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Book review:

PSEUDO-DIFFERENTIAL OPERATORS, SINGULARITIES, APPLICATIONS

by

Yuri V. Egorov, Bert-Wolfgang Schulze

The book is devoted to study of pseudo-differential operators. For the past 30 years theory of these operators has played an important role in many investigations of the linear and nonlinear partial differential equations. There are a number of textbooks on pseudo-differential equations in the literature, but this book is exceptional. It contains both the standard calculus of pseudo-differential operators and of the pseudo-differential operators on manifolds with conical and edge singularities.

The book is composed of nine chapters. Chapter I demonstrates basic properties of L^2-Sobolev spaces. Chapter II deals with standard pseudo-differential analysis. An operator P in $\mathbf{R}^n$ is pseudo-differential if it has the form

$$P(x.D)u = (2\pi)^{-n} \int p(x,\xi)\hat{u}(\xi)e^{ix\xi}dx$$

where $\hat{u}(\xi) = \int e^{-ix\xi}u(x)dx$. For linear differential operator $P(x,D) = \sum_{|\alpha|\leq m} a_\alpha(x)D^\alpha$ the function p (symbol of the operator P) is equal to $\sum_{|\alpha|\leq m} a_\alpha(x)\xi^\alpha$.

Calculus of pseudo-differential operators, pseudo-differential operators on closed manifolds and Garding inequality constitute the main subjects of Chapter II.

The third chapter presents elliptic pseudo-differential operators, construction of a parametrix, a priori estimates, and ends up with the theorem that if X is a simply-connected closed smooth manifold of dimension $n > 2$ and P is a scalar elliptic pseudo-differential operator on X, then *ind* $P = 0$. In Chapter IV some basic questions of existence and regularity of solutions to elliptic boundary value problems on bounded domain $\Omega \subset \mathbf{R}^n$ with a smooth (C^∞) boundary are considered.

Chapter V is devoted to Kondratiev's theory in its classical formulation. This time Ω is a bounded domain in $\mathbf{R}^n$. The boundary Γ of Ω is assumed to be a surface infinitely differentiable everywhere except of a one point q, and in a neighbourhood of q it coincides with a cone. Existence and regularity of solutions to elliptic boundary value problem

$$\begin{aligned} P(x,D)u &= f(x) \quad \text{in } \Omega \\ B_j(x,D)u &= g_j(x) \quad \text{on } \Gamma \end{aligned}$$

in usual and weithed Sobolev spaces are discussed. Chapter VI entitled "Non-elliptic operators; propagation of singularities" deals with such notions as Fourier integral operators, operators of principal type, wave fronts of distributions, and the Cauchy problem for hyperbolic equations. Fourier integral operators are operators of a class more general than the pseudo-differential operators. Such operators are useful in theory of hyperbolic equations. Chapter VI provides, in particular, a proof of the theorem on propagation of singularities.

Chapter VII presents the general background to study pseudo-differential operators on manifolds with conical and edge singularities. It is a part of the authors' program of the analysis on manifolds with singularities. This program contains the following points:

1. an algebra of pseudo-differential operators,
2. symbolic structures,
3. adequate Sobolev spaces,
4. subspaces of distributions with asymptotics,
5. index theory.

Chapter VIII entitled "Pseudo-differential operators on manifolds with conical singularities" consists of two sections:

1. the cone algebra with asymptotics,
2. the algebra on the infinite cone.

Chapter IX presents for the first time in detail the edge pseudo-differential algebra. The last section of Chapter IX is a set of concluding remarks to the material on edge pseudo-differential operators, applications of such operators, an examples.

The book is recommended for all people dealing with partial differential equations but not only for them. Theory of the pseudo-differential operators on C^∞-manifolds and on manifolds with singularities has many applications in mathematical physics, mechanics, engineering and also in such branches of pure mathematics as geometry and topology. The authors present many new ideas, especially in the area of pseudo-differential operators on manifolds with singularities.

According to the Preface the theory stated in the first part of the book may be the subject of special university courses in partial differential equations.

To sum up: the book "*Pseudo-differential operators, singularities, applications*" presents modern theory of pseudo-differential operators on C^∞ manifolds and manifolds with singularities. It is very carefully edited, with an extensive bibliography (71 items), and a well done index.

Henryk Kołakowski

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