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A satisficing method of introducing the concepts of fuzzy goal and fuzzy constraints into the DEA

by

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Abstract: Evaluation of efficiency of the DMUs (Decision Making Units) in a company can be carried out with the help of DEA (Data Envelopement Analysis). The efficiency calculated for a DMU_0 with the help of DEA is, however, higher than the maximum efficiency among the single object-single output results. It can be postulated that total efficiency is constrained between the limits thus obtained. Within this context we introduce the concepts of fuzzy goal and fuzzy constraints into the DEA formulation, propose the satisficing method following the precepts of the maximizing decision introduced by Bellman and Zadeh, and the improvement procedure for the satisficing solution using the dialogue with the DM over the tradeoff rate for two inputs, developed by Sakawa.

Keywords: DEA, fuzzy goal, fuzzy constraint, satisficing method

1. Introduction

Evaluation of efficiency of the DMUs (Decision Making Units) in a company can be carried out with the help of DEA (Data Envelopement Analysis), Charnes, Cooper, Rhodes (1978), Tone (1993). The efficiency calculated for a DMU_0 with the help of DEA is, however, higher than the maximum efficiency among the single object-single output results. It can be postulated that total efficiency is constrained between the limits thus obtained. Within this context we introduce the concepts of fuzzy goal and fuzzy constraints into the DEA formulation, propose the satisficing method following the precepts of the maximizing decision introduced by Bellman and Zadeh, and the improvement procedure for the satisficing solution using the dialogue with the DM over the tradeoff rate for

2. The DEA based on the possibility production set

The original formulation of DEA involved fractional mathematical programming and the direct estimation of the efficiency of DMUs. Besides this, though, DEA has been also formulated on the basis of the concept of constructing the possibility production set, Tone (1993). We will introduce now the approach to DEA modeling based on the possibility production set.

Let us have n input sets $X_j = (x_{j1}, \ldots, x_{jm})$ in DMU_j $(j = 1, \ldots, n)$ and the common output set $Y_j = (y_{j1}, \ldots, y_{js})$. We denote the input vector $\vec{X} = [X_1, \ldots, X_n]$ and the output vector $\vec{Y} = [Y_1, \ldots, Y_n]$. We define the possibility production set (x, y) as the set satisfying the following constraints:

$$\begin{array}{rcl} x & \geq & \vec{X}\lambda \\ y & \leq & \vec{Y}\lambda \\ \lambda & \geq & 0 \\ L & \leq & e^T\lambda \leq U \end{array} \tag{1}$$

where $x \in R^m$, $y \in R^s$, $\lambda \in R^n$, and $e^T = (1, \ldots, 1)$.

By (1), we constrain the possibility production set in DEA. Therefore, various DEA models were proposed by varying L and U. Specifically, when $L = 0$, $U = \infty$, this is the CCR (Charnes-Cooper-Rhodes) model, and when $L = U = 1$, this is the BCC (Banker-Charnes-Cooper) model. Let us denote efficiency by Θ, and then the CCR and BCC models are formulated as follows:

CCR model 1

$$\begin{array}{l} \min \Theta \\ \text{s.t. } \Theta x_0 \geq \vec{X}\lambda \\ y \leq \vec{Y}\lambda \\ \lambda \geq 0 \end{array} \tag{2}$$

Here, for $\Theta^* = 1$ we may again have surplus or shortage of inputs or outputs. Therefore, we generally formulate the CCR model using the slack variables as follows:

CCR model 2

$$\begin{array}{l} \min \Theta - e(e^T s^+ + e^T s^-) \\ \text{s.t. } \Theta x_0 - s^+ = \vec{X}\lambda \\ y + s^- = \vec{Y}\lambda \\ \lambda, s^+, s^- \geq 0 \end{array} \tag{3}$$

BCC model 1

$$\begin{aligned}
&\min \Theta \\
&\text{s.t. } \Theta x_0 \geq \vec{X}\lambda \\
&e^T\lambda = 1 \\
&\lambda \geq 0
\end{aligned} \tag{4}$$

When $\Theta^* = 1$ in formulation (4), we may have surplus or shortage of inputs or outputs. Therefore, we generally formulate the BCC model using the slack variables as follows:

BCC model 2

$$\begin{aligned}
&\min \Theta - e(e^T s^+ + e^T s^-) \\
&\text{s.t. } \Theta x_0 - s^+ = \vec{X}\lambda \\
&y + s^- = \vec{Y}\lambda \\
&e^T\lambda = 1 \\
&\lambda, s^+, s^- \geq 0
\end{aligned} \tag{5}$$

where Θ^* is the solution for (5), and is referred to as the DEA efficiency.

3. Fuzzy satisficing method introducing fuzzy goal and fuzzy constraints into DEA

In this section, let us assume that we focus on the BCC model 2 of (5), because the discussion for the CCR model is the same as for the BCC model. We obtain one objective solution Θ_k^* for an individual output y_{kj} $(k = 1, \ldots, s)$ in some DMU_j as follows:

$$\begin{aligned}
&\min \Theta_k - e(e^T s^+ + e^T s^-) \\
&\text{s.t. } \Theta_k x_{ij} - s_i^+ = \vec{X}_i\lambda \\
&y_{ij} + s_i^- = \vec{Y}_i\lambda \\
&e^T\lambda = 1 \\
&\lambda, s_i^+, s_i^- \geq 0
\end{aligned} \tag{6}$$

Let us denote $\max_k \Theta_k^*$ by Θ_1, and $\min_k \Theta_k^*$ by Θ_2, where Θ_k^* is the solution to (6).

THEOREM 3.1 *On the nature of DEA efficiency:*

$$\Theta_1 \leq \Theta^*$$

According to the BCC model 2 of (6), we can obtain a fuzzy goal ($\mu_G: \Theta \rightarrow [0,1]$) as follows:

$$\mu_g(\Theta) = \begin{cases} 0, & (\Theta \leq \Theta_2) \\ \frac{(\Theta - \Theta_2)}{(\Theta_1 - \Theta_2)} & (\Theta_2 \leq \Theta \leq \Theta_1) \\ 1, & (\Theta_1 \leq \Theta) \end{cases} \tag{7}$$

Making full use of the extension principle for binary operations, we can obtain fuzzy constraint ($\mu_C(\Theta_k): \Theta_k \rightarrow [0,1]$) for the first constraint in (6) according to inputs as follows:

$$\mu_C(\Theta_k) = \begin{cases} 1, & (\Theta_k \leq \Theta_2 \cdot x_{ij}) \\ \frac{(\Theta_1 \cdot x_{ij} - \Theta_k)}{(\Theta_1 \cdot x_{ij} - \Theta_2 \cdot x_{ij})} & (\Theta_2 \cdot x_{ij} \leq \Theta_k \leq \Theta_1 \cdot x_{ij} \\ 0, & (\Theta_1 \cdot x_i j) \leq (\Theta_k) \end{cases} \tag{8}$$

Let us note that we do not introduce any other fuzzy constraints into (6).

Bellman and Zadeh (1970) defined the maximizing decision for LP with a fuzzy goal and fuzzy constraints. We apply this maximizinig decision to DEA as follows:

$$\mu_d(\hat{\Theta}) = \max\{\min(\mu_G(\Theta), \mu_C(\Theta_1), \ldots, \mu_C(\Theta_s)\}$$

On the basis of the maximizing decision, we can formulate the Fuzzy Satisficing DEA model as follows (see Bellman and Zadeh, 1970, Sakawa, 1993):

$$\begin{aligned} &\min \alpha \\ &\text{s.t. } \alpha \leq \mu_G(\Theta) \\ &\alpha \leq \mu_C(\Theta_k),\ k = 1, \ldots, s \\ &y + s^- = \vec{Y}\lambda \\ &e^T\lambda = 1 \\ &\lambda, s^- \geq 0 \end{aligned} \tag{9}$$

with (9) solved by LP, and the fuzzy satisficing solution denoted $\hat{\Theta}$. After having evaluated DMU_js in terms of the fuzzy satisficing efficiency, we obtain the improvement plan for inputs according to the following theorem:

THEOREM 3.2 *Improvement plan for inputs:*

$$\begin{aligned} &\hat{\Theta} \cdot x_{ij} && (\Theta_k \leq \hat{\Theta}) \\ &1 \cdot x_{ij} && (\hat{\Theta} \leq \Theta_k) \end{aligned}$$

On the other hand Sakawa (1993) developed the interactive method for improvement of the fuzzy satisficing solutions from the fuzzy LP problems. We apply this method to the interactive fuzzy DEA, where we change the following tradeoff rate instead of the one described in Chapter 4 of Sakawa (1993):

$$\frac{\mu_C(\Theta_a)}{\mu_C(\Theta_b)} = \frac{\pi_a}{\pi_b}$$

where π_a and π_b are simplex multipliers

4. Conclusions

The DEA-defined efficiency is higher than the max efficiency among the single-object solutions for individual outputs. As we consider to be natural that total efficiency is located between min efficiency and max efficiency, we introduce the concept of fuzzy goal and fuzzy constraints into the DEA, propose the satisficing method conform to the maximizing decision defined by Bellman and Zadeh, and then the improvement method for the satisficing solution helped by dialogue with the DM over the tradeoff rate for two inputs.

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