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Coalitions and rationality[1]

by

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Let us consider a cooperative game $G = (N, W)$ where $N = \{1, \ldots, n\}$ is the set of players, W is an arbitrary family of subsets of N. Subsets of N are called coalitions, elements of W are called winning coalitions. One of most popular example of cooperative game is a voting game, where each player has a weight w_i, where w_i is a natural number. Coalition C is a winning coalition if the sum of weights of members of coalition C is greater than r, where r is defined by the game. Usually, r is greater than a half of the sum of all weights. Voting games are used to describe a situation in a parliament. Players are parties. Weight of a party is the number of members of the parliament who represent the party.

In weighted games the power of a coalition is connected with the number of its members. Addition of a new member to a winning coalition forms another winning coalition. So, a family of winning coalitions is monotonic. If we describe a situation in a parliament and use the voting game we assume that the following statements are true:

(i) All members of a party vote in the same way in the parliament

(ii) Each coalition can be formed.

In real life these statements are not always fulfilled. So, a family of winning coalitions observed in real life may be not monotonic. Monotonicity of a family of winning coalitions assumes some kind of rationality of political decisions. Monotonic family of winning coalitions may be constructed in many ways. A structure of such family may have various properties. Some of them may be corresponding to specific properties for a system of players' group decision.

Voting systems are tools invented by people for group decision making. Some intuitions are foundations of constructions of voting systems but these intuitions are rather fuzzy than well defined. For example, let us recall "liberum veto" rule in Polish parliamentary system in the 17^{th} and 18^{th} centuries. It was a case of minority voting. Each person could break a session of the parliament and annul all decision when he was voting in a special way against one of decisions which

[1] This note is a comment on the article by Josep Freixas and Gianfranco Gambarelli and on the note by Manfred Holler published in this volume.

were taken during this session. The rule seems quite irrational, but intuitions of freedom and responsibility of members of parliament were the basis of the "liberum veto" rule. Effects of applying this rule were different from the assumed purposes but nobody predicted it. So, it would be difficult to describe a rational intuition connected with this rule. The following problem is open to discussion. Can we base on intuitional properties in the analysis of voting systems?

If we exclude intuition from studies of voting systems we have to construct tools of analysis. Power indices are such tools. So, they can have unexpected properties connected with their construction. It may be interesting to study which structures of a family of winning coalitions assure special forms of intervals, for example one point interval or the same interval for monotonic and strictly monotonic property.

Desirability relation describes in which sense rationality is realized by a family of winning coalitions. Monotonicity of a power index means that a rationality connected with a structure of a family of winning coalitions is consistent with a rationality which is the basis for construction of a power index.

Here ends the discussion referring to power indices and their properties. We return to the presentation of regular papers.