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On power indices and minimal winning coalitions

by

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1. Introduction

Theoretically speaking, measuring means assigning numbers (or other symbols, as the case may be) to objects or other foci of interest. Without further requirements, measurement does not help us in achieving other objectives apart from possibly impressing or bewildering our fellow citizens. To be useful the results of measurement have to represent the properties measured so that the measurements can in some contexts be used instead of the properties themselves. Particularly convenient are measures that allow for mathematical operations to be performed on the measurement values so that the results of those operations are meaningful.

The standard way of going about establishing that the measurements represent some property in the objects themselves is to prove a representation theorem. The starting point is some qualitative property whose presence or absence in objects or relationships can at least in principle be established. In the case of probability measurement this the property of "being at least as probable as", in the case of utility it is the property of "being as good as", etc. In representation theorems one then proceeds to pin down those conditions of these properties or relationships that guarantee the existence of a measure whose results represent the properties in question. Moreover, the conditions determine to what extent the measurements yield unique results and, thus, which formal operations on measurements allow for meaningful interpretations.

2. Power indices as theoretical notions

In the case of power measurements one should bear in mind that the current power indices – i.e. the Shapley-Shubik, Penrose-Banzhaf, Holler and Johnston indices - are all meant to be theoretical *a priori* concepts. The earliest one of them, the Shapley-Shubik index, was originally designed to correspond to what in two-person games is called the value of the game. However, there

is an important difference between the value of a two-person game and the Shapley-Shubik index (or its more general counterpart – the Shapley value). To wit, while the former indicates the payoff that a player is unilaterally able to guarantee for himself/herself, the latter indicates the expected value of such a payoff, provided that certain probabilistic assumptions hold.

The power indices relate to empirical observations in the same way as probabilistic statements in general relate to empirical frequency counts, viz. almost any sequence of observations is in principle compatible with any probabilistic statement concerning the phenomena about which the observations have been made. This is a result of the definition of probability as the limit of relative frequency in an infinitely long sequence of events and the definition of power index values as probabilities of belonging to certain types of winning coalitions, provided that certain probabilistic assumptions concerning the coalition formation hold.

There are two ways in which a statement concerning the power index value of a player may be wrong:

1. The real coalition formation process differs from that postulated by the index.
2. The observation sequence is too short for the convergence of the empirical frequency estimates to the probability values.

3. Which coalitions should be considered?

One controversy regarding the power indices pertains to what kind of coalitions ought to be considered in defining the indices. The Shapley-Shubik and Penrose-Banzhaf indices focus on all winning coalitions, whereas the Holler and Johnston indices concentrate on minimal winning coalitions. If one knows that the coalitions being formed are likely to be of either one of these types, then obviously the corresponding power index family is more appropriate than the other. There is very little point in arguing the pros and cons of the indices if the the coalition formation process type is known. One should simply pick an index with the known type of underlying coalition formation type. Of course, this type is not often known.

In political contexts, however, the power measures based on minimal winning coalitions seem less plausible than those based on winning coalitions for the simple reason that the political actors in general seem to be interested in increasing their support (seats). In terms of the Holler and Johnston indices this might in some contexts entail a decrease in power. Thus, either the actors are not primarily interested in increasing their power (which would certainly be surprising) or the measures based on minimal winning coalitions are inappropriate.

4. Yet another interpretation

There exists a relatively extensive literature on interpretation of power indices (see *e.g.* Nurmi, 1980, for references). Moreover, the critics of power indices have pointed out that the dispositional properties of the intuitive notion of power are not captured by power indices (see Barry, 1991, Dowding, 1997). This is, indeed, the case and it is very difficult to see how any *a priori* index could accomplish this. Power as an intuitive notion contains the idea that an actor is able to make another actor to do something that the latter would not otherwise do. The indices, in turn, are based on what is observable, i.e. has already happened. Thus, the counterfactual aspect ("had actor i not made actor j do x, x would not have been done by j") cannot possibly be captured in terms of observables only. What we see is simply that certain coalitions have formed, some winning and some non-winning.

It would, however, be too hasty to say that the power indices are useless because they cannot capture the dispositional aspect of power. One should bear in mind that the power indices are *a priori* concepts. They measure the probability of an actor's being on the winning side, given certain assumption concerning coalition formation. Rather than measuring his/her influence on the opinions of the others and, consequently, on the outcomes of the collective decision making, they reflect the probability that his/her views are represented in the outcomes (see Nurmi, 1997, for further discussion). More specifically, they measure the probability that an actor with a given amount of resources, given the resource distribution over the other actors and the decision rule, has views that coincide with those expressed in collective decisions assuming that the coalition formation takes place in accordance with the principles underlying the index. No causal attribution is thereby made with regard to whose opinions are dominant.

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