

Power, power indices and intuition¹

by

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I very much agree with Manfred Holler's statement (Holler, 1997) about intuition with respect to measuring of power (or, perhaps better, voting power). He says: "If we could trust our intuition, then power indices in general would be rather useless. The number of paradoxes related to the application of these measures, which are the result of a deviation from intuition, indicate that our intuition most likely needs help when it comes to evaluating power - or forming 'reasonable expectations' with respect to power". I am not stating that public good index is not suitable to measure power because it does not satisfy monotonicity. The only thing we probably disagree on is what power indices are about.

Power is a fuzzy, not well defined concept. In a general framework, anybody will agree that the real power in real committee might be non-monotonic (and there is a lot of empirical evidence of that). What we are speaking about is a priori evaluation of voting power in a specific model of a committee: it relates to a "direct expression of voting power" as a simple general economic equilibrium model (in conditions of perfect competition) to the real world economic equilibrium. Power index speaks about the properties of a model, not about the properties of the power as such.

Using mathematics, we are operating in a wonderful world of virtual reality. The sense of that is, of course, to identify, describe, touch, some properties of real phenomena, but certainly not directly. What is the question the power indices pretend to answer?

We have an extremely simple model of a weighted voting game, described by quota and distribution of votes, or, by a super-additive simple game. Then, assuming nothing about preferences, restrictions on coalition formation etc., we ask the following question: having no additional information that is not

¹Few comments on the discussion, initiated by Manfred Holler, by his note *Power, monotonicity, and expectations* (see this volume).

described by this model, and assuming that an infinite (or very large) number of voting events will take place, what is the probability (or, the reasonable expectation) that an actor's YES will be decisive?

There are two ways of approaching the problem.

- a) One way is to assume that the model is complete: there are no a priori preferences and no aversions and restrictions on coalition formation (a sort of perfect voting competition).
- b) The model is incomplete, and we just do not have the additional information.

In the first case we are measuring an abstract voting power in an abstract model of a committee. In the second case we have to add something to the model and classify possible cases. I would propose a new terminology: *unconstrained power indices* (first case) and *constrained power indices* (second case).

The most of power indices are presented as unconstrained ones. There is no explicit amendment of the model in Shapley-Shubik, Banzhaf-Coleman, Holler-Packel, Johnston, Deegan-Packel etc. In this case the question arises: why, measuring the same phenomena, they are generating values with different properties?

Axiomatization gives some guide as to how to evaluate and distinguish properties of different indices. With respect to the original model the traditional intuition seems to be acceptable. Perhaps, the word *properties* would be more appropriate than *axioms*, because the use of the notion of axioms creates an impression that they are expressing some inherent characteristics of power as such. Implication of my analysis raises just a question mark over Banzhaf, Holler and perhaps other indices as abstract evaluations of a priori voting power in a simple model mentioned, that is considered to be complete.

What Manfred Holler probably has in mind is what I call constrained power indices. (Holler uses sometimes a nice expression *power story* in a very similar context). We have a nice example of constrained power indices: a priori unions of Guillermo Owen (1982, 1997). What I believe is that public good index is one of them. It implicitly introduces additional behavioral assumption: that coalition value in voting games is a public good, what can be true in some cases, but not always. By the way, working with minimal critical winning coalitions is not probably the only way of implementing this assumption (If we define power index on the basis of minimal critical winning coalitions of the minimal size, it will have monotonicity property. By minimal critical winning coalition of the minimal size I mean a subset of minimal critical winning coalitions having minimal number of members.). Then the question is: what does an individual actor benefit from this public good? But this is about something else than about a priori abstract evaluation of a voting power in an unconstrained committee.

The future of power indices as measures closer to the real world committees is undoubtedly in the development of the concept of constrained power indices. Behavioral assumptions, preferences and coalition formation restrictions, as well as more sophisticated voting rules, have to be added to the original simple model and this is the reason I prefer weighted majority game design of the model to

simple games: it creates more space for formulating of structural properties and “constraints”. Then, each model will have its own intuition that might be axiomatized.

My monotonicity analysis (Turnovec, 1997) has no ambition of answering the straightforward question: which index is right? It rather indicates that there is something missing in power analysis: a unified approach to the problems of modelling and evaluating of voting and decisional power.

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