

Power and expectations¹

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Expanding Europe has provoked a wide discussion on the need for changes in the decision-making procedures. This entails the necessity of reflecting in the adopted decision-making techniques both what can be said in most general terms about a given country using the word *power* and the related *expectations*. Notwithstanding the inherent difficulties in determining indices of power, it would seem that the choice of a particular decision-making technique should ensure the achievement of two basic objectives, (a) to enable the correct manifestation of one's preferences on a set of alternatives, and (b) to guarantee that the position of a decision-maker (country) - understood as the degree of influence on the group decision and the distribution of the resulting 'profit' - within the given decision-making technique should not be 'worse' than that of other decision makers of an equal status.

1. Power measures

A number of different indices of power are used to measure the position of the i -th decision-maker. First, there is the set of static indices (*a priori* power indices): the Shapley-Shubik index (1954) - $SS(i)$; the Banzhaf index (1965, 1968) - $B(i)$; the Coleman coalition maintenance index (1971) - $CP(i)$; the Coleman action initiation index (1971) - $CI(i)$; the Coleman group capacity index (1971) - $C(A)$; the Rae index (1969) - $R(i)$; the Zipke index (Nevison, 1979) - $Z(i)$; the Brams-Lake index (1977) - $BL(i)$; the Deegan-Packel index (1979) - $DP(i)$; the Holler index (1982) - $H(i)$; and the Johnson index (1978) - $J(i)$.

¹This note contains impressions from the Józefów (July 1996) 3rd Bi-annual Meeting of Working Group WG-7 'Group Decision Making and Power Indices' and the discussion there with Gianfranco Gambarelli (Bergamo), Manfred Holler (Hamburg), Jerzy Hołubiec (Warsaw), Annick Laruelle (Brussels), Hannu Nurmi (Turku), Honorata Sosnowska (Warsaw), Frantisek Turnovec (Prague), Mika Widgren (Helsinki), and is published as a comment on the article by Josep Freixas and Gianfranco Gambarelli (in this volume).

However, the views presented here are exclusively those of the author.

As an alternative, there are dynamic indices based on the probability of specified events: the Gehrlein-Fishburn index (1986) - $GF(i)$; and the cabinet power index (Mercik, 1996).

The *a priori* power indices can be reduced to a certain extent by showing their mutual relationships. For instance, the following relationships can be found:

1. $CP(i) = \frac{2^n}{w} \left(Z(i) - \frac{w}{2^n} \right)$ (Nevison, Schoepke, Zicht, 1978),
2. $B(i) = 2 \left(Z(i) - \frac{w}{2^n} \right)$ (Nevison, 1979),
3. $B(i) = 2 \left(BL(i) - \frac{1}{2} \right)$ (Brams, Lake, 1977),
4. $BL(i) = \frac{Z(i) + Z^*(i)}{2}$ (Nevison, 1979),
5. $\frac{1}{B(i)} = \frac{1}{2} \left(\frac{1}{CI(i)} + \frac{1}{CP(i)} \right)$ (Shubik, 1982),
6. $C(A) = \frac{w}{2^n}$ (Coleman, 1971),

where w denotes the total number of winning coalitions; n denotes the number of voters, and $Z^*(i)$ denotes the value of the Zipke index for a dual game.

As a result of the partial mutual substitutability of the *a priori* indices of power, despite their great diversity, two are used in practice: the Shapley-Shubik index of power and the Banzhaf absolute power index. The former evaluates the power of a given voter related to coalition formation, the latter evaluates the power of a voter related to his ability to overturn particular coalitions.

A separate role is played by those *a priori* indices that either reject the assumption of equal chances for each coalition (e.g. Owen, 1977; Nurmi, 1980) or are connected with the notion of Public Good (Holler, Packel, 1983; Holler, Li, 1995) or assume a particular coalition structure (e.g. Johnson, 1978; Turnovec, 1995; Widgren, 1995). In the opinion of their proponents, each of those indices is, in particular situations, the best measure of the power of a voter.

Dynamic power indices provide a better reflection of the unpredictability of the behaviour of participants in a decision-making process but require very strong assumptions concerning probability distributions or 'escape' into boundary distributions based on the Gaussian distribution, which in a way 'smooths out' the real behaviour of decision-makers and requires substantial statistical data.

2. Evaluation and expectations

It would seem to be expected that particular power indices should behave in accordance with the expectations of their proponents. However, this is not the case. There are certain phenomena that are described in the power index literature as paradoxes. They include, for instance, the Paradox of Quarrelling (Kilgour, 1974; Straffin, 1976; Brams, 1975, 1976; Nevison, 1979), the Paradox of Size (Brams, 1975; Holubiec, Mercik, 1994; Turnovec, 1997), and the Paradox of Cohesiveness (Gehrlein, Fishburn, 1986; Mercik, 1990). In particular the paradoxes involve violations of the monotonicity of power indices, as understood in the Freixas and Gambarelli paper that sparked off this note. It is reasonable

Country	%-6	SS-6	%-9	SS-9	%-10	SS-10	%-12	SS-12	%-15	SS-15
Belgium	11.76	15.00	8.62	8.10	7.94	7.46	6.58	6.40	5.75	5.50
Denmark			5.17	5.70	4.76	5.67	3.95	4.90	3.45	3.30
Germany	23.53	23.33	17.24	17.90	15.87	16.39	13.16	13.40	11.49	12.10
Greece					7.94	7.46	6.58	6.40	5.75	5.50
Spain							10.53	11.10	9.20	9.00
France	23.53	23.33	17.24	17.90	15.87	16.39	13.16	13.40	11.49	12.10
Ireland			5.17	5.70	4.76	5.67	3.95	4.20	3.45	3.30
Italy	23.53	23.33	17.24	17.90	15.87	16.39	13.16	13.40	11.49	12.10
Luxembourg	5.88	0.00	3.45	1.00	3.17	0.71	2.63	1.20	2.30	2.00
Netherlands	11.76	15.00	8.62	8.10	7.94	7.46	6.58	6.40	5.75	5.50
Austria									4.60	4.30
Portugal							6.58	6.40	5.75	5.50
Finland									3.45	3.30
Sweden									4.60	4.30
UK			17.24	17.90	15.87	16.39	13.16	13.40	11.49	12.10
Total	100	99.99	100	100.2	100	99.99	100	100.6	100	99.9

Table 1. (After Turnovec, 1996)

to pose the question whether, given that the paradoxes are as common as it is observed, it is possible to find an index of power that is resistant at least to most, if not to all, of the power paradoxes. In my opinion such power indices do not exist in the class of static indices. The problem is partly solved by the introduction of indices of power as interval values or, even more generally, fuzzy values, but the problem that then emerges is the match between the power as measured by a given index and the expectations with regard to the 'payoff'. Perhaps an approach based on Holler's Public Good Index might be a compromise?

As has been mentioned, the fundamental problem of voters' expectations is the match between the power of a given voter and his claim to the distribution of the 'payoff' resulting from the decision. This is closely related to power indices, but in view of the fact that different indices of power yield different results for the same decision-making body, an unequivocal distribution of the 'payoff' on their basis is impossible. If we look at the European Union data, Table 1 (Turnovec, 1996), we can come to the firm conclusion that the system of allocating votes to particular states used by the EU Council of Ministers preserves the best match between the number of votes and the resulting power of a given state if power is measured by means of the Shapley-Shubik index. Is this result not a strong argument in favour of using this very index of power exclusively?

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