	2	644.000000	8	7	10
10	1	5	675.000000	9	0	0

Dendrogram using Single Linkage

```
                         Rescaled Distance Cluster Combine

    C A S E      0         5        10        15        20        25
  Label     Num  +---------+---------+---------+---------+---------+

  PEACE       8   --┐
  FAMILY      9   -┘├-----┐
  TASKS      11   ---┘    ├-------------┐
  LIFE        3   ---------┘             ├-----┐
  LEARN      10   -----------------------┘     ├---┐
  FUTURE      2   -----------------------------┘   ├-------------┐
  SURVIVE     4   ---------------------------------┘             ├-┐
  LUXURY      1   ---------------┬---------------------------┐   │ │
  ADVENTURE   7   ---------------┘                           ├-┘ │
  CONNECT     6   -------------------------------------------┘   │
  GOD         5   -----------------------------------------------┘
```

Figure 3. Agglomeration Schedule using Single Linkage

Stage	Clusters Combined Cluster 1	Cluster 2	Coefficient	Stage Cluster 1st Appears Cluster 1	Cluster 2	Next Stage
1	8	9	66.000000	0	0	2
2	8	11	117.000000	1	0	3
3	3	8	184.000000	0	2	5
4	1	7	245.000000	0	0	7
5	3	10	399.000000	3	0	6
6	2	3	480.000000	0	5	9
7	1	6	645.000000	4	0	10
8	4	5	675.000000	0	0	9
9	2	4	870.000000	6	8	10
10	1	2	1428.000000	7	9	0

Dendrogram using Complete Linkage

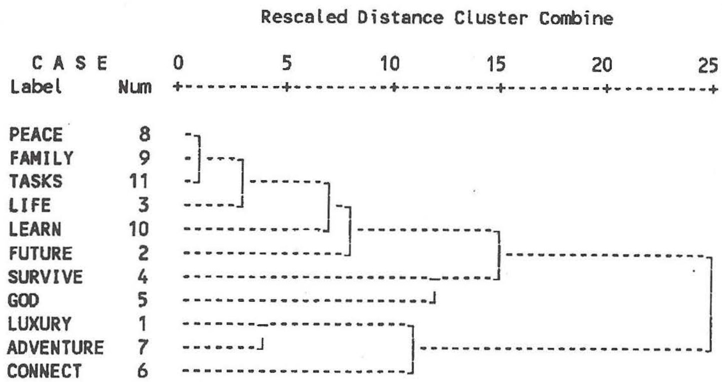

Figure 4. Agglomeration Schedule using Complete Linkage

Stage	Clusters Combined Cluster 1	Cluster 2	Coefficient	Stage Cluster 1st Appears Cluster 1	Cluster 2	Next Stage
1	8	9	66.000000	0	0	2
2	8	11	95.500000	1	0	3
3	3	8	145.777786	0	2	5
4	1	7	245.000000	0	0	8
5	3	10	330.000031	3	0	6
6	2	3	341.200012	0	5	7
7	2	4	437.277802	6	0	10
8	1	6	569.250000	4	0	9
9	1	5	623.000000	8	0	10
10	1	2	615.559937	9	7	0

Dendrogram using Centroid Method

```
                     Rescaled Distance Cluster Combine

 C A S E       0         5        10        15        20        25
Label     Num  +---------+---------+---------+---------+---------+

PEACE       8   -+
FAMILY      9   -+-+
TASKS      11   ---+---+-----------------+
LIFE        3   -------+                 |
LEARN      10   -------------------------+-------+
FUTURE      2   -------------------------+       +---------------+
SURVIVE     4   ---------------------------------+               |
LUXURY      1   -----------------+---------------------------+   |
ADVENTURE   7   -----------------+                           +---+
CONNECT     6   ---------------------------------------------+   |
GOD         5   -------------------------------------------------+
```

Figure 5. Agglomeration Schedule using Centroid Method

Stage	Clusters Combined Cluster 1	Cluster 2	Coefficient	Stage Cluster 1st Appears Cluster 1	Cluster 2	Next Stage
1	8	9	66.000000	0	0	2
2	8	11	95.500000	1	0	3
3	3	8	147.375000	0	2	5
4	1	7	245.000000	0	0	9
5	3	10	328.843750	3	0	6
6	2	3	340.210938	0	5	7
7	2	4	503.615234	6	0	8
8	2	5	564.153809	7	0	10
9	1	6	569.250000	4	0	10
10	1	2	468.448608	9	8	0

```
Dendrogram using Median Method

                         Rescaled Distance Cluster Combine

   C A S E      0         5        10        15        20        25
  Label     Num +---------+---------+---------+---------+---------+

  PEACE       8  -+-+
  FAMILY      9  -+ +-------+
  TASKS      11  ---+       +---------------------+
  LIFE        3  -----------+                     +-+
  LEARN      10  ---------------------------------+ +-------------+
  FUTURE      2  -----------------------------------+             |
  SURVIVE     4  -------------------------------------------------+
  GOD         5  -------------------------------------------------+
  LUXURY      1  -----------------------+-------------------------+
  ADVENTURE   7  -----------------------+                         |
  CONNECT     6  -------------------------------------------------+
```

Figure 6. Agglomeration Schedule using Median Method

Stage	Clusters Combined Cluster 1	Cluster 2	Coefficient	Stage Cluster 1st Appears Cluster 1	Cluster 2	Next Stage
1	8	9	33.000000	0	0	2
2	8	11	96.666672	1	0	3
3	3	8	206.000000	0	2	6
4	1	7	328.500000	0	0	8
5	2	10	568.500000	0	0	6
6	2	3	876.833374	5	3	9
7	4	5	1214.333374	0	0	9
8	1	6	1593.833374	4	0	10
9	2	4	2202.125000	6	7	10
10	1	2	3665.272949	8	9	0

```
Dendrogram using Ward Method

                    Rescaled Distance Cluster Combine

   C A S E      0         5        10        15        20        25
  Label    Num  +---------+---------+---------+---------+---------+

  PEACE      8   -┐
  FAMILY     9   -┤-┐
  TASKS     11   -┘ ├-----┐
  LIFE       3   ---┘     ├-----------┐
  FUTURE     2   -------┬-┘           ├---------------------------┐
  LEARN     10   -------┘             │                           │
  SURVIVE    4   -----------┬---------┘                           │
  GOD        5   -----------┘                                     │
  LUXURY     1   ---┬----------┐                                  │
  ADVENTURE  7   ---┘          ├----------------------------------┘
  CONNECT    6   --------------┘
```

Figure 7. Agglomeration Schedule using Ward Method

Iteration history for the 2 dimensional solution (in squared distances)

Iteration	S-stress	Improvement
1	.06193	
2	.04776	.01417
3	.04000	.00776
4	.03468	.00532
5	.03114	.00354
6	.02886	.00228
7	.02730	.00156
8	.02622	.00108
9	.02543	.00079

Iterations stopped because
S-stress improvement is less than .001000

For matrix
Stress = .05641 RSQ = .98855

Configuration derived in 2 dimensions

Stimulus Coordinates

Stimulus Number	Stimulus Name	Dimension 1	Dimension 2
1	LUXURY	2.2842	.1105
2	FUTURE	-.5470	.6076
3	LIFE	-1.0964	.0649
4	SURVIVE	-.5844	-.7927
5	GOD	.6375	-1.3350
6	CONNECT	.9736	.8007
7	ADVENTURE	2.3358	-.0120
8	PEACE	-1.1657	-.0503
9	FAMILY	-1.1697	-.0138
10	LEARN	-.5417	.6328
11	TASKS	-1.1262	-.0128

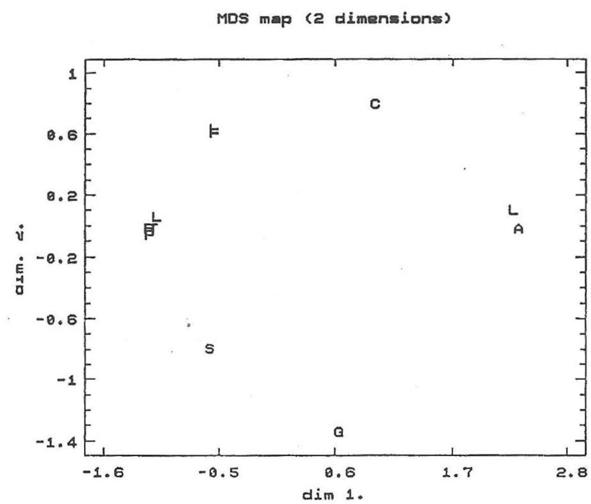

Figure 8. MDS (2 dimensional)

Iteration history for the 3 dimensional solution (in squared distances)

Iteration	S-stress	Improvement
1	.09491	
2	.03992	.05499
3	.02771	.01221
4	.02295	.00476
5	.01980	.00316
6	.01767	.00213
7	.01616	.00151
8	.01500	.00116
9	.01408	.00092

Iterations stopped because
S-stress improvement is less than .001000

Stress = .01326 RSQ = .99927

Configuration derived in 3 dimensions

Stimulus Coordinates

Stimulus Number	Stimulus Name	Dimension 1	2	3
1	LUXURY	2.6944	.2218	-.2013
2	FUTURE	-.6689	.7699	-.6127
3	LIFE	-1.3078	.0444	.1329
4	SURVIVE	-.6057	-1.0997	.3209
5	GOD	.7351	-1.4951	-.6863
6	CONNECT	1.1802	1.0056	.1871
7	ADVENTURE	2.8066	.0269	-.1268
8	PEACE	-1.4517	-.0748	.0810
9	FAMILY	-1.4292	-.0666	.1299
10	LEARN	-.5979	.6440	.6642
11	TASKS	-1.3550	.0236	.1112

Figure 9. MDS (3 dimensional)

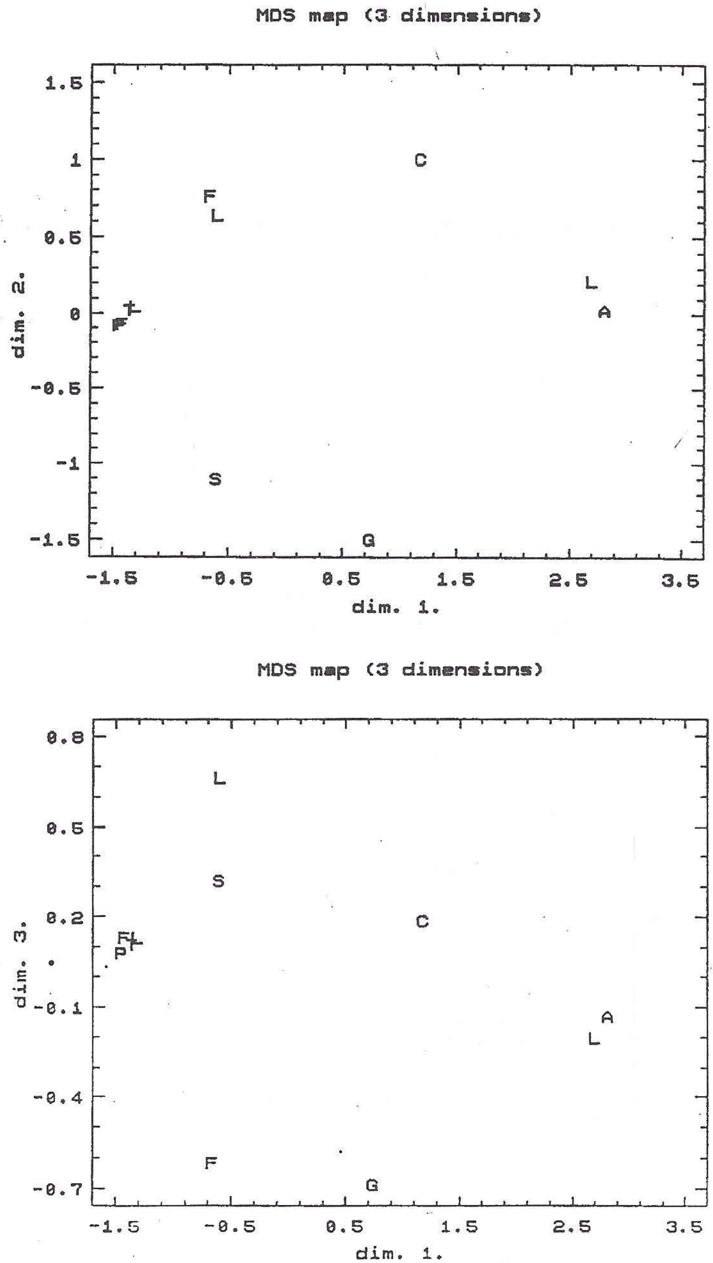

Figure 10. MDS map (3 dimensional)