Power, monotonicity and expectations[1]

by

Manfred J. Holler

Institute of Economics, University of Hamburg,
Von-Melle-Park 5, 20146 Hamburg, Germany

A popular view of power indices, applied to voting games, is that these measures "represent a reasonable expectation of the share of voting power given by the ability to contribute to the formation of winning configurations" (Turnovec, 1997). That is

(a) there is an "ability" called "voting power", which we cannot measure directly; but

(b) we can calculate indices which represent "reasonable expectations" of this ability.

Often it is argued that these expectations are considered reasonable if the index measure, which is used to quantify these *expectations*, satisfies *monotonicity* "to some degree".

An alternative view is that the index as such expresses voting power, *and not* expectations of it. In this case, the measure has to be directly related to the ability to contribute to forming winning coalitions. This train of thought uses the *Public Good Index* (PGI), also called Holler-Packel index, for cases in which the coalition values can be considered public goods (see Holler and Packel, 1983, for the underlying axioms of this measure and Holler and Li, 1995, for extensions). The *basic principles* underlying the PGI are: • the public good property, i.e. nonrivalry in consumption and • nonexcludability of access; and the non-free-riding property. It is immediate from these principles that the strict minimum winning coalitions (SMWCs) should be considered when it comes to measuring power. All other coalitions are either nonwinning or contain at least one member which does not contribute to winning. If coalitions of the second type form, then it is by luck, similarity of preferences, tradition, etc. – *but not because of power*. Note that the PGI does not maintain that only SMWCs will form. In a recent publication, Brams and Fishburn (1995) present a member-MWC index, which is *formally identical* to the PGI. The member-MWC index

---

assumes that only SMWCs will form. This is clearly different from the story behind the PGI.

Once we accept the two quoted principles underlying the PGI, the index follows from the axioms which reflect these principles (see Holler and Packel, 1983). If a measure which builds on the two *PGI principles* in a reasonable way suffers from nonmonotonicity then we have to conclude that power is nonmonotonic. The PGI gives values which are nonmonotonic in voting weights. The standard example to demonstrate the nonmonotonicity of the Public Good Index is as follows: given a voting game $v = (d, w)$ with decision rule $d = 51$ and weight distribution $w = (35, 20, 15, 15, 15, 15)$, the PGI is $h(v) = (4/15, 2/15, 3/15, 3/15, 3/15)$. We get the power values of player i, $h_i(v)$,
(a) by counting the number of SMWCs of which i is a member, which gives us the decisiveness c_i of i, and
(b) by dividing c_i by the sum of all c_i values of the game.

One might well argue that the PGI does not build on the underlying principles in a reasonable way. Then, of course, we should look for a more proper measure. Alternatively, we could argue that the public good property and the non-free-riding property are simply not relevant with respect to voting power. These arguments, however, are quite different from saying that the PGI is not suitable to measure power because it does not satisfy the axiom of nonmonotonicity.

When it comes to monotonicity of power with respect to voting weights, it is important to note that none of the existing measures guarantees that the power measure of a player i will *not* decrease if the voting weight of i increases. Fisher and Schotter (1978) demonstrate this result (i.e., the *paradox of redistribution*) for the Shapley-Shubik index and the normalized Banzhaf index. This paradox stresses the fact that power is a social concept: if we discuss the power of an individual member of a group in isolation from his or her social context, we may experience all sorts of paradoxical results. It seems that sociologists are quite aware of this problem and nonmonotonicity of an individual's power with respect to his or her resources does not come as a surprise to them (see, e.g., Caplow, 1968). Political scientists, however, often see the nonmonotonicity of power as a threat to the principle of democracy. To them it is hard to accept that increasing the number of votes a group has could decrease its power, although it seems that there is ample empirical evidence for it (see Brams and Fishburn, 1995, for references.) In general, economists also assme that more resources is more likely to mean more power than less. However, they also deal with concepts like monopoly power, bargaining, and exploitation which stress the social context of power and the social value of resources (assets, money, property, etc.). Note that in the discussion of power indices voting weights are a proxy for resources.

There are, of course, different concepts of monotonicity, and the concept to which the *paradox of redistribution* refers is a rather strong one. Turnovec (1997) analyzes various monotonicity concepts which are satisfied by, e.g., the Shapley-Shubik index but not by the PGI. This kind of study could help to

distinguish differences in the power indices.

Needless to say, the PGI is unsuitable to express our *expectations* if our intuition, which is at the heart of these expectations, implies monotonicity. However, if we could trust our intuition, then power indices in general would be rather useless. The number of paradoxes related to the application of these measures, which are the result of a deviation from intuition, indicates that our intuition most likely needs help when it comes to evaluating power - or forming "reasonable expectations" with respect to power.

References

BRAMS, S.J., FISHBURN, P.C. (1995) When size is liability? Bargaining power in minimal winning coalitions. *Journal of Theoretical Politics*, **7**, 301-316.

CAPLOW, T. (1968) *Two Against One: Coalitions in Triads.* Prentice-Hall, Englewood Cliffs, N.J.

FISCHER, D., SCHOTTER, A. (1980) The inevitability of the paradox of redistribution in the allocation of voting weights. *Public Choice*, **33**, 46-67.

HOLLER, M.J., LI, X. (1995) From public good index to public value: an axiomatic approach and generalization. *Control and Cybernetics*, **24**, 257-270.

HOLLER, M.J., PACKEL, E.W. (1993) Power, luck and the right index. *Zeitschrift für Nationalökonomie (Journal of Economics)*, **43**, 21-29.

TURNOVEC, F. (1997) *Monotonicity and Power Indices.* Institute for Advanced Studies, Vienna (East European Series, No. 41).