Control and Cybernetics

vol. **26** (1997) No. 2

Audit decisions using belief functions: a review[1]

by

Rajendra P. Srivastava

Ernst & Young Center for Auditing Research and Advanced Technology
Division of Accounting and Information Systems,
School of Business, The University of Kansas,
Lawrence, KS 66045, USA,
E-mail: rajendra@falcon.cc.ukans.edu

Abstract: This article provides an overview of the audit process along with the belief-function approach to audit decisions. In particular, the article highlights the advantages of using belief functions for representing uncertainties in audit evidence and discusses the audit risk model of the American Institute of Certified Public Accountants as a plausibility model. Also, the article discusses the use of belief functions to represent the strength of audit evidence under various situations: positive evidence, negative evidence, mixed evidence, evidence bearing on one variable, and evidence bearing on more than one variable with the same or different level of support. The article discusses the process of audit planning and evaluation under belief functions in a complex situation with all the interdependencies among the audit evidence, and among the assertions and related accounts. Finally, the article discusses how considering the network structure of audit evidence and integrating statistical and non-statistical items of evidence objectively will lead to an efficient audit.

Keywords: belief functions, evidence aggregation, audit decision, uncertainty

1. Introduction

The main purpose of this article is to provide an overview of the audit process along with a belief-function approach to audit decisions. In particular, I plan to discuss the following: (1) the audit planning model (i.e., the audit risk model) of the American Institute of Certified Public Accountants (AICPA, 1983) and its interpretation as a plausibility model in belief functions, (2) representation

[1]The author would like to thank C. Joseph Coate and Theodore J. Mock for their insightful comments.

of the strength of audit evidence in terms of belief functions, (3) audit planning and evaluation in a complex situation with all the interdependencies among the audit evidence, and among the accounts and their relevant assertions, and (4) the importance of formally integrating statistical and non-statistical items of evidence using belief functions. I will assume readers to have a basic understanding of the Dempster-Shafer theory of belief functions (Shafer, 1976, see also, Yager, Kacprzyk, and Fedrizzi, 1994).

In essence, an audit is the process[2] of collecting, evaluating, and aggregating evidence pertaining to the audit engagement. The items of evidence collected by the auditor invariably have uncertainties associated with them. Traditionally, both practitioners and researchers have used probabilities[3] to represent such uncertainties and have used combination rules based on probability theory for aggregating them (see, e.g., AICPA, 1981, 1983, Boritz, 1990, Boritz and Jensen, 1985, Boritz and Wensley, 1990). Some researchers have proposed using Bayes' theorem to combine these uncertainties (see, e.g., Kinney, 1984, 1989, Leslie, 1984, Sennetti, 1990). However, there are limitations with the probability framework in representing uncertainties encountered by auditors as discussed by Shafer and Srivastava (1990, see also Akresh, Loebbecke and Scott, 1988). I will discuss how using belief functions for representing uncertainties in the audit evidence helps overcome these limitations (see, e.g., Srivastava and Shafer, 1992, and Srivastava, 1993).

It is evident from the auditing literature that the audit evidence forms a network with variables being the accounts in the financial statements, audit objectives[4] of the accounts, and the financial statements as a whole (e.g., see

[2]There are several types of audits: financial statements audit, compliance audit, income tax audit, operational audit, and assertion audit. In principle, they are all the same; they all involve collection, evaluation, and aggregation of evidence to form an opinion. However, the nature of evidence may differ from one type of audit to another. In this article, I will focus on the audit of financial statements (see, e.g., Arens and Loebbecke, 1996, for details on the definitions of various types of audit).

[3]Most frequently, in practice, a linguistic scale such as *high, medium*, or *low* has been used to measure the level of risk associated with various risk components.

[4]Financial Statements consist of a set of four statements in the USA: balance sheet, income statement, statement of cash flows, and statement of retained earnings. The balance sheet represents the account balances of the assets, liabilities, and the owner's equity of the company at the end of a fiscal period. The income statement presents the revenue, cost of goods sold, administrative expenses, taxes, and the net income of the company for the period. The other two statements are related to the balance sheet and income statement.

It is implied that, through the financial statements, the management is making certain assertions about the assets, liabilities, revenues, and expenses of the company. These assertions are known as "Management Assertions". Statement on Auditing Standards No. 31 (AICPA, 1980) classifies them into five categories: 'Existence or Occurrence', 'Completeness', 'Rights and Obligation', 'Valuation or Allocation', 'Presentation and Disclosure' (see, e.g., Arens & Loebbecke, 1996, for details). In order to facilitate accumulation of evidence to determine whether each management assertion is met, the AICPA has developed its own set of objectives called audit objectives, e.g., Existence, Completeness, Rights and Obligation, Accuracy, Classification, Cutoff, Realizable-Value, Detail Tie-in, and Presentation and Disclosure. These objectives are closely related to the management assertions. For example, the audit objec-

Arens and Loebbecke, 1996, and Srivastava, Dutta and Johns, 1996). The network structure occurs because certain items of evidence support more than one account or audit objective. For example, confirmations of accounts receivable from customers provide support to two audit objectives: 'Existence' and 'Valuation'. Furthermore, if we express uncertainties in the audit evidence by belief functions, then combining various items of evidence is essentially a problem of propagating beliefs in a network (see, e.g., Shenoy and Shafer, 1988, 1990, Srivastava, 1995a, 1995b). Also, one might expect in a network structure that an item of evidence supporting more than one variable could provide a different level of support for each variable. This situation is quite common in auditing (Srivastava, Dutta, and Johns, 1996). We need to convert these assessments of beliefs on individual variables to beliefs on the joint space of the variables in order to (1) combine all the evidence in the network, and (2) preserve the interdependencies among the evidence. However, the conversion process can become quite complex depending on the number of variables involved and the nature of the individual support, whether it is positive or negative (see, e.g., Dubois and Prade, 1986, 1987, 1992, and 1994). I will describe a heuristic algorithm developed[5] by Srivastava and Cogger (1995) to convert such individual beliefs into beliefs on the joint space for a fully dependent case. Also, I will show how to combine audit evidence in a network of variables and discuss how such an approach would affect the efficiency and effectiveness of an audit using the computer program AUDITOR'S ASSISTANT developed by Shafer, Shenoy and Srivastava (1988).

We know that the auditor collects both statistical and non-statistical items of evidence on every engagement. An example of statistical evidence is the procedure to determine the value of inventory using a statistical sampling technique (see, e.g., Arens and Loebbecke, 1981, and Bailey, 1981). Currently, an auditor uses an intuitive approach to combine such items of evidence. The problem with such an approach is that the auditor is not able to properly take advantage of the strength of evidence that may be non-statistical in nature to save on the extent of work done for statistical tests. I will describe how such items of evidence can be formally combined using the approach developed by Srivastava and Shafer (1994). Also, I will discuss how such an approach should lead to an

tives Existence, Completeness, and Rights and Obligation, respectively, correspond to the management assertions Existence or Occurrence, Completeness, Rights and Obligation. The audit objectives Accuracy, Classification, Detail Tie-in, and Realizable-Value relate to the 'Valuation' assertion because they all deal with the valuation of the account balance.

[5]Determining beliefs on the joint space of the variables from the beliefs on the individual variables is similar to determining the joint probability distribution from the marginal probabilities. In general, the solution is not unique. It depends on the assumed relationship between the two sources of beliefs. This relationship may vary from fully dependent to fully independent situations. In our case, since the two sets of beliefs are coming from the same evidence, we must treat them to be fully dependent. The approach of Srivastava and Cogger (1995) provides the joint beliefs for such a situation which is a unique solution for fully dependent case.

efficient audit.

The remaining part of the article is divided into seven sections and an appendix. Section 2 provides the belief-function plausibility interpretation of the audit risk model of the American Institute of Certified Accountants. Section 3 deals with representing the strength of evidence using belief functions. Section 4 deals with the planning and evaluation of an audit with the network structure of evidence. Section 5 discusses how statistical items of evidence can be combined with non-statistical items of evidence using belief functions and how this process would result in an efficient audit. Section 6 provides a summary of other works not discussed elsewhere in the article that deal with applications of belief functions to audit decisions. Section 7 provides a summary and conclusion of the article and also lists some of the potential research problems. Finally, Appendix describes Srivastava and Cogger's (1995) algorithm.

2. The AICPA planning model: a plausibility model

As mentioned earlier, the essence of a financial audit is to accumulate sufficient evidence related to the financial statements in order to express an opinion whether the financial statements present fairly the financial position of the company in accordance with GAAP (Generally Accepted Accounting Principles). In principle, if all the management assertions of all the accounts are met, that is, if all the audit objectives of all the accounts are met (see Footnote 2 for management assertions and audit objectives) then the financial statements are fairly stated. In order to facilitate accumulation of evidence, the AICPA has developed an audit planning model known as the audit risk model through SAS 39 and SAS 47 (AICPA 1981, 1983). This model is used primarily to decide how much evidence to gather in an audit for a particular account for a given assertion. The model is usually expressed as:

$$AR = IR.CR.DR \tag{1}$$

where

AR – Audit risk: The risk that the auditor is willing to accept that the financial statements may be materially misstated when he or she has given a clean (unqualified) opinion after the audit.

IR – Inherent risk: The auditor's assessment of the likelihood that the account may be materially misstated due to inherent factors without considering the effects of internal controls.

CR – Control risk: The risk that internal controls fail to detect and correct material errors given that such errors exist in the account.

DR – Detection risk: The risk that the auditor's detection procedures (test of details of balances and analytical procedures) fail to detect material errors in the account given that such errors exist and controls failed to detect and correct them.

The risks in the above model are usually considered at each account or transaction stream level for planning the audit. In fact, the present thinking is that these risks should be assessed at each audit objective or management assertion level (see, e.g., AICPA, 1988, and Arens and Loebbecke, 1996).

The auditor first makes a judgment about AR based on the users of the financial statements and the type of client, whether a private company or a public company. Next he or she assesses the inherent and control risks. Of course, the auditor collects relevant items of evidence to make the assessments of these risks. The assessed values of IR and CR are then used to determine DR for a desired level of AR. This value of DR, in turn, determines the extent of testing to be performed for the account balance. Actually, DR is further divided into two components: $DR = APR.TDR$. APR represents the risk that the auditor's analytical procedures will fail to detect material misstatements in the account given that such errors exist and control procedures have failed to detect and correct them. Similarly, TDR is the risk that the auditor's tests of details of balance will fail to detect material misstatements in the account given that such errors exist, internal control procedures have failed to detect and correct them, and the analytical procedures have also failed to detect them.

The multiplicative structure of the audit risk model makes intuitive sense. The auditor will fail to detect material errors in an account and give a clean opinion when all of the following conditions are met: (1) material misstatements do exist in the account ($1 \geq IR > 0$), (2) internal controls fail to detect and correct such errors ($1 \geq CR > 0$), (3) the auditor's analytical procedures fail to detect them ($1 \geq APR > 0$), and (4) the auditor's tests of details fail to detect them ($1 \geq TDR > 0$).

Although the profession has been using the above model for planning an audit, there are several problems with the model as mentioned in the introduction, including the following. First, the SAS 47 model is too simplistic; it fails to incorporate the network structure of the audit evidence. As discussed by Srivastava and Shafer (1992), ignoring the network structure will make the audit process inefficient. Also, it fails to incorporate the logic usually considered in the audit process when combining direct and indirect items of evidence[6]. Although not explicitly mentioned in the audit risk model, traditionally, it has been assumed that a financial statement such as the balance sheet is related

[6]To illustrate direct and indirect items of evidence consider the following situation. The ending accounts receivable balance (A/R_E) is related to the beginning accounts receivable balance (A/R_B), Sales, and Cash receipts (C/R), as follows (assuming that sales returns and cash discount on sales are not significant):

$$A/R_E = A/R_B + Sales - C/R$$

We know from the above relationship that when A/R_B, Sales, and Cash Receipts are fairly stated then the ending accounts receivable balance (A/R_E) is fairly stated. An item of evidence pertaining to the ending balance of A/R will be a direct piece of evidence such as confirmation of accounts receivable from customers. The evidence pertaining to Sales and C/R will serve as indirect evidence for the ending balance of A/R.

to all its accounts through an 'and' relationship. That is, the balance sheet is fairly stated if and only if all the accounts on it are fairly stated. However, Srivastava, Dutta and Johns (1996) question the use of "and" relationship between FS and the accounts, and between an account balance and the transaction streams. From Footnote 4, we know that when A/R_B, Sales, and Cash Receipts are fairly stated then the ending accounts receivable balance (A/R_E) is fairly stated. However, we can not infer in the reverse direction that if A/R_E is fairly stated then both Sales and Cash Receipts are fairly stated; there are infinitely many possibilities. However, auditing researchers have used "and" relationship in absence of anything better. Gillett (1996b) and Srivastava (1997b) have investigated this problem in great details.

Second, the use of probabilities to model an auditor's judgment about risks is not appropriate. For example, SAS 47 suggests that if the auditor does not want to depend on the inherent factors, then he or she should set the inherent risk equal to 1 ($IR = 1$). If we interpret this number as probability then we conclude that it is certain that the account is materially misstated. But this is not what the auditor has in mind when he or she decides not to depend on inherent factors for the audit. The auditor's feeling is represented better by a belief-function plausibility of 1 for material misstatements. A plausibility value of 1 implies that the auditor lacks evidence based on inherent factors. Srivastava and Shafer (1992) analyze this issue further and state the following:

> "... the auditor may believe, on the basis of inherent factors, that the account is fairly stated and yet be unwilling to rely on these factors past a certain point. In this case, the auditor may, as SAS No. 47 suggests, assign a value less than the maximum, say 70 percent, to inherent risk. If interpreted in probability terms, this number says that the inherent factors give a 30 percent chance that the account is not materially misstated and a 70 percent chance that it is materially misstated. This suggests that the evidence is negative, contrary to the auditor's intuition. The probability interpretation is even more confusing if the auditor sets the inherent risk at 50 percent. What does this mean? Does it mean that the auditor is completely ignorant about the state of the account, or does it mean there is more evidence that the account is not being materially misstated than when only 30 percent assurance was assumed?"

A belief function interpretation of the risks in the audit risk model makes more intuitive sense than the probability interpretation. In fact, as shown by Srivastava and Shafer (1992, see also, Srivastava, 1993), the risks in the audit risk model can be interpreted as belief-function plausibilities provided all the evidence is positive.

The third deficiency in the audit risk model, in my opinion, is the lack of explicit consideration of the risk of material misstatements due to management fraud. It is implicitly assumed that the auditor will accept only those engagements where there is no potential for such material misstatements. That is,

the auditor will first perform certain preliminary procedures to make reasonably sure that financial statements are not fraudulent and then only accept the engagement. However, in practice, this has not happened, as evidenced from the increasing number of cases where financial statements were found fraudulent after the auditor had given clean opinions (Fanning, Cogger and Srivastava, 1995). However, the situation has changed recently because of escalating litigation; the auditors are now putting much more effort in assessing the risk of material misstatements due to management fraud. Dutta, Harrison, and Srivastava (1996) have discussed this issue and shown how the current practice of the firms concerned with such material misstatement can be modeled analytically. Also, they discuss the consequence of not considering the risk of fraud in the audit risk model. Furthermore, they show the impact of negative items of evidence on the audit process using belief functions.

3. Strength of evidence in belief functions

In this section, I want to show how belief functions help us model uncertainties encountered in audit evidence. As discussed earlier, there are some items of evidence that bear on only one variable, but there are some that bear on more than one variable. Representing uncertainties in belief functions associated with those items of evidence that bear on just one variable is quite straightforward. However, for those items of evidence that bear on more than one variable, we need to use a special technique to convert the beliefs defined on individual variables to m-values for the joint space. This process keeps the interdependencies among all items of evidence and allows us to use Dempster's rule for aggregating all the evidence in the network. I will discuss these issues in detail below.

3.1. Evidence bearing on one variable

In a situation where the evidence bears on only one variable such as an account or an audit objective or a management assertion of an account, the representation of the strength of evidence is straightforward. Consider that the auditor accumulates an item of evidence, evaluates its strength, and, based on professional judgment, concludes that the evidence provides, say 0.6 level of support on a scale 0-1, that assertion A is met ('a') and no support for its negation ('$\neg a$')[7]. We can express this judgment in terms of belief functions as $\text{Bel}(a) = 0.6$, and $\text{Bel}(\neg a) = 0$. This can be expressed in terms of m-values[8] (the *basic probability*

[7] As a convention, I will use the first letter of the variable's name in the upper case to represent the variable and the lower case to represent its values. For example, a variable named 'existence' will be represented by 'E' and its values by 'e' and '$\neg e$' implying that it is met and not met, respectively.

[8] In the belief-function formalism, uncertainty is not only assigned to the single elements of a frame but also to all other proper subsets of the frame and to the entire frame. These uncertainties are called m-values or the *basic probability assignment function*. For a frame of n elements, we will have, in general, m-values for each individual elements, each set of two

assignment function) as $m(a) = 0.6$, $m(\neg a) = 0$, and $m(\{a, \neg a\}) = 0.4$. These m-values suggest that the auditor has (1) direct evidence that the assertion is true (i.e. 'a' is true) with 0.6 degree of support, (2) no evidence that the assertion is not true ('$\neg a$'), and (3) 0.4 degree of uncommitted support.

Consider now a situation where the auditor feels that the evidence is negative. That is, in the auditor's judgment, the evidence supports '$\neg a$', the negation of the assertion, say, at 0.3 level, and there is no support for 'a' that the assertion is met. This feeling can be expressed as: $m(a) = 0$, $m(\neg a) = 0.3$, and $m(\{a, \neg a\}) = 0.7$. There are situations where the auditor might have several items of evidence, some might be positive and some negative. Rather than evaluating individually the level of support from each item of evidence, the auditor may make an intuitive judgment about the overall strength of the combined evidence. Suppose the auditor's combined evaluation of all the evidence pertinent to this assertion is that 'a' is true with a medium level of support, say, 0.4, '$\neg a$' is true with a low level of support, say, 0.1. This feeling can be expressed as: $m(a) = 0.4$, $m(\neg a) = 0.1$, and $m(\{a, \neg a\}) = 0.5$. We can not express such feelings using probabilities.

3.2. Evidence bearing on more than one variable

It is common, on almost every audit engagement, that the auditor collects items of evidence that bear not only on one variable but on many. For example, in the audit of sales transactions, the evidence that duties of billing, recording sales, and handling cash receipts are separated bears on two variables: (1) Recorded sales are for shipments made to non-fictitious customers (existence), and (2) Existing sales transactions are recorded (completeness). The evidence that 'Batch totals are compared with computer summary reports for cash receipts' bears on three variables: existence, completeness, and valuation of cash receipts (for more examples see Arens and Loebbecke, 1996).

In general, the level of support from such items of evidence for each variable may differ. For example, suppose in the case of the latter item of evidence above, the auditor's assessment of the levels of support are as follows: (1) 0.6 degree of support that 'existence' objective is met ('e'), and no support for its negation ('$\neg e$'), (2) 0.4 degree of support that 'completeness' objective is met ('c'), and no support that it is not met ('$\neg c$'), and (3) 0.3 degree of support that 'valuation' objective is met ('v') and 0.1 degree of support that it is not met ('$\neg v$'). This feeling can be easily written[9] in terms of belief functions on each variable as: $\text{Bel}(e) = 0.6$ and $\text{Bel}(\neg e) = 0$; $\text{Bel}(c) = 0.4$ and $\text{Bel}(\neg c) = 0$; $\text{Bel}(v) = 0.3$ and

elements, each set of three elements, and so on, to the m-value for the entire frame. All such m-values add to one, i.e., $\sum_{A \subseteq \Theta} m(A) = 1$, where A represents all the proper subsets of the frame Θ. The m-value for the empty set is zero.

[9] I emphasize that to express such a feeling on the strength of evidence in classical probabilities will not be easy. One has to consider several conditional probabilities which are not known any way.

$\text{Bel}(\neg v) = 0.1$. We need to express the above judgment in terms of m-values in order to combine various items of evidence using Dempster's rule. Since all the beliefs above come from the same evidence, we need to convert these beliefs into m-values on the joint space of the three variables $(\{e, \neg e\} \times \{c, \neg c\} \times \{v, \neg v\})$. This task is not trivial. Dubois and Prade (1986, 1987, see also 1992, and 1994) have discussed a formal approach to determining the m-values on the joint space. It should be pointed out that the transformation of beliefs on separate variables to m-values on the joint space is not unique. However, these m-values are unique for a given relationship among the sources of individual beliefs. For example, there will be one set of m-values on the joint space if the sources are independent. Such a set of m-values are determined using Dempster's rule to combine the individual set of m-values. However, our interest here is in a fully dependent case because the individual beliefs come from the same source. For such a case, Srivastava and Cogger (1995) have developed a heuristic algorithm for determining m-values on the joint space of the variables from the beliefs defined on each variable.

Using the algorithm of Srivastava and Cogger (1995) for our example above, one gets the following non-zero m-values on the joint space (see Table 2 in the Appendix for details):

$$\begin{aligned} m(\{ec\neg v, \neg ec\neg v\}) &= 0.1, \\ m(\{ecv, \neg ecv\}) &= 0.3, \\ m(\{ecv, ec\neg v, e\neg cv, e\neg c\neg v, \}) &= 0.6. \end{aligned}$$

If we marginalize[10] the above m-values on the individual variable space then we do get the beliefs that the auditor had estimated. The Srivastava and Cogger approach is valid even for non-binary variables.

The traditional approach of representing uncertainties using probabilities demands much more detailed information in terms of conditional probabilities to represent the strength of support in the above situations where one item of evidence bears on many variables. Using belief functions, we can model the auditor's judgment easily by simply finding his or her individual beliefs. This approach becomes important when we want to model dependent items of evidence. In auditing, we have many dependent items of evidence where one

[10]Marginalization in belief functions is similar to the marginalization in probability theory; you simply sum over the variables that are not desired. For example, for the following m-values:

$$\begin{aligned} m(\{ec\neg v, \neg ec\neg v\}) &= 0.1, \\ m(\{ecv, \neg ecv\}) &= 0.3, \\ m(\{ecv, ec\neg v, e\neg cv, e\neg c\neg v, \}) &= 0.6. \end{aligned}$$

marginalizing them onto the space $\{e, \neg e\}$, yields $m(e) = 0.6$, $m(\neg e) = 0$, and $m(\{e, \neg e\}) = 0.4$, marginalizing onto $\{c, \neg c\}$ yields $m(c) = 0.4$, $m(\neg c) = 0$, and $m(\{c, \neg c\}) = 0.6$, and onto $\{v, \neg v\}$ yields $m(v) = 0.3$, $m(\neg v) = 0.1$, and $m(\{v, \neg v\}) = 0.6$.

item of evidence bears on several variables. If such dependencies are not treated properly, it will lead to an inefficient audit.

To illustrate the above point, let us consider the evidence 'Batch totals are compared with computer summary reports for cash receipts.' Assume that the auditor assesses 0.6 degree of support in favor of all the three objectives being met, i.e., $\text{Bel}(e) = 0.6$, $\text{Bel}(c) = 0.6$, and $\text{Bel}(a) = 0.6$, and no support to their negations. If one assumes all these assessments to be independent of each other as if they are coming from independent sources, then the total belief that all the objectives are met based on just these items of evidence will be $(0.6)^3 = 0.216$. However, if we treat them as interdependent, then the total belief that all the three are met is 0.6, a much higher belief than when the three beliefs were assumed to come from three independent sources. This can become a serious problem for the auditor, especially when he or she has to worry about efficiency. Moreover, there are numerous situations where such interdependencies exist in the audit evidence (see the evidential network in Section 4). Modeling uncertainties associated with such items of evidence using the approach described here allows us to preserve the interdependencies among the evidence and still use Dempster's rule for aggregating all the evidence.

4. Planning and evaluation of an audit with the network structure of evidence

In this section, I plan to discuss how various uncertain items of audit evidence with all their interrelationships can be aggregated using Dempster's rule for determining whether the financial statements are fairly stated in order to give a clean (unqualified) opinion. Let me restate the objective of an audit of the financial statements. The objective of such an audit is to give an opinion on the financial statements that they represent fairly the financial position of the company. In other words, the auditor has to be confident with a certain level of assurance that there are no material misstatements in the financial statements. In order to achieve this objective, he or she accumulates relevant items of evidence, evaluates their strengths, and aggregates them to form the overall opinion. However, the question is when do we say the financial statements are fairly stated, that is, they do not contain material misstatements? One possible and generally accepted answer is that the financial statements are fairly stated when all the accounts constituting the financial statements are fairly stated. For example, the balance sheet of a company will be fairly stated if all the accounts constituting it such as cash, accounts receivable, inventory, accounts payable, etc. are fairly stated. Such a relationship is represented by a categorical relationship 'and' between the balance sheet and all its accounts. This relationship implies that the balance sheet is fairly stated if and only if all the accounts constituting the balance sheet are fairly stated. The next question is, how do we determine that each account is fairly stated? Again, the generally accepted answer is that an account is fairly stated if all its management assertions, and

in turn, all the related audit objectives, have been met. For example, accounts receivable will be fairly stated when all the relevant management assertions, i.e., all the corresponding audit objectives such as 'existence', completeness', 'accuracy', 'classification' etc. have been met. Again, this is an 'and' relationship between the account and its audit objectives.

Furthermore, we all know that accounts on the balance sheet are the aggregate of certain transaction streams for the fiscal period. For example, the ending balance of accounts receivable on the balance sheet is equal to the beginning balance of accounts receivable plus the sales minus the cash receipts on sales for the fiscal period assuming that sales returns and cash discounts are not significant. (If these activities are significant then the auditor can simply add these terms in the equation). Moreover, we know that individual audit objectives of the balance sheet accounts are related to audit objectives of the corresponding transactions streams (Leslie, Aldersley, Cockburn and Reiter, 1986). For example, the 'existence' objective of accounts receivable is related to the 'existence' objective of sales and the 'completeness' objective of cash receipts on sales. Thus, we can further build a relationship between a specific audit objective of the balance sheet account and the related objectives of the transaction streams. In fact, this relationship is again assumed to be an 'and' relationship. Thus, in the case of accounts receivable, the 'existence' objective will be met if and only if sales 'exist' and cash receipts are 'complete'. We can represent the above relationships among various variables (e.g., balance sheet, accounts on the balance sheet, audit objectives of the accounts and transaction streams) through a tree of variables. Fig. 1 shows such a tree of variables for accounts receivable with only three audit objectives for simplicity of presentation.

The next step in the audit process is to identify relevant items of evidence bearing on various variables. As discussed in Section 2, in general, we have audit evidence at all levels of the financial statements. Table 1 represents a list of procedures relevant to the audit of accounts receivable presented in Figure 1. Figure 2 represents the evidential network based on the procedures in Table 1, and the structural relationships among the variables in Figure 1 for the audit of accounts receivable with only three audit objectives.

The auditor, at the planning stage, will make a judgment about the appropriate level of assurance to obtain from various items of evidence. This judgment will depend on (1) the extent, nature and timing of the evidence to be collected, (2) an overall assurance that the accounts receivable balance is fairly stated, and (3) the cost of accumulating the evidence. The cost of the audit is an important factor in determining the extent, nature, and timing along with the mix of evidence.

As we can see in Figure 2, aggregating all the evidence in an audit is really a problem of propagating beliefs in the network, whether it is at the planning stage, during the audit process, or at the evaluation stage. As is evident from Figure 2, even for a simple case with only three audit objectives, the network has become so big that it is almost impossible to aggregate all these items of

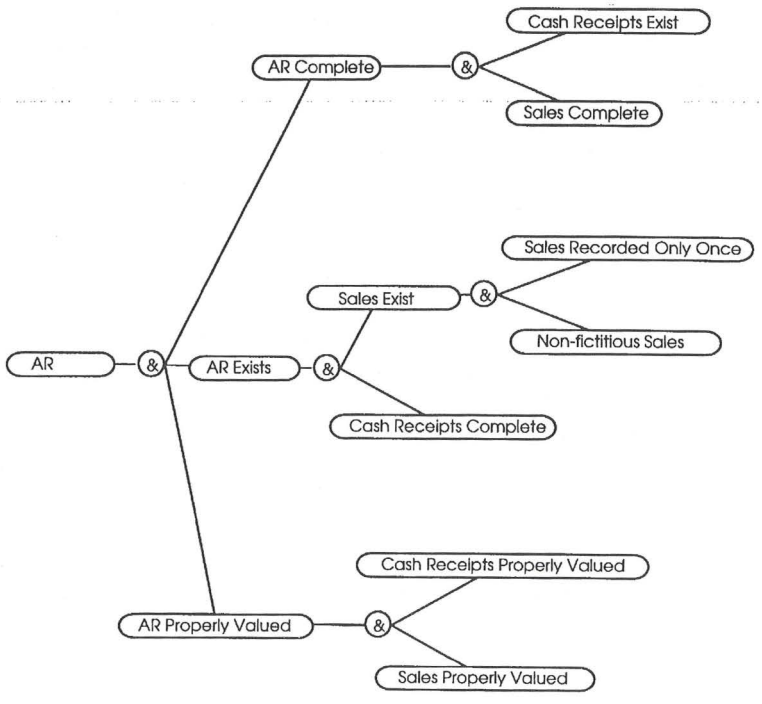

Figure 1. Tree of variable nodes for accounts receivable (AR) audit. For simplicity of presentation, only three audit objectives of AR are considered in this case. A circle with '&' represents an 'and' relationship between the variable on its left and the variables on its right. For example, variable 'AR complete' is related to 'Cash Receipts Exist' and 'Sales Complete' through an 'and' node. This implies that AR will be complete if and only if cash receipts 'exist' and sales are 'complete'.

Table 1*.

1 (Inherent Factors)	Prior years' experience with the account, related accounting system, and the control environment. Also, the knowledge about the competence and trustworthiness of accounting personnel working in the sales and collection cycle, and other relevant inherent factors.
2 (*AP*)	(i) Review accounts receivable trial balance for large and unusual receivables. (ii) Calculate ratios indicated in carry-forward working papers (not included here) and follow up any significant changes from prior years.
3 (*STT*)	Review the sales journal and ledger for unusual transactions and amounts.
4 (*STT*)	(i) Review the cash receipts journal and the ledgers for unusual transactions and amounts. (ii) Review the subsidiary ledger for miscellaneous credits.
5 (*TC*)	Observe for segregation of duties between receipt and recording of cash and also preparation of in dependent bank reconciliation statement.
6 (*TC*)	Observe whether a restrictive endorsement is used on cash receipts.
7 (*TC*)	Observe whether monthly statements are mailed.
8 (*TC*)	Observe whether the accountant reconciles bank account.
9 (*TC*)	Account for a sequence of shipping documents.
10 (*STT*)	Trace selected shipping documents to duplicate sales invoice and the sales journal for assurance that each one has been billed and included in the journal.
11 (*TC*)	Account for a sequence of sales invoices in the sales journal.
12 (*TC*)	For selected duplicate invoice numbers from the sales journal, examine underlying documents for indication of internal certification that the total amount recorded in the journal, date, customer name, pricing, extension, and footings have been checked.

* taken from Srivastava (1995a, Table 2)

13 (STT)	Trace selected duplicate invoice numbers from the sales journal to (a) Duplicate sales invoice, and test for the total amount recorded in the journal, date, customer name. Check the pricing, extensions, and footings. (b) Bill of lading, and test for customer name, product description, quantity, and date. (c) Duplicate sales order, and test for customer name, product description, quantity, date, and internal approval. (d) Customer order, and test for customer name, product description, quantity, date, and credit approval by the credit manager.
14 (STT)	Trace recorded sales from the sales journal to the file of supporting documents, which includes a duplicate sales invoice, bill of lading, sales order, and customer order.
15 (STT)	Obtain the prelisting of cash receipts, and trace amounts to the cash receipts journal, testing for name, amount, and date.
16 (STT)	Compare the prelisting of cash receipts with the duplicate deposit slip, testing for names, amounts, and dates. Trace the total from the cash receipts journal to the bank statement, testing for dates, amounts of deposit, and delay in deposit.
17 (TDB)	Confirm accounts receivable using positive confirmations above a given amount and perform alternative procedures for all confirmations not returned on the first and second request.

Table 1. The audit procedures used in Fig. 2 (Arens and Loebbecke, 1996: 368-370).

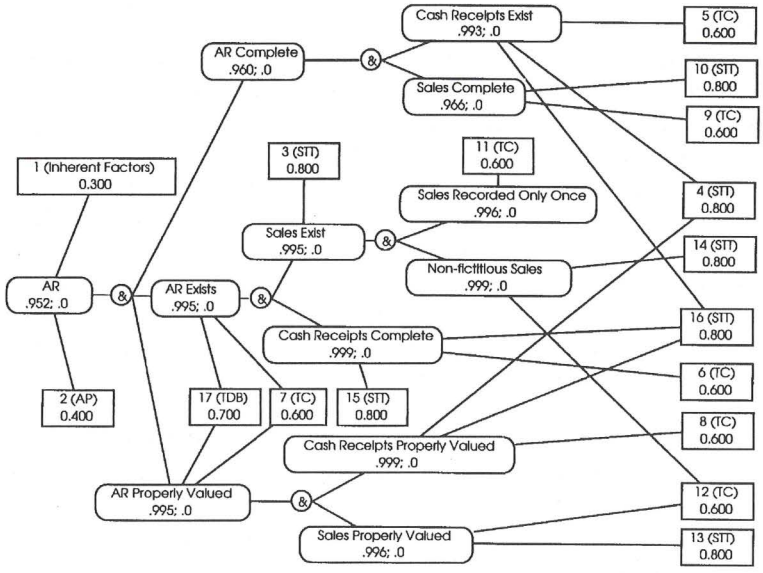

Figure 2. Evidential network for accounts receivable ($\grave{A}R$) as the main node. All the procedures have been performed and the corresponding 'beliefs' have been aggregated by AUDITOR'S ASSISTANT (Shafer, Shenoy and Srivastava, 1988).

evidence without a computer program. There are several computer programs currently available for propagating beliefs in networks: (1) 'DELIEF' developed by Zarley, Hsia, and Shafer (1988), (2) 'AUDITOR'S ASSISTANT' developed by Shafer, Shenoy, and Srivastava (1988), (3) 'BELIEF' by Almond (1989), (4) 'PULCINELLA' developed by Saffiotti and Umkehrer (1991), and (5) 'TresBel' by Xu (1991).

I have used AUDITOR'S ASSISTANT to create Figure 2. This system allows the user to draw the evidential network pertinent to the situation. I suggest readers see Srivastava, Dutta, and Johns (1996) for details (see also, Srivastava 1995c, 1995d, and Shafer, Shenoy, and Srivastava 1988). The system can be used at all the three stages of an audit: planning stage, during the audit, and at the end of the audit.

At the planning stage, the auditor can use the system to perform a sensitivity analysis to determine what combination of evidence will give the desired level of overall belief for the account under consideration that it is fairly stated. During the audit, the auditor can use it to determine if the procedures performed provide the planned level of assurance that yields the desired overall belief. If certain procedures do not yield the planned level of assurance then the auditor can use the system to modify the subsequent audit procedures to achieve the desired level of overall belief. If certain items of evidence provide negative support, the system can be used to determine the effect of this negative evidence on the overall assurance. At the end of the audit, the system can be used to aggregate all the evidence to determine the overall belief that the account is fairly stated.

Srivastava, Dutta, and Johns (1996) have used AUDITOR'S ASSISTANT to evaluate the audit of accounts receivable of a health care unit using the audit program of a Big Six accounting firm. The choice of a health care unit was made for three reasons: (1) It provided a complex audit task where some traditional procedures such as confirmations of accounts receivable were not being performed. (2) The SAS 47 model gave little help in term of determining whether an adequate audit was being conducted, thus a good case for testing the system. (3) One of the authors had expertise in auditing health care units.

We all know that the structure of audit evidence is quite complex due to all the dependencies among the evidence and all the relationships among the accounts, the audit objectives of the accounts and the transaction streams. Thus, it is not possible to develop an analytical model for audit planning or evaluation. A computer program like AUDITOR'S ASSISTANT is the only answer for an efficient and effective audit. Such a system can be used to develop an interactive audit risk model. Also, I would like to point out that the belief-function approach provides the auditor with the appropriate impact of a negative item of evidence in the network on the overall belief that the account is fairly stated. The auditor can use this information to plan and adjust the audit procedures accordingly to achieve the desired level of belief. Several alternatives are available to the auditor if he or she encounters a negative item of evidence

and as a consequence the overall belief is below the desired level of overall assurance: (1) perform an increased level of testing, (2) perform some other relevant procedures, (3) propose an adjustment to the account balance, or (4) issue a qualified opinion. In each alternative situation, the auditor will re-evaluate the assessment of beliefs or level of assurance for or against the affected variables and aggregate all the evidence.

5. Integrating statistical and non-statistical evidence

As we have seen in Section 4, the audit process in general involves both statistical and non-statistical items of evidence. For example, procedure 17 in Table 1 is a statistical item of evidence because it uses statistical sampling techniques whereas procedure 7 is a non-statistical item of evidence. Both of these items of evidence bear on variables 'AR Exist' and 'AR Properly Valued.' The question is how do we integrate the two items of evidence to determine the overall belief that the financial statements are fairly stated? In discussing AUDITOR'S ASSISTANT in Section 4, I did not raise this question. I simply assumed that the auditor has evaluated the strength of each item of evidence, whether it is statistical in nature or not, then used the program to aggregate all the evidence. In this section, I want to discuss the approach proposed by Srivastava and Shafer (1994) that provides beliefs from statistical evidence. Once we know beliefs from statistical evidence then we can combine them with the beliefs from non-statistical evidence using Dempster's rule.

There are two main decisions regarding statistical evidence that the auditor has to make during an audit. One, he or she must determine the sample size, n, based on an acceptable risk of incorrect acceptance (ARIA or type II error) and an acceptable risk of incorrect rejection (ARIR or type I error). The auditor uses the audit risk model to determine ARIA, which is simply the risk associated with the test of details of balance (TDR) in the model as discussed in Section 2 ($TDR = AR \div (IR.CR.APR)$). Two, the auditor must determine the level of assurance obtained from the statistical evidence after the procedure has been performed.

5.1. Sample size determination

For the sample size determination, Srivastava and Shafer (1994) derive a formula that relates the sample size, n, with the desired level of belief 'x' that the objective is met for the variable sampling using mean per unit estimator:

$$n = \frac{\sigma^2}{TE^2}\left[Z_{\alpha/2} + \sqrt{-2\log_e(1-x)}\right]^2, \tag{2}$$

where TE is the tolerable error (the maximum amount of error the auditor is willing to tolerate and accept the account balance to be fairly stated), σ is the standard deviation of the population, and $Z_{\alpha/2}$ is the standard normal deviate.

Probability $(1-\alpha)$ represents the probability of achieving a belief of at least x that the account balance is not materially misstated. The significance level of the test that the recorded mean is equal to the true audited mean is still, by design, α. The minimum power of the test is $(1-\beta)$ where β is related to the desired belief x through the standard normal deviate Z_β as:

$$Z_\beta = \sqrt{-2\log_e(1-x)}. \tag{3}$$

EXAMPLE 5.1 *Let us consider an example to illustrate the above process. Suppose the auditor wants to determine the sample size for confirmations of accounts receivable. He or she has evaluated the inherent factors, the accounting system, and internal controls that relate to 'existence' and 'valuation' objectives of accounts receivable. Suppose that based on all other evidence planned on the audit, the desired level of belief from the confirmations test is 0.7 in order to achieve an overall belief of, say, 0.95 that the accounts receivable balance is fairly stated (see Figure 2). Also, assume that the recorded mean of accounts receivable is \$500, the estimated standard deviation of the population* $\sigma = \$75$*, the tolerable error* $TE = \$25$ *per account, and the desired level of risk of incorrect rejection* $\alpha = 20\%$*. The sample size in this case from (2) is* 72*, i.e.,* $n = 72$.

5.2. Evaluation of sample results

Let us consider that the auditor has performed the statistical test and wants to determine the level of belief whether the account is fairly stated or not based on the outcome of the test. Srivastava and Shafer (1994) have discussed this aspect using again the mean per unit estimator. There are two possible outcomes in evaluating the sample results. One, the sample (audited) mean, $\bar{y}$, may fall inside the tolerable interval, $[\mu_r - TE, \mu_r + TE]$, where μ_r is the recorded mean. Two, the audited mean may fall outside of the tolerable interval.

For the condition when the audited mean falls inside the tolerable interval, i.e., for $\mu_r - TE \le \bar{y} \le \mu_r + TE$, the belief that the account balance is not materially misstated is (Srivastava and Shafer 1994, (20)):

$$\text{Bel(no material errors)} = x = 1 - \exp\left(-\frac{n}{2S^2}\left(TE - |\bar{y} - \mu_r|\right)^2\right), \tag{4}$$

$$\text{Bel(material errors)} = 0, \tag{5}$$

where S is the sample standard deviation.

For the condition when the audited mean falls outside the tolerable interval (i.e., for $\bar{y} \ge \mu_r + TE$ or $\bar{y} \le \mu_r - TE$), the two beliefs are:

$$\text{Bel(no material errors)} = 0, \tag{6}$$

$$\text{Bel(material errors)} = 1 - \exp\left(-\frac{n(\bar{y} - \mu_r + TE)^2}{2S^2}\right), \text{ for } \bar{y} \ge \mu_r + TE \tag{7}$$

and

$$\text{Bel(material errors)} = 1 - \exp\left(-\frac{n(\mu_r - TE - \bar{y})^2}{2S^2}\right), \text{ for } \bar{y} \leq \mu_r - TE(8)$$

EXAMPLE 5.2 *Let us consider the same example discussed previously. Now however, the auditor decides to choose a sample size of 100 ($n = 100$ instead of 72) and obtains the following results: the audited mean, $\bar{y} = \$488$, and the sample standard deviation, $S = \$100$. Since $TE = \$25$, the tolerable interval in the present case is $[\$475, \$525]$. The audited mean falls within this interval and therefore the belief that the account balance is not materially misstated is given by (4):*

$$\begin{aligned} \textit{Bel(no material errors)} &= 1 - \exp\left(-\frac{100}{2(100)^2}(25 - |488 - 500|)^2\right), \\ &= 0.570, \end{aligned}$$

and

$$\textit{Bel(material errors)} = 0.$$

However, if the desired level of belief that the accounts receivable balance is not materially misstated was 0.7, the above result is not acceptable. This simply means that when this evidence (statistical test results) is combined with the other items of evidence the auditor has accumulated on the audit, the overall belief that the accounts receivable balance is not materially misstated is going to be less than the desired, say a 0.95 level of overall belief. What should the auditor do? The auditor has three options at this stage, as mentioned earlier: (1) Increase the sample size, perform additional testing and reevaluate the findings. (2) Propose an adjustment to the account balance in order to achieve the desired 0.7 level of belief. (3) Do not give an unqualified (clean) opinion.

Consider that the auditor is not too happy to increase the sample size, because the size is already 100, which is much larger than the planned sample size of 72 as computed earlier. Let us say the auditor prefers to propose an adjustment and issue a clean opinion. What should the adjustment be in the present case? In order to determine the amount of adjustment in recorded mean, we use (4) to determine a new recorded mean such that the belief that the account balance is not materially misstated is 0.7 with the given values: $\bar{y} = \$488$, $TE = \$25$, $S = \$100$, and $n = 100$. Equation (4) yields a value of \$497.48. This means that if the recorded mean is adjusted downward by \$2.52 to a value \$497.48 then the sample results will provide the desired 0.7 level of belief. Thus, if there were 1,000 total number of accounts receivable then the overall balance has to be decreased by $\$2,520$. The auditor can issue a clean opinion after this adjustment.

6. Other applications of belief functions in auditing

Krishnamoorthy, Mock, and Washington (1994) have used belief functions to analyze auditors' behavior in aggregating items of evidence to determine the likelihood of material misstatements in the valuation of inventory. Turner (1994) has examined in a quasi-experimental setting three models descriptive of the audit judgment process: the Audit risk model, the Dempster-Shafer Belief Function Model, and the Hogarth and Einhorn Belief Adjustment Model. Dusenbury, Reimers, and Wheeler (1996) have performed empirical studies to compare the audit risk model of SAS 47 with two other models: the firm-based model, and a belief-based model. Dutta and Srivastava (1992) have used belief functions to explain the auditor's behavior in aggregating evidence under the following conditions: (1) Sequential processing of evidence with all the items of evidence being positive. (2) Sequential processing of evidence with all the items of evidence being negative. (3) Sequential processing of mixed items of evidence starting with a positive item of evidence. (4) Sequential processing of mixed items of evidence starting with a negative item of evidence. (5) Simultaneously processing mixed items of evidence. Srivastava, Shenoy, and Shafer (1995) have developed propositions that make it easier to aggregate audit evidence in an 'and' tree. Gillett and Srivastava (1996) have discussed integration of statistical and non-statistical items of evidence in auditing for attribute sampling. Gillett (1996a, 1996b) has developed a method for integrating statistical and non-statistical items of evidence in monetary unit sampling.

Recently, Srivastava (1997a, 1996) has applied belief functions to explain (1) decision making behavior under ambiguity, and (2) how value judgments are made. This work has relevance to audit decision making because auditors do make value judgments and judgments under ambiguity. It is interesting to note that belief-function approach models these behaviors more naturally than the traditional approach based on probability theory. In fact, in the case of decision making under ambiguity, Ellsberg's paradox arises due to our inability to model ambiguity in probability theory (see Srivastava, 1997a).

7. Summary and conclusion

In summary, I have shown how the audit process is really a problem of aggregation of various items of evidence accumulated on the engagement. I have discussed some of the limitations of the SAS 47 model and how under certain situations it represents a plausibility model in belief functions. Also, I have shown how belief functions can be used to represent the strength of audit evidence under various scenarios: positive evidence, negative evidence, mixed evidence, one item of evidence bearing on one variable in the network, and one item of evidence bearing on more than one variable. It is well known that the structure of the audit evidence, in general, forms a network. The aggregation of evidence in a network becomes a complex problem without the help of a computer pro-

gram. I have discussed how the system AUDITOR'S ASSISTANT can be used to aggregate all the evidence on an audit for planning and evaluation. Also, I have shown how statistical and non-statistical items of evidence can be aggregated in an audit, and shown how such an approach affects the efficiency of the audit process.

However, there are still many issues and problems that need further research. These problems include the following:

- First, is it appropriate to assume that the relationship between, say, the balance sheet and its accounts is an 'and' relationship, or that the relationship between a balance sheet account and its transaction streams is an 'and' relationship? We have been using such a relationship in practice but there is a problem. For example, if sales and cash receipts are fairly stated, then the accounts receivable balance is fairly stated. However, when the accounts receivable balance is fairly stated that does not necessarily mean that sales and cash receipts are fairly stated (there could be off-setting errors). But, an 'and' relationship dictates it so.
- Second, how will the program of an audit be affected when the risk of fraud is explicitly considered in the evidential network?
- Third, how can costs be integrated with the whole evidence aggregation process?
- Fourth, how do we determine the level of assurance obtained from different items of evidence?
- Fifth, what characteristics determine the strength of evidence?
- Sixth, how can we integrate belief functions and decision theory for audit decisions? Concerns about source reliability in auditing can be easily treated in auditing. We need to formally look at this issue in belief functions.

References

ALMOND, R. (1989) Fusion and propagation of graphical belief models: an implementation and examples. *Research Report S-130*, Department of Statistics, Harvard University.

AMERICAN INSTITUTE OF CERTIFIED PUBLIC ACCOUNTANTS (1980) *Statement on Auditing Standards, No. 31: Evidential Matter.* New York: AICPA.

AMERICAN INSTITUTE OF CERTIFIED PUBLIC ACCOUNTANTS (1981) *Statement on Auditing Standards, No. 39: Audit Sampling.* New York: AICPA.

AMERICAN INSTITUTE OF CERTIFIED PUBLIC ACCOUNTANTS (1983) *Statement on Auditing Standards, No. 47: Audit Risk and Materiality in Conducting an Audit.* New York: AICPA.

AMERICAN INSTITUTE OF CERTIFIED PUBLIC ACCOUNTANTS (1988) *Statement on Auditing Standards, No. 55: Consideration of the Internal Control Structure in a Financial Statement Audit.* New York: AICPA.

AKRESH, A.D., LOEBBECKE, J.K. and SCOTT, W.R. (1988) Audit approaches and techniques. In: *Research Opportunities in Auditing: The Second Decade*, edited by A.R. Abdel-khalik and Ira Solomon. Sarasota, FL: AAA, 13-55.

ARENS, A.A. and LOEBBECKE, J.K. (1996) *Auditing: An Integrated Approach.* Englewood Cliffs, NJ: Prentice-Hall.

ARENS, A.A. and LOEBBECKE, J.K. (1981) *Applications of Statistical Sampling to Auditing.* Englewood Cliffs, NJ: Prentice-Hall.

BAILEY, JR., A.D. (1981) *Statistical Auditing: Review, Concepts, and Problems.* Harcourt Brace Jovanovich.

BORITZ, J.E. (1990) Appropriate and inappropriate approaches to combining evidence in an assertion-based auditing framework. Working Paper, School of Accountancy, University of Waterloo, Canada.

BORITZ, J.E. and JENSEN, R.E. (1985) An hierarchical, assertion-oriented approach to planning audit evidence-gathering procedures. Presented at the Symposium on Audit Judgment and Evidence Evaluation, University of Southern California.

BORITZ, J.E., JENSEN, R.E. and WENSLEY, A.K.P. (1990) Structuring the assessment of audit evidence: An expert system approach. *Auditing: A Journal of Practice and Theory*, **9**, (Supplement), 49-109.

DUBOIS, D. and PRADE, H. (1986) The principles of minimum specificity as a basis for evidential reasoning. *Lecture Notes in Computer Science.* edited by G. Goos and J. Hartmanis. Springer-Verlag.

DUBOIS, D. and PRADE, H. (1987) The principles of minimum specificity as a basis for evidential reasoning. In: *Uncertainty in Knowledge-based systems* (Bouchon B., Yager R. R. eds.), Springer-Verlag, *LNCS*, **286**, 74-84.

DUBOIS, D. and PRADE, H. (1992) Evidence, knowledge, and belief functions. *Internal Journal of Approximate Reasoning*, **6**, 295-319.

DUBOIS, D. and PRADE, H. (1994) Focusing versus updating in belief function theory. In: *Advances in the Dempster-Shafer Theory of Evidence*, (Yager, R.R., Kacprzyk, J, and Fedrizzi, M. eds.), Wiley, 71-95.

DUSENBURY, R., REIMERS, J. and WHEELER S. (1996) An empirical study of belief-based and probability-based specifications of audit risk. *Auditing: A Journal of Practice and Theory*, **15**, 2, Fall, 12-28.

DUTTA, S.K., HARRISON, K. and SRIVASTAVA, R.P. (1997) The audit risk model under the risk of fraud. *Applications of Fuzzy Sets & The Theory of Evidence to Accounting* (forthcoming).

DUTTA, S.K. and SRIVASTAVA, R.P. (1992) Belief revision in auditing: a theoretical investigation. *Proceedings of the 21st Annual Western Decision Science Meeting*, Reno Nevada, March 24-27.

FANNING, K., COGGER, K. and SRIVASTAVA, R.P. (1995) Detection of management fraud: a neural network approach. *International Journal of Intelligent Systems in Accounting, Finance and Management*, **4**, 113-126.

GILLETT, P. (1996A) Integrating Statistical and Non-Statistical Audit Evi-

dence Using Belief Functions: A Case of Monetary Unit Sampling. Working Paper, School of Business, University of Kansas.

GILLETT, P. (1996B) A Comparative Study of Audit Evidence and Audit Planning Models using Uncertain Reasoning. Ph.D. Dissertation. School of Business, University of Kansas.

GILLETT, P. and SRIVASTAVA R. (1996) Integrating Statistical and Non-Statistical Audit Evidence Using Belief Functions: A Case of Attribute Sampling. Working Paper, School of Business, University of Kansas.

KINNEY, JR., W.R. (1984) Discussant's response to an analysis of the audit framework focusing on inherent risk and the role of statistical sampling in compliance testing. *Proceedings of the 1984 Touche Ross/University of Kansas Symposium on Auditing Problems.* Lawrence, KS: School of Business, University of Kansas, 126-32.

KINNEY, JR., W.R. (1989) Achieved audit risk and the audit outcome space. *Auditing: A Journal of Practice and Theory*, **8**, (Supplement), 67-97.

KRISHNAMOORTHY, G., MOCK, T.J. and WASHINGTON, G.M. (1994) A Comparative Evaluation of Belief Revision Models in Auditing. Working Paper, School of Business, Boston College, Boston.

LEA, R.B., ADAMS, S.J. and BOYKIN R.F. (1992) Modeling of the audit risk assessment process at the assertion level within an account balance. *Auditing: A Journal of Practice and Theory*, 11, (Supplement), 152-179.

LESLIE, D.A. (1984) An analysis of the audit framework focusing on inherent risk and the role of statistical sampling in compliance testing. *Proceedings of the 1984 Touche Ross/University of Kansas Symposium on Auditing Problems.* Lawrence, KS: School of Business, University of Kansas, 89-125.

LESLIE, D.A., ALDERSLEY, S.J., COCKBURN, D.J. and REITER, C.J. (1986) An assertion-based approach to auditing (discussant's remarks). *Proceedings of the 1986 Touche Ross/University of Kansas Symposium on Auditing Problems.* Lawrence, KS: School of Business, University of Kansas, 31-67.

SAFFIOTTI, A. and UMKEHRER, E. (1991) Pulcinella: a general tool for propagating uncertainty in valuation networks. *Proceedings of the Seventh National Conference on Artificial Intelligence*, University of California at Los Angeles, 323-331.

SENNETTI, J.T. (1990) Toward a more consistent model for audit risk. *Auditing: A Journal of Practice and Theory* (Spring), 103-112.

SHAFER, G. (1976) *A Mathematical Theory of Evidence.* Princeton University Press.

SHAFER, G., SHENOY, P.P. and SRIVASTAVA R.P. (1988) Auditor's Assistant: A knowledge engineering tool for audit decisions. *Proceedings of the 1988 Touche Ross/University of Kansas Symposium on Auditing Problems.* Lawrence, KS: School of Business, University of Kansas, 61-84.

SHAFER, G. and SRIVASTAVA R.P. (1990) The bayesian and belief-function

formalisms: A general perspective for auditing. *Auditing: A Journal of Practice and Theory*, **9**, (Supplement), 110-48.

SHENOY, P.P. and SHAFER, G. (1988) An axiomatic framework for bayesian and belief-function propagation. *Proceedings of the Fourth Workshop on Uncertainty in Artificial Intelligence*. St. Paul, MN: AAAI Press, 307-14.

SHENOY, P.P. and SHAFER, G. (1990) Axioms for probability and belief-function propagation. In: *Uncertainty in Artificial Intelligence*, **4**, edited by R. D. Shachter et al. Amsterdam: North-Holland, 169-98.

SRIVASTAVA, R.P. (1997A) Decision making under ambiguity: a belief-function perspective. *Archives of Control Sciences*, **6** (XLII) (forthcoming).

SRIVASTAVA, R.P. (1997B) An 'AND-NEOR' Node for Propagating Beliefs in a Network of Variables with Linear Algebraic Relationships. Working Paper. School of Business, The University of Kansas.

SRIVASTAVA, R.P. (1996) Value judgments using belief functions. *Research in Accounting Ethics*, **2**, 109-130.

SRIVASTAVA, R.P. (1995A) A general scheme for aggregating evidence in auditing: propagation of beliefs in networks. *Artificial Intelligence in Accounting and Auditing*, **3**, edited by Miklos A. Vasarhelyi, Princeton: Markus Wiener Publishers, 55-99.

SRIVASTAVA, R.P. (1995B) The belief-function approach to aggregating audit evidence. *International Journal of Intelligent Systems* (March), 329-356.

SRIVASTAVA, R.P. (1993) Belief functions and audit decisions. *Auditors Report*, **17**, 1, Fall, 8-12.

SRIVASTAVA, R.P. and COGGER, K. (1995) Algorithm for Determining *m*-values on the Joint Space of the Variables from Belief Functions Defined on Each Variable. Working Paper in progress. School of Business, The University of Kansas, Lawrence, Kansas.

SRIVASTAVA, R.P., DUTTA, S.K. and JOHNS, R. (1996) An expert system approach to audit planning and evaluation in the belief-function framework. *International Journal of Intelligent Systems in Accounting, Finance and Management*, **5**, 3, 165-183.

SRIVASTAVA, R.P., SHAFER, G. (1994) Integrating statistical and non-statistical audit evidence using belief functions: a case of variable sampling. *International Journal of Intelligent Systems*, **9**, 519-539.

SRIVASTAVA, R.P., SHAFER, G. (1992) Belief-function formulas for audit risk. *The Accounting Review*, **67**, 2 (April), 249-283.

SRIVASTAVA, R.P., SHENOY, P., SHAFER, G. (1995) Propagating beliefs in an 'AND' tree. *International Journal of Intelligent Systems*, **10**, 647-664.

TURNER, J.L. (1994) Order and Level Effects of Negative Evidence Regarding Internal Accounting Controls: An Examination of Alternative Models of Audit Judgment. Ph. D. Dissertation. Texas A & M University.

YAGER, R.R., KACPRZYK, J. and FEDRIZZI, M. (1994) *Advances in the Dempster-Shafer Theory of Evidence*. John Wiley.

XU, H. (1991) *Technical Report TR/IRIDIA/91-11*, Université Libre de Brux-

elles.

ZARLEY, D., YEN-TEH, HSIA and SHAFER, G. (1988) Evidential reasoning using DELIEF. *Proceedings of the Seventh National Conference on Artificial Intelligence*, **1**. Cambridge, MA: AAAI Press, 205-09.

Appendix

Algorithm for converting beliefs assessed on individual variables from a single item of evidence to m-values on the joint space

In order to illustrate the algorithm, let us consider the example described in Section 3.2 with the following beliefs on the three variables ('existence', 'completeness', and 'valuation'):

$$\text{Bel}(e) = 0.6 \text{ and } \text{Bel}(\neg e) = 0,$$

$$\text{Bel}(c) = 0.4 \text{ and } \text{Bel}(\neg c) = 0,$$

$$\text{Bel}(v) = 0.3 \text{ and } \text{Bel}(\neg v) = 0.1.$$

Step 1: Express the beliefs in terms of m-values on the individual frames of the variables.

$$m(e) = 0.6, m(\neg e) = 0, \text{ and } m(e, \neg e) = 0.4,$$

$$m(c) = 0.4, m(\neg c) = 0, \text{ and } m(c, \neg c) = 0.6,$$

$$m(v) = 0.3, m(\neg v) = 0.1, \text{ and } m(v, \neg v) = 0.6.$$

Step 2: List the m-values for each variable in a columnar form; columns for variables, and rows for their values (see Table 2).

Step 3: Select the smallest non-zero m-value in each column (i.e., for each variable). These values are written inside rectangular boxes in Table 2. These values define the elements of the joint space.

Step 4: Select the smallest m-value among the set obtained in Step 3. This value represents the m-value for the set of elements on the joint space generated by the product of individual elements corresponding to the m-values selected in Step 3.

Step 5: Subtract the m-value obtained in Step 4 from each selected m-value in Step 3.

Step 6: Repeat Steps 3 - 4 until all entries are zero.

For our example, the m-values generated on the joint space through the above algorithm are given in Table 2.

Iteration	Variables					
	Existence		Completeness		Valuation	
	Value of the variable	Corresponding m-value	Value of the variable	Corresponding m-value	Value of the variable	Corresponding m-value
1	e	0.6	c	**0.4**	v	0.3
	$\neg e$	0.0	$\neg c$	0.0	$\neg v$	**0.1**
	$\{e, \neg e\}$	**0.4**	$\{c, \neg c\}$	0.6	$\{v, \neg v\}$	0.6
	$m(\{ec\neg v, \neg ec\neg v\}) = 0.1$					
2	e	0.6	c	**0.3**	v	**0.3**
	$\neg e$	0.0	$\neg c$	0.0	$\neg v$	0.0
	$\{e, \neg e\}$	**0.3**	$\{c, \neg c\}$	0.6	$\{v, \neg v\}$	0.6
	$m(\{ecv, \neg ecv\}) = 0.3$					
3	e	**0.6**	c	0.0	v	0.0
	$\neg e$	0.0	$\neg c$	0.0	$\neg v$	0.0
	$\{e, \neg e\}$	0.0	$\{c, \neg c\}$	**0.6**	$\{v, \neg v\}$	**0.6**
	$m(\{ecv, ec\neg v, e\neg cv, e\neg c\neg v, \}) = 0.6$					
4	e	0.0	c	0.0	v	0.0
	$\neg e$	0.0	$\neg c$	0.0	$\neg v$	0.0
	$\{e, \neg e\}$	0.0	$\{c, \neg c\}$	0.0	$\{v, \neg v\}$	0.0

Table 2. m-values on the joint space from the beliefs on individual variables using Srivastava and Cogger (1995) algorithm