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Book review:

MULTIVARIATE APPROXIMATION AND SPLINES

by

G. Nürnberg, J. Schmidt, G. Walz,
Editors

This a selection of papers presented at the international conference "Multivariate Approximation and Splines" held in Mannheim, Germany, on September 7-10,1996. The selected 24 papers provide an overview of current research in multivariate approximation. Most of them are research papers devoted to approximation, interpolation, data fitting, splines, neural networks, wavelets. There are 3 survey papers.

- O.V. Davydov (Dnepropetrovsk), M. Sommer (Eichstätt), H. Strauss (Erlangen) – I. *On almost interpolation by multivariate splines*, The paper gives a survey of some recent results in multivariate interpolation related to almost interpolation sets with respect to finite dimensional spaces.
- W. Li (Norfolk) – *Unconstrained minimization of quadratic splines and applications.* This is a review of unconstrained minimization of multivariate quadratic splines.
- G. Nürnberg (Mannheim), O.V. Davydov (Dnepropetrovsk), G. Walz, F. Zeilfelder (Mannheim) – *Interpolation by bivariate splines on crosscut partition.* The survey is given of recent methods for constructing interpolation points for spaces of splines of arbitrary degree and arbitrary smoothness.

One paper is directly oriented towards applications in tomography

- W. Schempp (Siegen) – *Wavelet modelling of high resolution radar imaging and clinical magnetic resonance tomography*

The remaining papers are devoted to different aspects of multivariate approximation

1. V.F. Babenko, V.A. Kofanov, S.A. Piguchov (Dnepropietrovsk) – *Multivariate Inequalities of Kolmogorov type and their applications.* Kolmogorov type inequalities are proved for multivariate periodic functions and the connections are investigated between these inequalities and the abstract problem of approximation of a given class of function by another class of functions.
2. D. Bainov (Sofia), A. Dishliev (Sofia), S. Hristova (Plovdiv) – *Monotone iterative technique for impulsive differential-diffierence equations with variable impulsive perturbations.* Sequences of lower and upper solutions are constructed which converge to a solution of the initial value problem for nonlinear impulse differential difference equation

3. K. Bittner (Rostock), C.K. Chui (Austin), J. Prestin (Rostock) – *Multivariate cosine wavelets.* Bivariate folding operators are used to construct biorthogonal cosine wavelets on rectangular grids with bell functions not necessarily of the tensor type.
4. O. Davydov (Dnepropetrovsk), M. Sommer (Eichstätt), H. Strauss (Erlangen) – *Locally linearly independent systems and almost approximation.* A method is presented for constructing almost interpolation sets in the case of existence of locally linearly independent systems of basis functions.
5. F.-J. Delvos (Siegen) – *Exponential-type approximation in multivariate harmonic Hilbert spaces.* The concept of harmonic Hilbert space is introduced in the multivariate setting as an extension of periodic Hilbert spaces.
6. M. von Golitschek (Würzburg) – *Interpolation by continuous functions spaces.* Interpolation operators are investigated for which the operator norm grows with the dimension of the approximation space.
7. A. Kamont (Sopot) – *Discrete characterization of Besov spaces and its applications to stochastics.* Characterizations of multivariate Besov spaces are given which involve only the values of functions on dyadic points.
8. B. Lenze (Dortmund) – *One-sided approximation and interpolation operators generating hyperbolic sigma-pi neural networks.* A method is proposed for designing three-layer feedforward neural networks which are used for one-sided approximation and interpolation of regular gridded data.
9. W. Light (Leicester) – *Interpolation by translates of a basis function.* A straightforward approach to the variational principle is provided which allows to discuss the connection of the variational principle with multivariate approximation problems.
10. T. Lyche (Oslo), K. Scherer (Bonn) – *On the sup-norm condition number of the multivariate triangular Bernstein basis.* An upper bound is given for the condition number of the multivariate Bernstein basis with respect the the sup-norm.
11. J.C. Mason (Huddersfield), E. Venturino (Leeds) – *Integration method of Clenshaw-Curtis type based on four kinds of Chebyshev polynomials.* It is shown that discrete orthogonality formulae holds for four kinds of Chebyshev polynomilas. Each formula yields a quadrature formula of Clenshaw-Cirtis type for integrating weighted functions.
12. B. Mulansky (Dresden) – *Tensor products of convex cones.* It is shown that cones arising in shape preserving interpolation by tensor products are intersections of injective cones. In consequence, sufficient conditions for multivariate shape constraints are derived from the univariate conditions.
13. E. Novak, K. Ritter (Erlangen) – *The curse of dimension and a universal method for numerical integration.* The problem is studied how to compensate a high number of variables by a high degree of smoothness of the underlying functions or by a favorable structure of the problem.
14. G. Plonka (Rostock) – *Necessary and sufficient conditions for orthonor-*

mality of scaling vectors. Conditions for orthonormality of scaling vectors are investigated in terms of their two-scale symbols and the corresponding transfer operators.

15. D. Potts (Lübeck), G. Steidl (Mannheim), M. Tasche (Rostock) – *Trigonometric preconditioners for block Toeplitz systems* For positive definite double symmetric block-Toeplitz matrices (with Toeplitz bloks arising from a generating function of the Wiener class) optimal and Strang-type preconditioners are constructed by using Fejér and Fourier sum of the generating function.
16. M. Reimer (Dortmund) – *The average size of certain Gram-determinants and interpolation on noncompact sets.* It is proved that Gram determinants defined by reproducing kernels can be averaged with respect to the arguments occuring. As a consequence, it is shown that there exist interpolation points which yield small sup-norms of the corresponding Lagrange functions.
17. R. Schaback (Göttingen) – *Radial basis functions viewed from cubic splines.* For problems of interpolation by radial functions, by applying an optimality principle for quasi-interpolants which reproduce polynomials, local error bounds are given for interpolation by natural cubic splines.
18. G. Schmeisser, J.J. Voss (Erlangen) – *A new interpretation of the sampling theorem and its extensions.* Equivalence between sampling of signals and sampling of entire harmonic functions is proved which allows to give a new uniqueness theorem for entire harmonic functions of exponential type.
19. J.W. Schmidt, M. Walter (Dresden) – *Gridded data interpolation with restrictions on the first order derivatives.* Interpolation of data by biquadratic and biquatric splines on rectangular grids is investigated. The interpolation problem is shown to be solvable provided additional knots are added to the original grid.
20. J. Stöckler (Stuttgart) – *Affine frames and multiresolution.* Generalized Laurent operators are used to study multivariate affine frames which are generated by multiresolution with a single scaling function.

Ewa Bednarczuk

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