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Electric energy losses minimization in induction motor speed control taking into consideration the electromagnetic transients

by

Leszek Kawęcki, Tadeusz Niewierowicz

Postgraduate Studies Section, National Polytechnical Institute, Mexico, D. F.

Abstract: The paper presents a method of solving the minimization problem of the electric energy losses during the starting and speed control of induction motors, taking into consideration the electromagnetic transients. To solve this problem, the Pontryagin maximum principle is used. A solution example using a digital computer is presented.

1. Introduction

The starting and speed control of induction motors can be carried out in different ways, but the easiest and most effective method is the frequency control. Changing simultaneously the frequency and the amplitude of motor supplied voltage, not only the speed variation of the motor, but also, among others, minimization of the electric energy losses in the stator and rotor windings can be obtained.

The solution method of the optimization problem depends to a considerable extent, on the complexity of the accepted mathematical model of the motor. A supposition, permitting a great simplification in the mathematical description of the induction motor is to neglect the electromagnetic transients in the motor.

With this supposition and some others, it is possible to find the general mathematical description of the optimal controller that minimizes the electric energy losses Kawecki, Niewierowicz (1988).

The purpose of this work is to find, using a mathematical model of the induction motor that takes into consideration the electromagnetic transients, a mathematical description of the optimal control of induction motors that guarantees minimization of electric energy losses during the starting or speed control.

The problem is solved using the Pontryagin maximum principle.

2. Mathematical model of the motor

The mathematical model of the motor has been based on the following assumptions:

- The supplied voltage is sinusoidal and symmetric.
- The induction motor is symmetric.
- The inductances and resistances are constant.
- The motor is working on the linear segment of its magnetization curve.
- The magnetic losses can be neglected.

Besides, it is assumed that the control signal is the stator current which takes the following shape (indirect control):

$$\vec{i}_1 = i_1(\cos\xi + j\sin\xi) \tag{1}$$

where:

$\vec{i}_1$ stator current vector on the $d-q$ axes, rotating synchronously with the rotor angular frequency;

i_1 absolute value of the stator current vector;

ξ angle between the stator current vector and "d" axis.

The equations that describe the two-phase equivalent induction motor on the $d-q$ axes, rotating synchronously with the rotor, are:

$$\begin{aligned} \frac{d\Psi_{2d}}{dt} &= -A\Psi_{2d} + Bi_1\cos\xi \\ \frac{d\Psi_{2q}}{dt} &= -A\Psi_{2q} + Bi_1\sin\xi \\ \frac{d\omega_r}{dt} &= Ci_1(\Psi_{2d}\sin\xi - \Psi_{2q}\cos\xi) - \frac{1}{J}M_o \end{aligned} \tag{2}$$

where:

$$\begin{aligned} A &= \frac{R_2',\omega_n}{X_o+X_2'} \\ B &= \frac{X_oR_2'}{X_o+X_2'} \\ C &= \frac{3}{2}p^2\frac{X_o}{X_o+X_2'}\frac{1}{J} \end{aligned} \tag{3}$$

R_1, R_2' resistances of the stator winding and of the rotor winding related to stator circuit, respectively;

X_0 magnetization reactance of the two-phase equivalent motor calculated for the nominal frequency of the stator current;

X_1 dissipation reactance of one phase of the stator winding of the two-phase equivalent motor, calculated for the nominal frequency of the stator current;

X_2' dissipation reactance of one phase of the rotor winding of the two-phase equivalent motor, related to the stator circuit, calculated for the nominal frequency of the stator current;

p number of pairs of poles;

J rotor inertia torque;

M_o load torque;

t time;

Ψ_{2d}, Ψ_{2q} rotor magnetic flux components on the $d-q$ axes;
ω_n nominal angular frequency of stator current;
ω_r rotor angular speed of the motor of the one pair of poles ($\omega_r' = \omega_r/p$ for the motor of p-pairs of poles).

3. Optimization index

The electric energy losses in the motor windings can be expressed by:

$$Q = \frac{1}{2}\int_0^{t_r} (i_1^2 R_1 + {i_2'}^2 R_2')dt \tag{4}$$

where:
Q electric energy losses in the stator and rotor windings;
i_2' absolute value of the rotor current vector;
t_r control time.

Using the known dependence between the inside motor variables Krause (1987), the rotor current absolute value can be expressed in terms of the stator current absolute value:

$$i_2' = \frac{\sqrt{(\omega_n \Psi_{2d} - X_o i_1 \cos\xi)^2 + (\omega_n \Psi_{2q} - X_o i_1 \sin\xi)^2}}{X_o + X_2'} \tag{5}$$

Substituting (5) in (4) we obtain:

$$Q = \frac{1}{2}\int_0^{t_r} \{i_1^2 R_1 + D[(\omega_n \Psi_{2d} - X_o i_1 \cos\xi)^2 + (\omega_n \Psi_{2q} - X_o i_1 \sin\xi)^2]\}dt \tag{6}$$

where:

$$D = \frac{R_2'}{(X_o + X_2')^2} \tag{7}$$

Equation (6) describes the optimization index.

4. Solution of the problem

The optimization problem consists in finding control (1) that minimizes the optimization index (6).

That means, it is necessary to find how the control variables i_1 and ξ must change with the time for minimization of functional (6).

To solve this problem, the Pontryagin maximum principle is applied Athans, Falb (1966).

Taking into consideration the motor mathematical model (2) and the optimization index (6), the hamiltonian takes the following shape:

$$
\begin{gathered}
H = -\tfrac{1}{2} i_1^2 R_1 - \tfrac{1}{2} D(\omega_n \Psi_{2d} - X_o i_1 \cos\xi)^2 - \tfrac{1}{2} D(\omega_n \Psi_{2q} - X_o i_1 \sin\xi)^2 + \\
+ Q_1(-A\Psi_{2d} + B i_1 \cos\xi) + Q_2(-A\Psi_{2q} + B i_1 \sin\xi) + \\
+ Q_3 C i_1 (\Psi_{2d} \sin\xi - \Psi_{2q} \cos\xi) - Q_3 \tfrac{1}{J} M_o
\end{gathered} \tag{8}
$$

where:

Q_1, Q_2, Q_3 variables conjugated with the state variables Ψ_{2d}, Ψ_{2q}, ω_r, respectively.

In accordance with the Pontryagin maximum principle, the optimal control variables should satisfy:

$$
\begin{aligned}
\frac{\partial H}{\partial i_1} &= 0 \\
\frac{\partial H}{\partial \xi} &= 0
\end{aligned} \tag{9}
$$

Taking into consideration the first equation in (9) and hamiltonian (8) we obtain:

$$
i_1 = (G\Psi_{2d} + KQ_1 - EL\Psi_{2q}) \cos\xi + (G\Psi_{2q} + KQ_2 + EL\Psi_{2d}) \sin\xi \tag{10}
$$

where:

$$
\begin{aligned}
G &= \frac{R_2' X_o \omega_n}{R_1 (X_o + X_2')^2 + R_2' X_o^2} \\
K &= \frac{X_o R_2' (X_o + X_2')}{R_1 (X_o + X_2')^2 + R_2' X_o^2} \\
L &= \frac{(X_o + X_2')^2}{R_1 (X_o + X_2')^2 + R_2' x_o^2}
\end{aligned} \tag{11}
$$

Considering (8), the second equation in (9) takes the following shape:

$$
\sin\xi(-M\Psi_{2d} - BQ_1 + E\Psi_{2q}) + \cos\xi(M\Psi_{2q} + BQ_2 + E\Psi_{2d}) = 0 \tag{12}
$$

where:

$$
M = \frac{R_2' X_o \omega_n}{(X_o + X_2')^2} \tag{13}
$$

Considering equation (12) as a scalar product of two vectors and taking into account that the hamiltonian should obtain the maximum value, one can write:

$$
\begin{aligned}
\sin\xi &= \frac{M\Psi_{2q} + BQ_2 + E\Psi_{2d}}{\sqrt{(M\Psi_{2q} + BQ_2 + E\Psi_{2d})^2 + (-M\Psi_{2d} - BQ_1 + E\Psi_{2q})^2}} \\
\cos\xi &= \frac{M\Psi_{2d} + BQ_1 - E\Psi_{2q}}{\sqrt{(M\Psi_{2q} + BQ_2 + E\Psi_{2d})^2 + (-M\Psi_{2d} - BQ_1 + E\Psi_{2q})^2}}
\end{aligned} \tag{14}
$$

Equations (10) and (14) describe, in an implicit form, the optimal control which guarantees minimization of the electric energy losses during the induction motor speed control.

To build the optimal control system, it is necessary to know the optimal control description in a explicit form, therefore, it is necessary to know how the amplitude and the frequency of the supplied voltage, change in function of time (open-loop control system) or in function of angular speed of the rotor (closed-loop control system).

Mathematically, it can be expressed as:

- for the open-loop control system:

$$\begin{aligned} u &= f_u(t) \\ \omega &= f_\omega(t) \end{aligned} \tag{15}$$

- for the closed-loop control system:

$$\begin{aligned} u &= {f'}_u(\omega_r) \\ \omega &= {f'}_\omega(\omega_r) \end{aligned} \tag{16}$$

where:
u supplied voltage amplitude;
ω angular frequency of the supplied voltage.

To find the direct control it is necessary to solve state equations (2) substituting i_1, $\sin\xi$ and $\cos\xi$ by (10) and (14), which implies the solution of the conjugated equations and the state equations together.

The shape of the conjugated equations depends on the type of load with which the induction motor works. Therefore, we should define the kind of load of the motor. For example, if we assume that the load torque is null or constant, the conjugated variables keep the following conjugated equations system:

$$\frac{dQ_1}{dt} = F(\omega_n\Psi_{2d} - X_o i_1 \cos\xi) + AQ_1 - Q_3 C i_1 \sin\xi \tag{17}$$

$$\frac{dQ_2}{dt} = F(\omega_n\Psi_{2q} - X_o i_1 \sin\xi) + AQ_2 + Q_3 C i_1 \cos\xi \tag{18}$$

$$\frac{dQ_3}{dt} = 0 \rightarrow Q_3 = \text{const} \rightarrow CQ_3 = \text{const} = E \tag{19}$$

where:

$$F = \frac{R_2'\omega_n}{(X_o + X_2')^2} \tag{20}$$

Taking into account (19), we can write:

$$\begin{aligned} \tfrac{dQ_1}{dt} &= F(\omega_n\Psi_{2d} - X_o i_1 \cos\xi) + AQ_1 - E i_1 \sin\xi \\ \tfrac{dQ_2}{dt} &= F(\omega_n\Psi_{2q} - X_o i_1 \sin\xi) + AQ_2 + E i_1 \cos\xi \\ CQ_3 &= E = \text{const} \end{aligned} \tag{21}$$

To find out how the control variables i_1 and ξ should be changed in function of time, it is necessary to solve equations (2) and (21), substituting in them i_1, $\sin\xi$ and $\cos\xi$ by (10) and (14), which implies the knowledge of the initial conditions of the conjugated variables:

$$Q_1(0), Q_2(0), Q_3(0) \cup E(0) = CQ_3(0) \tag{22}$$

Only the initial conditions of the state variables are known:

$$\Psi_{2d}(0) = \psi_{2d}, \Psi_{2q}(0) = \psi_{2q}, \omega_r(0) = \omega_i \tag{23}$$

where:
$\psi_{2d}, \psi_{2q}, \omega_i$ have null value at the starting of motor.

Besides, the final value of the rotor angular velocity is known, and should be nominal for the start of the motor or should obtain a required value for the speed control:

$$\omega_r(t_r) = \omega_d \tag{24}$$

where:
ω_d final value of the rotor angular velocity (wished) ($\omega_d = \omega_n/p$ for the starting of motor).

From the transversability conditions Athans, Falb (1966) the final values of the conjugated variables Q_1 and Q_2 can be calculated:

$$\begin{aligned} Q_1(t_r) &= 0 \\ Q_2(t_r) &= 0 \end{aligned} \tag{25}$$

As seen, the problem to be solved, is the two-point boundary value problem and it consists in finding initial values of the conjugated variables (22) knowing the initial values of state variables (23) and the final values (24) and (25). To solve this problem it is necessary to use a strategy for the change of the initial values (22).

Therefore, in the presented case, a general solution of the optimization problem is impossible. Only a particular solution for a specific induction motor can be obtained using a computer.

As a result, the indirect mathematic description of the optimal control for a specific motor can be obtained:

$$\begin{aligned} i_1 &= f_i(t) \\ \xi &= f_\xi(t) \end{aligned} \tag{26}$$

Therefore, basing on (26), only the open-loop system control (15) can be found.

The mathematical equations that describe the direct control variables (u and ω) in function of the indirect control variables (i_1 and ξ) are Schreiner, Gildebrand (1973), Krause (1987):

$$\omega = \omega_r + \beta\omega_n \tag{27}$$

$$\beta = \cos\xi \frac{d\sin\xi}{dt} - \sin\xi \frac{d\cos\xi}{dt} \tag{28}$$

$$u_d = Ni_1\cos\xi - Pi_1\omega\sin\xi - M\Psi_{2d} - S\omega_r\Psi_{2q} + P\frac{di_1}{dt}\cos\xi \tag{29}$$

$$u_q = Ni_1 \sin\xi + Pi_1\omega\cos\xi - M\Psi_{2q} + S\omega_r\Psi_{2d} + P\frac{di_1}{dt}\sin\xi \tag{30}$$

$$u = \sqrt{u_d^2 + u_q^2} \tag{31}$$

where:
$\beta = \frac{\omega-\omega_r}{\omega_n}$ relative slip;

$$\begin{aligned} N &= R_1 + \tfrac{R_2' X_o^2}{(X_o+X_2')^2} \\ P &= \tfrac{(X_o+X_1)(X_o+X_2')-X_o^2}{(X_o+X_2')\omega_n} \\ S &= \tfrac{X_o}{X_o+X_2'} \end{aligned} \tag{32}$$

Knowing the initial conditions of the conjugated variables (20), and after solving the two-point boundary value problem, state equations (2) and conjugated equations (13) can be solved, applying indirect optimal control (15) and (19) and calculating, during the solution, the values of the direct control (equations (27)-(31)).

5. Results comparison

Comparing the results obtained in this paper with the optimal control in the closed-loop system obtained for the mathematical model of induction motor without the electromagnetic transients which description is Kawecki, Niewierowicz (1988):

$$\alpha = \nu + \beta \tag{33}$$

$$\beta = R_2'\sqrt{\frac{R_1}{A_1}} \tag{34}$$

$$\gamma = \begin{cases} \frac{i_1^o}{u_m}\sqrt{\frac{F_1R_1}{A_1+R_1(X_o+X_2')^2}} & \text{for} \quad i_1^o < i_{omax}\sqrt{\frac{A_1+R_1(X_o+X_2')^2}{A_1+R_1{X_2'}^2}} \\ \frac{i_{omax}}{u_m}\sqrt{\frac{F_1R_1}{A_1+R_1{X_2'}^2}} & \text{for} \quad i_1^o >= i_{omax}\sqrt{\frac{A_1+R_1(X_o+X_2')^2}{A_1+R_1{X_2'}^2}} \end{cases} \tag{35}$$

where:

$$\begin{aligned} F_1 \quad = \quad & \left\{[X_o - (X_o+X_2')(X_o+X_1)]\alpha + \frac{R_1R_2'}{\beta}\right\}^2 + \\ & \left[R_1(X_o+X_2') + \frac{R_2'\alpha}{\beta}(X_o+X_1)\right]^2 \end{aligned} \tag{36}$$

$$A_1 = R_2'X_o^2 + R_1(X_o+X_2')^2 \tag{37}$$

i_{omax} maximum value of the magnetizing current amplitude for which the motor operates in the linear part of its magnetizing curve;
i_1^o maximum admissible value of the stator current amplitude i_1;
u_m nominal value of the supplied voltage amplitude;
$\alpha = \frac{\omega}{\omega_n}$ relative angular frequency of the stator current;
$\gamma = \frac{u}{u_m}$ relative amplitude of the supplied voltage;
$\nu = \frac{\omega_r}{\omega_n}$ relative angular velocity of the rotor.

we can conclude then, that the electromagnetic transients cause the variations with the time of the relative slip β (28). This slip is constant when the electromagnetic transients are neglected (34). The curve shapes of the relative slip and other variables of the control (ω and u or α and γ) depend on the induction motor parameters. To determine these shapes we must solve the optimization problem as it is showed in the preceding chapter. Then, the form of the control variable curves may be different for the different motors. When the induction motor mathematical model is without the electromagnetic transients, the angular frequency of the control voltage (ω, α) is a linear function (33) and the amplitude of this voltage (u, γ) is almost a linear function (35) of the motor speed, then, because the electromagnetic torque of the motor during the control is constant Kawecki, Niewierowicz (1988), both variables are an aperiodic (almost linear) function of the time.

6. Example of the solution

To illustrate the solution method of the electric energy losses minimization during speed control of induction motors a numerical example is presented. In this example the idle starting ($M_o = 0$) of a grinding electrospindle is presented. The parameters of the grinding electrospindle are:

$$\begin{aligned} m &= 3,\ p = 1,\ u_m = 187.79[\mathrm{V}], \\ \omega_n &= 9420[\mathrm{rd/s}],\ R_1 = 0.8[\Omega],\ R_2' = 1[\Omega], \\ X_1 &= X_2' = 4.1[\Omega],\ X_o = 53.7[\Omega], \\ J &= 8.18 \times 10^{-6}[\mathrm{kgm}^2] \end{aligned}$$

where:
m number of the phases of the motor.

To solve the two-point boundary value problem, the parametric optimization algorithm was applied Kawecki, Niewierowicz (1991).

The initial condition for the conjugated variables found by PC-AT computer, for the motor parameters mentioned above are:

$$\begin{aligned} Q_1(0) &= -1.7167 \times 10^{-2} \\ Q_2(0) &= -7.862 \times 10^{-2} \\ E(0) = 202.413 \cup Q_3(0) &= 1.188 \times 10^{-3} \end{aligned}$$

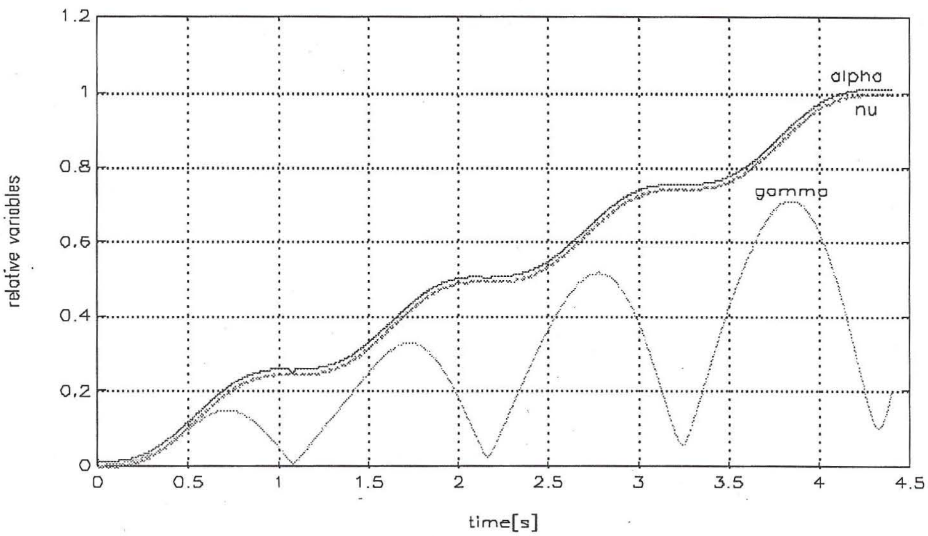

Figure 1. Control and controlled variables

Using the PC-MATLAB software package, equations (2), (13) with indirect control (15),(19) were solved. The direct control was calculated, using formulas (18) - (31) for the motor parameters and the initial conditions for the conjugated variables presented above. Additionally, the electric energy losses were calculated too, according to equation (6).

The results of the calculations are presented in the Figures 1 and 2.

The values of the starting time t_r and the electric energy losses Q obtained during the starting are:

$$t_r = 4.37[s]$$
$$Q = 11.192[J]$$

For comparison we present the calculation results obtained for the idle starting of the same motor for:

- the optimal control obtained with the mathematical model of induction motor without the electromagnetic transients:

$$t_r = 1.657[s]$$
$$Q = 11.188[J]$$

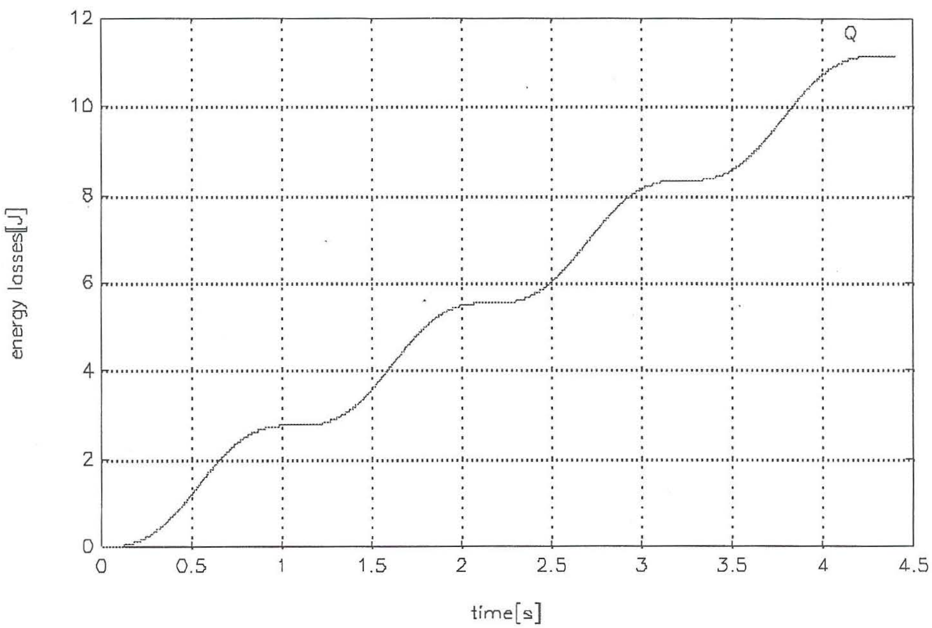

Figure 2. Electric energy losses

- ordinary starting

$$t_r = 0.616[\mathrm{s}]$$
$$Q = 238.316[\mathrm{J}]$$

7. Conclusions

This paper shows, that it is possible to find analytically the indirect form of the frequency control, which minimizes the electric energy losses during the starting or speed control of induction motors, taking into consideration the electromagnetic transients.

To find the direct description (explicit description form) of this control it is necessary to solve the two-point boundary value problem for a specific motor, using a computer.

The solution obtained in the paper can serve to build the optimal open-loop control system of the specific motor.

In the present stage of microprocessor speed development, the results obtained in this paper may be applied to the induction motor starting in the open-loop control system, generating previously the voltage or current amplitude and frequency control curves by a computer.

The simulation results, based on the optimal control described in this paper, may be also used to evaluate the other practical control systems. For that it is sufficient to compare the results of two control system simulations: optimal and evaluated.

For the investigated motor we can conclude then:

I. the electromagnetic transients cause that:
   a) the optimal control variables (α, γ) have the oscillation (Fig.1)
   b) the starting time is longer (above 164%)
   c) the electric energy losses do not change practically,

II. the optimal control of the motor starting which takes into consideration the electromagnetic transients, comparing with ordinary starting:
   a) decreases considerably the electric energy losses (this losses in the optimal starting are about 4.7% of the ordinary starting losses, then the energy advantages are very large)
   b) increases considerably the starting time (about 609%).

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