

Multicriteria decision support in negotiations

by

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Abstract: The paper deals with negotiation support in decision situations described by different game models, such as bargaining problem, noncooperative games, cooperative games without side payments, in case of multicriteria outcomes for players. A methodology is presented linking modern approaches of multicriteria decision analysis and new solution concepts to the multicriteria games. The methodology is developed as a base for construction of computer based systems supporting players in analysis of decision situations and aiding in finding a consensus in negotiations.

Keywords: multicriteria decision making, decision support systems, negotiations, bargaining problem.

1. Introduction

The paper deals with methods and techniques of negotiation support, in particular mediation support, with use of a computer based decision support systems. Multi-actor decision situations are considered which can be characterized in the following way:

- There are several parties called players further on.
- Each of them has multiple objectives and their interests conflict.
- It is assumed that a mathematical model is given (called a substantive model of the game) which describes the set of admissible decisions of the players and allows calculation of their payoffs as dependent on assumed values of the decision variables. The payoffs are measured by multiple criteria, in general different for each player. The model can relate to different decision situations described by different class of games, e.g. bargaining problem, noncooperative games, cooperative games without or with side payments.
- Due to practical reasons utility functions of the players are not assumed to be explicitly given. It is assumed, however, that the players have in mind preferences over the criteria.

The decision support is discussed in two aspects:

- in unilateral analysis of the problem: aiding particular player in the analysis of the game (learning the nature of the problem i.e. learning about possible decision options and consequences, learning the player's preferences),
- in multilateral analysis and mediation i.e. aiding all the players in joint analysis of the game and aiding in reaching the consensus, i.e. in finding the solution which will be accepted by all the players.

The paper reviews results obtained during the research on the subject carried out at the Systems Research Institute by Lech Krus (leader of the project), Piotr Bronisz and Bożena Lopuch. The collaboration with the International Institute for Applied System Analysis (IIASA) in Austria, in particular with the Systems and Decision Science Program, Project for Methodology of Decision Analysis, as well as with the Institute of Automatic Control, Warsaw University of Technology played an important role in the studies. Participation in the IIASA PIN Network of institutions and researchers dealing with methodology and practice of international negotiations, established within the PIN Project was also very useful. The research is still continued within research project in the Systems Research Institute of Polish Academy of Sciences and also supported by State Committee of Scientific Research in Poland (grant no. 8S505 009 904).

The following reasons motivated the studies when started in 80's:

- The development of multicriteria decision support methods and systems for problems with a single decision maker was observed (see, in particular, Wierzbicki, 1982, 1986; Sawaragi, Nakayama, Tanino, 1985; Steuer, Choo, 1983). It was natural to ask the question: why not use the analogous approach in case of multi-actors problems.
- The gaming - as opposed to game theory - was widely used in support of learning about conflict situations. Also the first computer based group decision support systems were constructed, see for example Korhonen, Moskowitz, Wallenius, Zionts, 1986; Jarke, Jelassi, Shakun, 1987; Kersten, 1988; DeSanctis, Gallupe, 1987.
- The most of attention in game theory was given to normative, axiomatic approach under assumptions that players' payoffs are characterized by utility functions. The normative approach could hardly be applied in decision support. Increasing reservations as to the practical applicability of utility theory as a basis for decision making were observed. They implied the importance of interactive procedures in decision support and the need for broadening of the normative approaches.
- Research results within the holistic decision making (see Dreyfus, 1985) indicated the need of considering analytic tools in decision analysis as a support in "learning to make decision".
- Research results obtained within evolutionary rationality (e.g. Axelrod, 1985) indicated the importance of an evolutionary development of cooperative strategies and the need of broadening the normative approach.

The technique proposed consists in application of interactive learning procedures linking modern approaches of multicriteria optimizations and new solution concepts of game theory. The general scope of the presented research is following:

- development of axiomatic theory of bargaining in case of multicriteria payoffs of players, in particular generalization of existing solution concepts, new iterative solution concepts, learning procedures supporting analysis and mediation in decision situations described as bargaining problem,
- construction of an experimental software,
- construction of exemplary models illustrating the bargaining problem,
- gaming experiments with use of the experimental software,
- development of noncooperative game theory in case of multicriteria outcomes of players,
- development of the theory of multicriteria cooperative games without side payments.

The research has been inspired by Raiffa's monograph on: *Art and science of negotiations* (Raiffa, 1982), methods of multicriteria optimization and support (Wierzbicki, 1982, 1986), ideas of decision support in conflicts (Wierzbicki, 1983), papers by Axelrod (1985), Dreyfus (1985), results within modelling and optimization of international cooperation (Ameljańczyk, 1979, Piasecki, Hołubiec and Ameljańczyk, 1982).

2. General formulation of the problem

The problem is formulated for the case of n players. Each player has some number of decision variables and some assumed criteria measuring his payoffs. The decision variables of the player i ($i = 1, 2, \dots, n$) form the vector $z^i = (z_1^i, z_2^i, \dots, z_{k^i}^i)$, where k^i is number of variables. The payoff of the player i is measured by the vector of criteria $x^i = (x_1^i, x_2^i, \dots, x_{m^i}^i)$, where m^i is the number of criteria. The vector of decision variables of all the players is $z = (z^1, z^2, \dots, z^n) \in R^K$, $K = \sum_i k^i$, where R^K is the multidimension space (of all the players). The vector of payoffs is $x = (x^1, x^2, \dots, x^n) \in R^M$, $M = \sum_i m^i$, where R^M is the space of all the criteria of all the players.

We assume that a substantive model of the game is given by the set of admissible decisions Z_0 and by an outcome mapping P . The set $Z_0 \subset R^K$ is assumed to be compact, and the mapping $P : Z_0 \rightarrow R^M$ to be continuous. In such a case the set of attainable outcomes denoted by Q_0 is compact. The set of outcomes Q_0 of the model defines attainable payoffs of the players.

In the multicriteria payoffs space of the players a partial ordering is introduced in standard way with use of the cone D .

$$D = \{x \in R^M : \begin{aligned} &x_j^i \geq 0, \quad j = 1, 2, \dots, m^i, \\ &x_j^i \leq 0, \quad j = m^i + 1, \dots, m^i, \quad \text{for } i = 1, 2, \dots, n \}, \end{aligned}$$

where, respectively, the objectives to be maximized are indexed by $j = 1, 2, \dots, m'^i$, and to be minimized — by $j = m'^i + 1, \dots, m^i$.

With use of the cone the Pareto and weakly Pareto optimal elements of the set Q_0 are defined.

Pareto optimal (nondominated) elements \hat{q} of the set Q_0 are defined as the elements which belong to the set:

$$\hat{Q}_0 = \{ \hat{q} \in Q_0 : Q_0 \cap (\hat{q} + D \setminus \{0\}) = \emptyset \}.$$

Weakly Pareto optimal (weakly nondominated) elements \hat{q}^w of the set Q_0 are defined as the elements which belong to the set:

$$\hat{Q}_0^w = \{ \hat{q} \in Q_0 : Q_0 \cap (\hat{q} + \text{int } D) = \emptyset \}.$$

The set of attainable payoffs is not given explicitly. With use of the model implemented on computer one can derive some elements of the the set for given decision variables. The model is assumed to be a base for decision analysis carried out by the players with the use of a decision support system.

In the following sections the research and results obtained will be characterized in relation to different decision situations described by different classes of games.

3. Multicriteria bargaining problems

The problem is defined for n players by a pair (S, d) where $S \subset Q_0 \subset R^M$ is the so called agreement set, and d is a disagreement point, called also status quo point. The agreement set defines the payoffs of the players in case of their unanimous agreement. If the agreement is not reached the payoffs are defined by the point d . The bargaining problem describes a decision situation in which players cooperating in realization of, for example, a joint project can obtain some benefits. The agreement set describes benefits of the players in case of cooperation in comparison to the status quo describing their payoffs in the case when each player acts independently. The problem consists in finding reasonable solution - a point in the set S accepted by all the players and close to their preferences. The solution is looked for in the form of negotiations. It is assumed that there is a mediator aiding the negotiations. Let us note that each player deals with a multicriteria decision making problem. He is looking for the solution possibly close to his preferences in his multicriteria payoff space. On the other hand the players' payoffs are mutually dependent. There is a problem of proper allocation of benefits being the result of cooperation.

The bargaining problem in the case of two players is illustrated in Figs. 1 and 2, under assumptions that the players maximize all criteria. In the case of classical bargaining problem (Fig.1) the players' payoffs are measured by utilities u_1, u_2 , i.e. are unicriterial. The ideal point I is defined by the maximal attainable utilities of the players. In the classical bargaining theory (Nash, 1950;

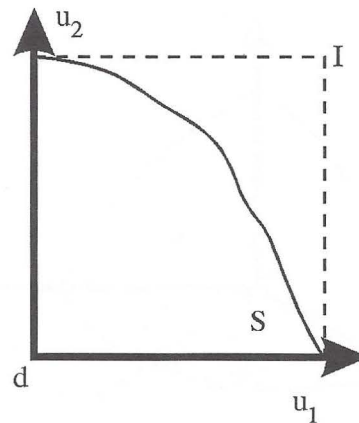


Figure 1.

Raiffa, 1953; Kalai, Smorodinsky, 1975; Roth, 1979; Thomson, 1980, and others) some axioms are formulated describing the assumed behavior of the players and their feeling of fairness. On the basis of the axioms different solution concepts are formulated and analyzed. Figure 2 illustrates a multicriteria bargaining problem with two players in which the first player has two criteria x_{11} , x_{12} . In comparison to the classical case, the first player has the additional problem of choosing a solution in his multicriteria space of payoffs. He has not one maximal value but a set of nondominated payoffs (a boundary of the set S) in the hyperplane defined by the coordinates x_{11} , x_{12} . The boundary is not given explicitly but by relations of the analytical model. The multicriteria decision analysis is useful in selection of the payoff being close to the preferences of the players. The final solution can be reached, however, under agreement of all the players. Axiomatic approach of the classical bargaining theory is very useful in looking for such a solution. However, the solutions of the classical theory are not transferred to the multicriteria case in a straight way. Therefore a theoretical research dealing with the generalization of the classical theory was really necessary.

The theory of multicriteria bargaining problem has been developed in our research as a basis for construction of interactive procedures supporting the players in analysis and negotiations.

Main results are as follows:

- New concept of utopia point related to the players' aspirations in the space of the players' multicriteria outcomes has been introduced.
- Some solution concepts for bargaining problem have been generalized on multicriteria case: the Raiffa – Kalai – Smorodinsky solution, the Imai solution and other concepts. The generalized solutions have been axiomatically characterized, and their properties have been analyzed.
- New iterative solution concept has been introduced based on the Fandel –

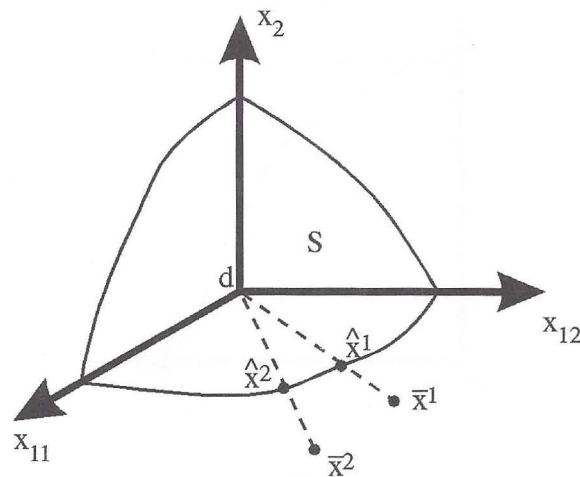


Figure 2.

Wierzbicki confidence principle and assumptions of the players' rationality.

- The iterative solution was the basis for construction of an interactive mediation procedure supporting the players in finding an agreeable Pareto optimal solution in the set of admissible outcomes. Convergence of the procedure has been proved. The iterative solution used in the procedure has very useful properties. In particular it is manipulation free, i.e. the players cannot benefit by cheating.

The results are presented in Kruś (1991a,1996), Kruś, Bronisz (1992a), Bronisz, Kruś, Wierzbicki (1989), Bronisz, Kruś (1988,1989).

Looking for a solution in the multicriteria bargaining problem we have to consider jointly two decision problems: the first one – the solution should be related to the preferences of all the players, the second one – the solution should fulfill some basic fairness rules. The theory developed in the paper is thought as a background for construction of decision support systems aiding the players in both the decision problems. The first decision problem relates directly to multicriteria decision making carried out by each of the players. An application of reference point approach (Wierzbicki, 1982, 1986) is proposed. Considering the second decision problem, solution concepts of the multicriteria bargaining problem are proposed satisfying properties (called axioms) that could be accepted by rational players.

The reference point approach is assumed to be applied by each player independently. According to the approach the player specifies the reference points in his multicriteria space (for example points \bar{x}^1, \bar{x}^2 in Fig.2), and the computer based system responds with nondominated points (\hat{x}^1, \hat{x}^2) being close to the reference points. Using the approach the player can explore the set of

nondominated points in his multicriteria space of payoffs. It is assumed that the exploration should be finished with a selection of the nondominated payoff preferred by the player.

The **utopia point relative to the players aspirations** (RA utopia) is defined as a composition of the preferred nondominated payoffs selected by all the players. The RA utopia point has been used for construction of new solutions concepts and interactive learning procedures.

To illustrate the theoretical results the concept of iterative solution is presented. For simplicity of notation, without loss of generality, it is assumed that all the criteria of the players are maximized.

Let a multicriteria bargaining problem (S, d) satisfy the following set of conditions B :

- S is compact, there is $x \in S$ such that $x \gg d$.
- S is comprehensive, i.e. for $x \in S$ if $d \leq y \leq x$ then $y \in S$.
- For any $x \in S$, let $Q(S, x) = \{k \in M : y \geq x, y_k > x_k \text{ for some } y \in S\}$. Then for any $x \in S$, there exists $y \in S$ such that $y \geq x, y_k > x_k$ for each $k \in Q(S, x)$.

It is assumed that the solution of the multicriteria bargaining problem (S, d) is looked for in some number of rounds (iterations) $t = 1, 2, \dots, T$, in which payoffs $d^t \in S$ are determined. The final, admissible and accepted by the players, payoff d^T is the solution of the problem.

The following postulates that should be fulfilled by the process $\{d^t\}$, $t = 1, 2, \dots, T$, are proposed:

- P1. The process starts at the disagreement point, and all outcomes belong to the agreement set, i.e. $d^0 = d$, $d^t \in S$ for $t = 1, 2, \dots, T$,
- P2. The process is progressive, i.e. $d^t \geq d^{t-1}$ for $t = 1, 2, \dots, T$,
- P3. The final outcome is Pareto optimal, i.e. d^T ($= \lim_{t \rightarrow \infty} d^t$ if $T = \infty$) is a Pareto optimal point in S .
- P4. *Principle of α -limited confidence.* Let $0 < \alpha_i^t \leq 1$ be a given confidence coefficient of the i -th player at round t . Then, acceptable demands are limited by:

$$d^t - d^{t-1} \leq \alpha_{\min}^t [u(d^{t-1}) - d^{t-1}]$$

for $t = 1, \dots, T$, where α_{\min}^t is a minimal confidence coefficient at the round t , $\alpha_{\min}^t = \min \{\alpha_1^t, \dots, \alpha_n^t, \alpha_{\max}^t\}$,
 $\alpha_{\max}^t = \max \{a \in R : d^{t-1} + a[u(d^{t-1}) - d^{t-1}] \in S\}$,
 α_{\max}^t defines a maximum value of the confidence coefficient at the round t , resulting from the requirement that the outcome d^t should belong to the set S .

$u(d^{t-1})$ is the RA utopia point which reflects the preferences of the players in the subset $\{x \in S : x \geq d^{t-1}\}$ of the set S .

- P5. *Principle of rationality.* Each player is assumed to behave in rational way, trying to maximize his outcomes in particular rounds according to his preferences expressed with the use of RA utopia points. It is assumed that at each round t , each player i explores a set of his individually

nondominated points in the set $S^t = \{x \in S : x \geq d^{t-1}\}$, and defines his preferred point $x^{it}, i \in N$. Let $u(d^{t-1})$ denote RA utopia point of the set S^t defined on the basis of the preferred by the players individually nondominated points in the round t , i.e.

$$u(d^{t-1}) = (u_1(d^{t-1}), u_2(d^{t-1}), \dots, u_n(d^{t-1})), \quad u_i(d^{t-1}) = x_i^{it}, \quad i \in N.$$

Formally, the players' rationality is formulated as follows:

For any d^t , at each round t , there is no such an outcome $x \in S$, $x > d^t$ that fulfills the condition $x - d^{t-1} \leq \alpha_{\min}^t * [u(d^{t-1}) - d^{t-1}]$.

THEOREM 3.1 *For any multicriteria bargaining problem (S, d) satisfying the conditions B and for any confidence coefficients α_i^t such that $0 < \varepsilon \leq \alpha_i^t \leq 1$, $t = 1, 2, \dots, T$ there is a unique process d^t , $t = 0, 1, \dots, T$, $T \leq \infty$, satisfying the postulates P1, P2, P3, P4, P5. The process is defined as follows:*

$$\begin{aligned} d^0 &= d, \\ d^t &= d^{t-1} + \alpha_{\min}^t * [u(d^{t-1}) - d^{t-1}] \quad \text{for } t = 1, 2, \dots, T \end{aligned}$$

where T is a minimal number t for which $d^t = d^{t-1}$ or $T = \infty$.

The proof can be found in Kruś, Bronisz (1992a).

The process d^t , $t = 0, 1, \dots, T$ is called the iterative solution concept. The concept has been proposed under inspiration of the single negotiation text procedure originally proposed by Roger Fisher and described by Raiffa (1982), and of the principle of limited confidence Fandel (1979) and Fandel, Wierzbicki (1985).

The solution has very interesting and important properties. It is independent of affine transformation of criteria. It assures anonymity of players, i.e. it does not depend on the order of the players. The process is manipulation free.

4. MCBARG system supporting multicriteria bargaining

The obtained theoretical results, in particular the described concept of iterative solution and the interactive mediation procedure formulated according to the concept, formed the basis for construction of the MCBARG computer based system. It is a system supporting decision analysis in situations described by multicriteria bargaining problem. A general scheme of the interactive mediation procedure is presented in Fig. 3. It is assumed that the analytical model is defined in the space of multicriteria payoffs of all the players by a given status-quo point and by a system of inequalities describing the agreement set. The system enables generation and edition of the analytical model, analysis of the bargaining problem, and performing interactive negotiation session.

The interactive negotiation session is performed in some number of rounds according to the iterative solution concept. In each round the system supports:

- Initial multiobjective analysis of the bargaining problem. It gives an information about bounds of payoffs. It calculates an example of cooperative solution – the so called neutral solution.

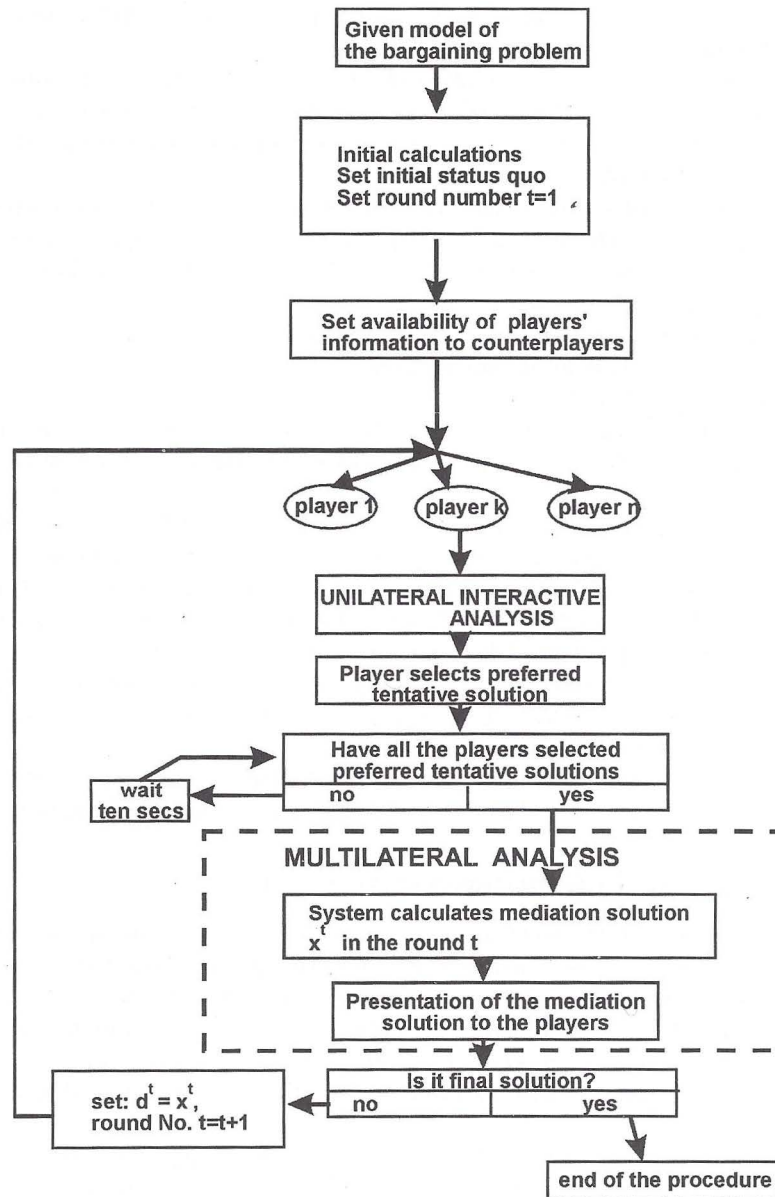


Figure 3. MCBARG system - a general scheme of the mediation procedure

- Unilateral, interactive analysis based on reference point approach. It is made in the form of learning procedure. For reference points and confidence coefficients assumed by the particular player the system generates anticipated cooperative payoffs.
- Multilateral support, calculation of single text mediation proposals. According to the preferences of all the players the system derives the cooperative solution d^t of the round t , treated as a mediation proposal. The solution is analyzed by the players.

The system is used under supervision of a system analyst who generates the model and conducts the session. During the session the players interact with the system. The system is operated with use of a "menu" system. It has built-in documentation and "help" option.

The main menu includes the following options:

INFORMATION – built-in documentation,

MODEL – editor of the model,

NEGOTIATION – activating the interactive negotiation-mediation session,

OLD SESSION – viewer of the results of all the performed and saved sessions.

Two tutorial examples of multicriteria bargaining models have been elaborated related to cooperation of agricultural farms, and to international negotiations on the acid rains problem. The second one has been constructed under inspiration of RAINS model of acidification elaborated at IIASA, and described by Alcamo, Shaw, Hordijk (1990). Some number of experiments with human players have been performed confirming applicability of the proposed approach.

The system works on IBM PC or compatible computers. It is distributed by IIASA and used for educational purposes.

The system documentation is given in Kruś, Bronisz, Łopuch (1990). The exemplary models are presented in Kruś (1992), Kruś, Łopuch, Bronisz (1989).

5. Multicriteria noncooperative games

The research deals with decision support problems in the case of conflict situations which can be described as noncooperative games. The theory of noncooperative games has already been intensively developed by Nash (1951), Arrow, Debreu (1954), Arrow, Hurwicz (1958), Aubin (1979) and many other researchers, as a base for analysis of conflict situations, under the general assumption that the players outcomes are measured by one-dimensional goal functions.

In our research the problem of decision analysis and support in case of multicriteria outcomes of the players has been undertaken. According to the general formulation of the problem (see point 2), a set of n players is considered. With each of the players a strategy space R^{k_i} is associated, where strategies are defined by the vector of decision variables z^i (i is the number of the player). The multicriteria noncooperative game is described by a multistrategy set defined

by the set Z_0 of admissible decisions of all the players and by the multicriteria outcome mapping P . In the research we follow ideas and results obtained by Wierzbicki (1985, 1990).

The scope of the research and obtained results include:

- Formulation of the n -person multicriteria noncooperative games in strategic form and definition of the noncooperative and weak noncooperative equilibria.
- Theorem on existence of equilibria in multicriteria games. It has been shown under assumptions of convex and compact set of multistrategies and continuity and concavity of gain functions of the players. In the proof the Ky-Fan theorem has been utilized.
- Analysis of properties of the equilibria. It is typical that there exists not the unique equilibrium but a set of equilibria. It has been shown that the analysis of a multicriteria noncooperative game cannot be replaced by the analysis of the game in which each criterion of every player is treated as a "player" in a classical noncooperative game. The sets of the equilibria in the two classes of games are different.
- Analysis of relations between the multicriteria game and a related classical game obtained with use of independent scalarization of the objectives of the players. It has been shown that if the multistrategy is the Nash equilibrium in the scalarized game, then it is also an equilibrium in the multicriteria game; in case of linear scalarization (with use of weights), and in more general case, for any scalarizing functions having properties of strong (strict) monotonicity.
- Construction of a two-criteria duopoly model. It is an extension of the classical duopoly model presented by Aubin (1979). With use of the model, features of the multicriteria noncooperative game and the theoretical results have been illustrated. Some number of noncooperative equilibria in multicriteria game have been calculated with use of the scalarization. It has been also shown that the equilibria can be not Pareto optimal in the set of attainable outcomes.

Decision support problems in case of noncooperative multicriteria games have been discussed. The features of the game and its equilibria have special significance for construction of decision support systems. The decision support systems in this case are considered as tools: first - supporting the players in analysis of the game: in analysis of noncooperative equilibria and possible conflict escalation, and second - aiding selection of a mutually acceptable, cooperative, Pareto optimal outcome. Application of the reference point approach of multicriteria optimization and bargaining, interactive mediation procedures seems to be useful in construction of the systems, further theoretical research is, however, required. Obtained results are presented in Kruś, Bronisz (1992b, 1994).

6. Multicriteria cooperative games without side payments

Cooperative games without side payments describe another decision situation, in which players can cooperate and create different coalitions to obtain some benefits. The classical game theory has been also developed under general assumptions that the players outcomes are measured by one-dimensional goals.

Our research deals with n -person cooperative games without side payments in the case of vector payoffs of players. The theory of cooperative games is developed as a basis for construction of interactive procedures of decision support. The scope of the research and obtained results include:

- Formulation of multicriteria cooperative games without side payments.
- Formulation and analysis of general concepts, such as core, excess function, nucleolus. The concepts are a generalization of the classical solution concepts on the multicriteria case.
- Theorems recording the existence of the core and the nucleolus, and also relations between the solution concepts.
- Proposition of new parametric excess function and nucleolus solution concept based on relative utopia concept. The excess function is formulated as dependent on "reference points" of the players. It is shown that the nucleolus generated by the excess function is invariant on affine transformations of criteria. It coincides with the Schmaidler (1969) nucleolus in case of unicriteria games without side payments. It coincides with the generalized Raiffa-Kalai-Smorodinsky solution concept proposed by Krusí and Bronisz (1992a) in the case of multicriteria bargaining problem. In the case of classical bargaining problem it coincides with the solution concept formulated by Raiffa (1953) and axiomatized by Kalai, Smorodinsky (1975).

Further research includes formulation of an interactive procedure to support players in analysis of the multicriteria cooperative game and in selection of an agreeable outcomes, consistent to the players preferences. The proposed nucleolus seems to be a good candidate for selection of such payoffs. The results are presented in Krusí, Bronisz (1996).

7. Final remarks

The methodology developed within this study can be characterized in the following points:

- The model-based approach is used in decision support. An analytical "substantial" model of the problem is constructed first, and implemented in a computer decision support system used for decision analysis. The model does not include preferential structure of the decision makers.
- The decision support system is considered as a tool supporting first of all learning to make decision.
- The system should assure sovereignty of the decision maker.

- The reference point approach of multicriteria optimization is utilized.
- New iterative solution concepts of multicriteria game theory are applied.
- The decision support in multi-actors problems deals with unilateral analysis, as well as with multilateral analysis and mediation support.

The research on decision support in multicriteria multi-actor problems is being developed for different classes of decision situations modelled by different classes of games: multicriteria bargaining problem, multicriteria noncooperative games, cooperative games without side payments.

In the case of multicriteria bargaining problems the research included necessary extensions of the theory, formulation of new solution concepts, construction of interactive procedures supporting unilateral and multilateral decision analysis as well as a mediation in negotiations. The prototype software - MCBARG system has been constructed on the basis of theoretical results. Exemplary models have been elaborated related to international negotiations on acid rains problem and to cooperation of agricultural farms. Some experiments with human players are conducted.

In the cases of decision situations described as multicriteria noncooperative games and described as multicriteria cooperative games without side payments the results are mainly methodological. The methodology is developed first of all as a basis for construction of computer systems supporting negotiations. The presented direction of research seems to be fruitful in further studies.

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