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Design of a robust adaptive fuzzy controller globally stabilizing the multi-input nonlinear system with state-dependent uncertainty

by

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Abstract: In this paper a novel robust adaptive fuzzy controller is proposed for the nonlinear system with state-dependent uncertainty. Compared with the conventional adaptive fuzzy controller that determines the function which bounds the uncertainty in the system dynamics by off-line calculation on the local state space, the proposed method determines that function by the fuzzy inference so that guarantees the stability of the closed loop system globally on the whole state space. In addition the method is applied to the multi-input system. We applied the proposed method to the Burn Control of the Tokamak fusion reactor whose dynamical equations contain the state-dependent uncertainty and proved the effectiveness of the scheme by the simulation results.

Keywords: adaptive fuzzy controller, robustness, multi-input nonlinear system, state-dependent uncertainties, fuzzy inference

1. Introduction

Recently active studies have been conducted on the nonlinear control schemes. Many researchers have been interested in the feedback linearization technique, see Isidori (1989). But the merits and demerits of that approach are clear now. For instance, the input-output or input-state linearization performance improvement is the main advantage of the feedback linearization scheme, whereas its major drawback is that it has difficulties in dealing with the system with parathe control of system with parametric uncertainty. Hence, we have seen active studies in the field of adaptive nonlinear control, like Sastry and Kokotovic (1986), Sastry and Isidori (1989), Taylor, Kokotovic, Marino and Kanellakopoulos (1989), Kanellakopoulos, Kokotovic and Marino (1991), which enlarge the applicability of the feedback linearization technique to the systems with parametric uncertainty.

But when the system contains state-dependent uncertainty or the internal disturbances, see Narendra and Annaswamy (1989), even the scheme of adaptive nonlinear control mentioned above cannot be easily applied to such a system. Meanwhile, many papers have demonstrated that intelligent control is effective in dealing with the systems whose modeling is not easy. And some of adaptive fuzzy control methods, Wang (1994), Wang, Mendel (1992), Wang (1996), have shown successful applicability to the systems with state-dependent uncertainty.

In this paper we propose a novel adaptive fuzzy control scheme which guarantees the global closed-loop system stability in the face of the system uncertainty due to the internal disturbances. Compared with the conventional adaptive fuzzy controller that guarantees only local stability, the proposed controller guarantees the global closed-loop system stability by introduction of new method of estimating the function which gives the upper bounds of the uncertain term in the system dynamics by mean of the fuzzy inference. And to prove the effectiveness of the proposed scheme, we applied it to the Burn Control of the Tokamak fusion reactor, Park, Young-Hwan, Park, Gwi-Tae (1995), Dolan (1982), Haney, Perkins, Mandrekas, Stacey (1990), whose dynamics contains state-dependent uncertainty.

The simulation results show that the proposed scheme is superior to the conventional adaptive fuzzy control methods. The organization of this paper is as follows. Section 2 presents the design of a robust adaptive fuzzy controller. Application of the proposed method to the Burn Control of the Tokamak fusion reactor is dealt with in Section 3. Finally, Section 4 concludes the paper.

2. Design of robust adaptive fuzzy controller

2.1. Control objectives

Consider the nth-order nonlinear systems of the form

$$\begin{aligned} \dot{\bar{x}} &= \bar{f}(\bar{x}) + \bar{u} \\ \bar{y} &= \bar{x} \\ \bar{x} &= [x_1,\, x_2,\, \cdots,\, x_n]^T \\ \bar{f} &= [f_1(\bar{x}),\, f_2(\bar{x}),\, \cdots,\, f_n(\bar{x})]^T \\ \bar{u} &= [u_1,\, u_2,\, \cdots,\, u_n]^T \end{aligned} \tag{1}$$

where f_i is unknown continuous functions, $u_i \in \Re$ is the input of the system be available for measurement. The control objective is to determine a control $\bar{u}$ such that the closed-loop system must be globally stable and the tracking error $\bar{e} = \bar{y}_m - \bar{y}$ should be as small as we determine.

A reference model is defined as:

$$\begin{aligned} \dot{\bar{x}}_m &= A_m\bar{x}_m + B_m\bar{r} \\ \bar{y}_m &= \bar{x}_m \\ \bar{x}_m &= [x_{m1}, x_{m2}, \cdots, x_{mn}]^T \\ \bar{r} &= [r_1, r_2, \cdots, r_n]^T \end{aligned} \tag{2}$$

where $\bar{x}_m,\bar{r} \in \Re^n$, $A_m,B_m \in \Re^{n\times n}$ and A_m is a stable matrix.

2.2. Controller design

If f_i is known functions, the control law

$$\bar{u} = -\bar{f}(\bar{x}) + A_m\bar{x} + B_m\bar{r} \tag{3}$$

applied to (1) results in

$$\dot{\bar{x}} = A_m\bar{x} + B_m\bar{r} \tag{4}$$

which implies that $\lim_{t\to\infty}\bar{e}(t) = 0$, a main objective of control. However, $\bar{f}$ is unknown.

To solve this problem, we follow the approach from Wang (1994), Wang, Mendel (1992), Wang (1996), **i.e.** indirect adaptive fuzzy control, where unknown function f_i is replaced by a fuzzy logic system $\hat{f}_i$.

To present the detailed design steps, we make an assumption.

ASSUMPTION 2.1 *There are the following linguistic descriptions about the unknown functions $f_i(\bar{x})$ (from human experts):*

$$\begin{aligned} R_f{}^{(r)}: \quad & IF\ x_1\ is\ A_1{}^r\ and\ \cdots\ and\ x_n\ is\ A_n{}^r \\ & THEN\ f_1(\bar{x})\ is\ C_1{}^r\ and\ \cdots\ and\ f_n(\bar{x})\ is\ C_n{}^r \end{aligned} \tag{5}$$

where $A_i{}^r$ and $C_i{}^r$ are fuzzy sets in $\Re$, $r = 1, 2, \cdots, L_f$.

Step 1: Initial Controller Construction

- Define U_c as the neighborhood of the point $\bar{x}_m(t)$ in $\Re^n$.
- Define M_i fuzzy sets $F_i{}^{l_i}$ whose membership functions $\mu_{F_i{}^{l_i}}$ uniformly cover U_{c_i} which is the projection of U_c onto the ith coordinate, where $l_i = 1, 2, \cdots, M_i$ and $i = 1, 2, \cdots, n$. We require that $F_i{}^{l_i}$'s include the $A_i{}^r$'s in (5).
- From the linguistic description (5), construct the fuzzy rule base for the fuzzy logic systems $\hat{f}_i(\bar{x}|\bar{\theta}_i)$'s, each of which consists of $M_1, M_2, \cdots, M_n$ rules whose IF parts comprise all the possible combinations of the F_i's for $i = 1, 2, \cdots, n$. And $\bar{\theta}_i$ is defined later as in (8) of fuzzy basis function

Specifically, the fuzzy rule base of $\hat{f}_i(\bar{x}|\bar{\theta}_i)$ consist of the rule

$$R_f{}^{(l_1,\cdots,l_n)}: \quad IF \quad x_1 \; is \; F_1{}^{l_1} \; and \; \cdots \; and \; x_n \; is \; F_n{}^{l_n} \qquad (6)$$
$$THEN\, \hat{f}_1(\bar{x}|\bar{\theta}_1) \; is \; G_1{}^{(l_1,\cdots,l_n)} \; and \; \cdots \; and \; \hat{f}_n(\bar{x}|\bar{\theta}_n) \; is \; G_n{}^{(l_1,\cdots,l_n)}$$

where $l_i = 1, 2, \cdots, M_i$ and $i = 1, 2, \cdots, n$, and $G_i{}^{(l_1,\cdots,l_n)}$'s are fuzzy sets in $\Re$ which are specified as follows : if the IF part of (6) agrees with the IF part of (5), set $G_i{}^{(l_1,\cdots,l_n)}$'s equal to the corresponding $C_i{}^{r}$'s ; otherwise, set $G_i{}^{(l_1,\cdots,l_n)}$ arbitrarily with the constraint that the centers of $G_i{}^{(l_1,\cdots,l_n)}$ be inside the constraint set Ω. Therefore, the initial adaptive fuzzy controller is constructed from the linguistic rules (5) and (6).

Construct the fuzzy basis functions

$$\xi^{(l_1,l_2,\cdots,l_i,\cdots,l_n)}(\bar{x}) = \frac{(\prod_{i=1}^{n}\mu_{F_i{}^{l_i}}(x_i))}{\sum_{l_1=1}^{M_1}\sum_{l_2=1}^{M_2}\cdots\sum_{l_n=1}^{M_n}(\prod_{i=1}^{n}\mu_{F_i{}^{l_i}}(x_i))} \qquad (7)$$

and collect them into a $\prod_{i=1}^{n} M_i$ - dimensional vector $\bar{\xi}(\bar{x})$ in a natural ordering for $l_1 = 1, 2, \cdots, M_1$ and $l_n = 1, 2, \cdots, M_n$.

Collect the points at which $\mu_{G_i{}^{(l_1,\cdots,l_n)}}$'s achieve their maximum values, in the same ordering as $\bar{\xi}(\bar{x})$, into vectors $\bar{\theta}_i$'s. The $\hat{f}_i(\bar{x}|\bar{\theta}_i)$ and $\hat{\bar{f}}_i(\bar{x}|\bar{\theta}_1, \cdots, \bar{\theta}_n)$ are constructed as

$$\hat{f}_i(\bar{x}|\bar{\theta}_i) = \bar{\theta}_i^T \bar{\xi}(\bar{x}) \qquad (8)$$

$$\hat{\bar{f}}(\bar{x}|\bar{\theta}_1, \cdots, \bar{\theta}_n) := [\bar{\theta}_1, \cdots, \bar{\theta}_n]^T \bar{\xi}(\bar{x}) = \Theta\bar{\xi}(\bar{x}) \qquad (9)$$
$$\Theta := [\bar{\theta}_1, \bar{\theta}_2, \cdots, \bar{\theta}_n]^T$$

Step 2 : Determination of $f^u(\bar{x})$ that bounds $|\bar{f}(\bar{x})|$
Let us make an assumption for $\bar{f}(\bar{x})$.

ASSUMPTION 2.2 *The norm of $\bar{f}(\bar{x})$ is bounded for all $\bar{x}$ in the compact subset U_c of $\Re^n$. i.e.,*

$$|\bar{f}(\bar{x})| < f^u(\bar{x}), \quad \forall \bar{x} \in U_c \subset \Re^n \qquad (10)$$

where $f^u(\bar{x})$ in $\Re$.

Subject to Assumption 2, it is axiomatic that there exists an angle $\vartheta(0 \le \vartheta < \frac{\pi}{2}[rad])$ such that $(\tan\vartheta)|\bar{x}|$ bounds $|\bar{f}(\bar{x})|$ for all $\bar{x}$ in the compact subset U_c of $\Re^n$, i.e.,

$$\exists\vartheta \;\; s.t. \;\; |\bar{f}(\bar{x})| \le f^u(\bar{x}) \le \tan\vartheta \cdot |\bar{x}| < \infty, \quad \forall \bar{x} \in U_c \subset \Re^n \qquad (11)$$

Then to estimate ϑ satisfying the inequality in (11), we propose fuzzy sets "large", "middle", "small" denoted by $\tilde{e_L}$, $\tilde{e_M}$, $\tilde{e_S}$ respectively in Fig. 1 and fuzzy rule base(R) and a fuzzy logic system (12) in connection with tracking

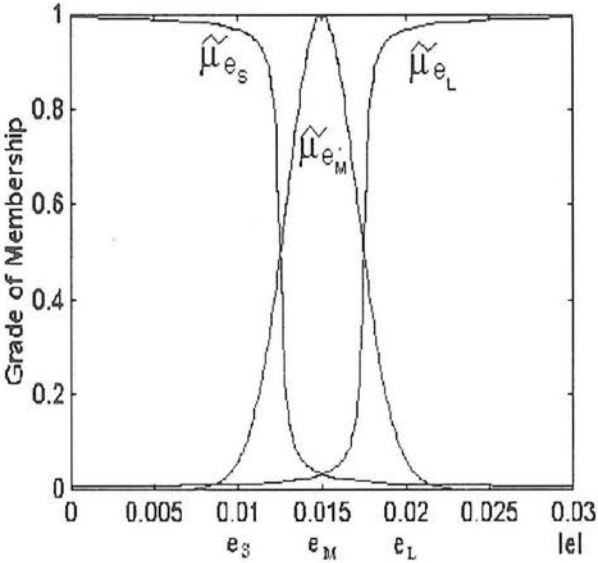

Figure 1. Membership functions for $\tilde{e_S}$, $\tilde{e_M}$, $\tilde{e_L}$

Here detailed descriptions of the membership functions for $\tilde{e_L}$, $\tilde{e_M}$, $\tilde{e_S}$ are

$$\begin{aligned}
\mu_{\tilde{e_L}}(|\bar{e}|) &= \frac{1}{1+e^{-\{\frac{|\bar{e}|-a}{b}\}}}, \quad a=\frac{e_L+e_M}{2} \\
\mu_{\tilde{e_M}}(|\bar{e}|) &= exp(-\frac{1}{2}(\frac{|\bar{e}|-e_M}{\sigma})^2) \\
\mu_{\tilde{e_S}}(|\bar{e}|) &= 1-\frac{1}{1+e^{-\{\frac{|\bar{e}|-a'}{b}\}}}, \quad a'=\frac{e_S+e_M}{2}
\end{aligned}$$

where e_L, e_M, e_S are chosen arbitrarily at the designer's disposal and b and σ are constants that determine the slope of the graph. In this paper, for the control of Tokamak fusion reactor in Section 3, e_L, e_M, e_S are chosen as 0.02, 0.015, 0.01, respectively, as shown in Fig. 1.

$$(R) \quad \begin{array}{ll}
\text{If } |\bar{e}| \text{ is small then } \vartheta \text{ is} & \frac{\tilde{\pi}}{4}[rad] \\
\text{If } |\bar{e}| \text{ is middle then } \vartheta \text{ is} & \frac{3\tilde{\pi}}{8}[rad] \\
\text{If } |\bar{e}| \text{ is large then } \vartheta \text{ is} & \frac{\tilde{\pi}}{}[rad]
\end{array}$$

$$\vartheta = \frac{\mu_{\tilde{e_S}}(|\bar{e}|)\frac{\pi}{4} + \mu_{\tilde{e_M}}(|\bar{e}|)\frac{3\pi}{8} + \mu_{\tilde{e_L}}(|\bar{e}|)\frac{\pi}{2}}{\mu_{\tilde{e_S}}(|\bar{e}|) + \mu_{\tilde{e_M}}(|\bar{e}|) + \mu_{\tilde{e_L}}(|\bar{e}|)} \tag{12}$$

$$f^u(\bar{x}) := \tan\vartheta \cdot |\bar{x}|,$$
$$\forall \bar{x} \in U_c \subset \Re^n \quad f^u(\bar{x}) \in \Re \tag{13}$$

From (R) and Fig.1 we can expect that ϑ approaches $\frac{\pi}{2}$ as $|\bar{e}|$ is greater than e_L. And we can see that when $|\bar{e}| > e_L$, ϑ determined from (12) sufficiently satisfies the inequality in (11) since $\vartheta \to \frac{\pi}{2}$ and $\tan\vartheta \to \infty$ as $|\bar{e}| > e_L$. (12) and (13) will be used later for the proof of boundedness of $|\bar{e}|$ in Step 4.

Step 3: Error dynamics equation

Define a constrained set Ω for Θ specified by the designer. For Ω, we require that Θ is bounded, that is,

$$\Omega = \{\Theta : tr\{\Theta\Theta^T\} \leq M\} \tag{14}$$

where M is positive constant specified by the designer.

Now we replace $\bar{f}(\bar{x})$ in (3) by the fuzzy logic system $\hat{\bar{f}}(\bar{x})$ in the form of (8) and (9). The resulting control law

$$\bar{u}_c = -\hat{\bar{f}}(\bar{x}) + A_m\bar{x} + B_m\bar{r} \tag{15}$$

is the so-called certainty equivalent controller in the adaptive control literature (Sastry and Isidori, 1989). We append another control term, $\bar{u}_s$, to the $\bar{u}_c$ such that the final control is

$$\bar{u} = \bar{u}_c + \bar{u}_s \tag{16}$$

This additional control term $\bar{u}_s$ is called (Sastry and Isidori, 1989) supervisory control for the reasons given at the end of Step 4.

Substituting (15) and (16) into (1) and after manipulation with (4), we obtain the error equation

$$\dot{\bar{e}} = A_m\bar{e} + [\hat{\bar{f}}(\bar{x}) - \bar{f}(\bar{x})] + u_s \tag{17}$$

where $\bar{e} := \bar{x}_m - \bar{x}$.

Since A_m is a stable matrix, we know that there exists a unique positive definite symmetric $n \times n$ matrix P which satisfies the Lyapunov equation:

$${A_m}^T P + PA_m = -Q \tag{18}$$

where Q is an arbitrary $n \times n$ positive definite matrix.

Step 4: Determination of supervisory control $\bar{u}_s$ and Lyapunov stability analysis

Let a Lyapunov function candidate be $V_e = \frac{1}{2}\bar{e}^T P\bar{e}$ and choose the supervisory control $\bar{u}_s$ as

$$\bar{u}_s = \begin{cases} -(kP\bar{e}/|P\bar{e}|)(|\hat{\bar{f}}(\bar{x})| + f^u(\bar{x})), \quad k > 1, \quad if\ V_e > V_s \end{cases} \tag{19}$$

where V_s is the value of V_c when $|\bar{e}| = e_s$.

Then using (17) and (18) we have the derivative of V_c as

$$\begin{aligned}
\dot{V}_c &= \frac{1}{2}\left[\bar{e}^T({A_m}^T P + PA_m)\bar{e} + 2\bar{e}^T P(\hat{\bar{f}}(\bar{x}) - \bar{f}(\bar{x})) + 2\bar{e}^T P\bar{u}_s\right] \\
&= -\frac{1}{2}\bar{e}^T Q\bar{e} + \bar{e}^T P(\hat{\bar{f}}(\bar{x}) - \bar{f}(\bar{x})) + \bar{e}^T P\bar{u}_s \\
&\leq -\frac{1}{2}\lambda_{Q_{min}}|\bar{e}|^2 + \bar{e}^T P(\hat{\bar{f}}(\bar{x}) - \bar{f}(\bar{x})) + \bar{e}^T P\bar{u}_s \qquad (20) \\
&\leq -\frac{1}{2}\lambda_{Q_{min}}|\bar{e}|^2 + |\bar{e}^T P||\hat{\bar{f}}(\bar{x}) - \bar{f}(\bar{x})| + \bar{e}^T P\bar{u}_s \\
&\leq -\frac{1}{2}\lambda_{Q_{min}}|\bar{e}|^2 + |\bar{e}^T P|(|\hat{\bar{f}}(\bar{x})| + |\bar{f}(\bar{x})|) + \bar{e}^T P\bar{u}_s
\end{aligned}$$

where $\lambda_{Q_{min}}$ is the minimum eigenvalue of Q.

If $|\bar{e}| > e_L$, $|\bar{f}(\bar{x})| < f^u(\bar{x})$ is guaranteed by the fuzzy logic system (12) and (13). Also in connection with $\bar{u}_s$ in (19), we have the following equation:

$$\begin{aligned}
&|\bar{e}^T P|(|\hat{\bar{f}}(\bar{x})| + |\bar{f}(\bar{x})|) + \bar{e}^T P\bar{u}_s = \\
&|\bar{e}^T P|(|\hat{\bar{f}}(\bar{x})| + |\bar{f}(\bar{x})|) - k|\bar{e}^T P|(|\hat{\bar{f}}(\bar{x})| + f^u(\bar{x})) \\
&= |\bar{e}^T P|(1-k)\cdot|\hat{\bar{f}}(\bar{x})| + |\bar{e}^T P|(|\bar{f}(\bar{x})| - kf^u(\bar{x})) \\
&< 0 \quad (if|\bar{e}| \neq 0) \qquad (21)
\end{aligned}$$

Therefore from (20) and (21), we have the following:

$$If\ |\bar{e}| > e_L, \ \ then\ \ \dot{V}_c < 0\ and\ V_c \leq V_L < \infty \qquad (22)$$

where V_L is the value of V_c when $|\bar{e}| = e_L$.

From (22) we can conclude that $|\bar{e}| \leq \sqrt{(2\frac{V_L}{\lambda_{P_{min}}})}$, $\forall \bar{x} \in \Re^n$, since

$$\begin{aligned}
V_c &\leq V_L \\
\tfrac{1}{2}\lambda_{P_{min}}|\bar{e}|^2 &\leq \tfrac{1}{2}\bar{e}^T P\bar{e} \leq V_L \\
|\bar{e}|^2 &\leq 2\frac{V_L}{\lambda_{P_{min}}} \\
|\bar{e}| &\leq (2\frac{V_L}{\lambda_{P_{min}}})^{\frac{1}{2}}, \ \ \forall \bar{x} \in \Re^n.
\end{aligned}$$

Step 5: On-line adaptation with projection algorithm

If we choose the adaptation law based on projection algorithm (Sastry and Isidori, 1989)

$$\begin{aligned}
\dot{\Theta} &= -\gamma P\bar{e}\bar{\xi}^T(\bar{x}) \\
&\text{if } \ (\ tr(\Theta\Theta^T) < M\ ) \text{ or } (\ tr(\Theta\Theta^T) = M \text{ and } \bar{e}^T P\Theta\bar{\xi}(\bar{x}) \geq 0\ ) \\
\dot{\Theta} &= -\gamma P\bar{e}\bar{\xi}^T(\bar{x}) + [\gamma\bar{e}^T P\Theta\bar{\xi}(\bar{x})/tr(\Theta\Theta^T)]\Theta \\
&\text{if } \ (\ tr(\Theta\Theta^T) = M \text{ and } \bar{e}^T P\Theta\bar{\xi}(\bar{x}) < 0\ ) \qquad (23)
\end{aligned}$$

then we can guarantee

$$tr(\Theta\Theta^T) \le M \tag{24}$$

Proof:
From (23) we see that if $tr(\Theta\Theta^T) = M$ and $\bar{e}^T P\Theta\bar{\xi} \ge 0$, then we have

$$\begin{aligned}
\frac{d}{dt}[tr(\Theta\Theta^T)] &= tr(\dot{\Theta}\Theta^T + \Theta\dot{\Theta}^T) \\
&= tr(-\gamma P\bar{e}\bar{\xi}^T\Theta^T - \Theta\bar{\xi}\bar{e}^T P\gamma) \\
&= -2\gamma\bar{e}^T\Theta\bar{\xi} \\
&\le 0
\end{aligned}$$

And from (23), if $tr(\Theta\Theta^T) = M$ and $\bar{e}^T P\Theta\bar{\xi} < 0$, then we have

$$\begin{aligned}
\frac{d}{dt}[tr(\Theta\Theta^T)] &= tr(\dot{\Theta}\Theta^T + \Theta\dot{\Theta}^T) \\
&= 2tr(-\gamma P\bar{e}\bar{\xi}^T\Theta^T + \gamma\frac{\bar{e}^T P\Theta\bar{\xi}}{tr(\Theta\Theta^T)}\Theta\Theta^T) \\
&= -2\gamma tr\{P\bar{e}\bar{\xi}^T\Theta^T\} + 2\gamma\bar{e}^T P\Theta\bar{\xi} \\
&= 0
\end{aligned}$$

Hence (23) guarantees (24). □

The overall scheme of indirect adaptive fuzzy control systems is shown in Fig.2.

3. Application to the burn control of the Tokamak fusion reactor

3.1. The Tokamak fusion reactor model

The simplest model for the Tokamak fusion reactor is the zero dimensional model shown below after Park, Park (1995):

$$\begin{aligned}
\dot{x}_1 &= f_1(\bar{x}) + g_{11}(\bar{x})u_1 + g_{12}(\bar{x})u_2 \\
\dot{x}_2 &= f_2(\bar{x}) + g_{21}(\bar{x})u_1 + g_{22}(\bar{x})u_2 \\
y_1 &= x_1\ , \qquad y_2 = x_2
\end{aligned} \tag{25}$$

$$\begin{aligned}
f_1(\bar{x}) = -c_1x_1\ , &\qquad f_2(\bar{x}) = c_2 < \sigma v > x_1 - c_3x_1{x_2}^{0.5} - c_4x_2 \\
g_{11}(\bar{x}) = 1\ , &\qquad g_{12}(\bar{x}) = 0 \\
g_{21}(\bar{x}) = -x_2/x_1\ , &\qquad g_{22}(\bar{x}) = 100/(4.8x_1) \\
c_1 = \tfrac{1}{\alpha\beta{x_1}^l{x_2}^m}\ , &\qquad c_2 = (3.52 \times 10^{23})/12 \\
c_3 = 0.1042Z_{eff}\ , &\qquad c_4 = \frac{1}{\alpha{x_1}^l{x_2}^m} - \frac{1}{\alpha\beta{x_1}^l{x_2}^m}
\end{aligned}$$

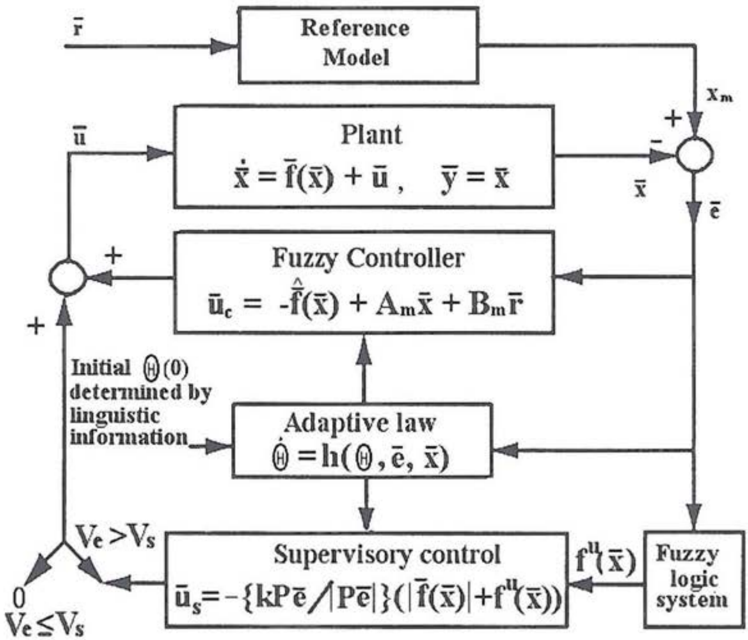

Figure 2. The overall scheme of indirect adaptive fuzzy control systems

where
x_1 : the particle density in the Tokamak fusion reactor, n$[10^{20}/m^3]$
x_2 : the temperature in the Tokamak fusion reactor, T[keV]
u_1 : the particle supply, S$[10^{20}/m^3 s]$
u_2 : the auxiliary heating, Paux $[MW/m^3]$
α, β, Z_{eff} : known constants
l, m : unknown constants

The uncertainty of parameters l and m is classified as follows, Wang (1994):

- constant confinement law : $l = m = 0$
- Bohm diffusion : $l = 0$, $m = -1$
- neoclassical diffusion : $l = -1$, $m = 0.5$

Clearly, the system (25) has state-dependent uncertainty due to the uncertainty of l and m.

3.2. Controller design and simulation

The system (25) is two-dimensional, so we choose $n = 2$, $M_1 = M_2 = 5$ in the fuzzy rule $R_f{}^{(l_1,\cdots,l_n)}$ in the Subsection 2.2. The reference model (2) is chosen as $A_m = -50I$, $B_m = 50I$, where I is the identity matrix. The fuzzy sets in Fig. 1 are chosen as $e_S = 0.01$, $e_M = 0.015$, $e_L = 0.02$. The system (25) is assumed such that there is no modeling error in the terms $g_{ij}(\bar{x})$, $i = 1, 2$, $j = 1, 2$. Hence we assume that the matrix $G(\bar{x}) = \begin{bmatrix} g_{11}(\bar{x}) & g_{12}(\bar{x}) \\ g_{21}(\bar{x}) & g_{22}(\bar{x}) \end{bmatrix}$ is nonsingular on $\bar{x} \neq 0$. Positive definite matrix Q and P is chosen appropriately. Subject to these choices we use a controller and adaptation algorithm of the form shown in Section 2.

$$
\begin{aligned}
\bar{u}(\bar{x}) &= G^{-1}(x)[-\hat{\bar{f}}(\bar{x}) + A_m\bar{x} + B_m\bar{r} - \bar{u}_s(\bar{x})] \\
\bar{u}_s(\bar{x}) &= \begin{cases} -(kP\bar{e}/|P\bar{e}|)(|\hat{\bar{f}}(\bar{x})| + f^u(\bar{x})), \quad k > 1, & if\ V_e > V_s \\ 0 & if\ V_e \leq V_s \end{cases}
\end{aligned}
$$

We apply this controller to the system (25) with varying l and m, see Fig. 3. We choose $M = 0.92 \times 10^3$ considering the operation range of $\bar{y}_m$, the output vector of the reference model. Fig. 4 shows the state x_1 and its desired value x_{m1}. Fig. 5 shows the state x_2 and its desired value x_{m2}. Fig. 6 shows the tracking errors e_1 and e_2 and confirms that the errors are bounded by the determined value $e_L = 0.02$. Fig. 7 shows the control inputs u_1 and u_2. Fig. 8 shows that $tr(\Theta\Theta^T)$ is bounded by M. Fig. 9a and Fig. 9b compare $\bar{f}(\bar{x})$ with $f^u(\bar{x})$.

4. Conclusion

We extend the approach of Wang (1994), Wang, Mendel (1992) to the multivari-

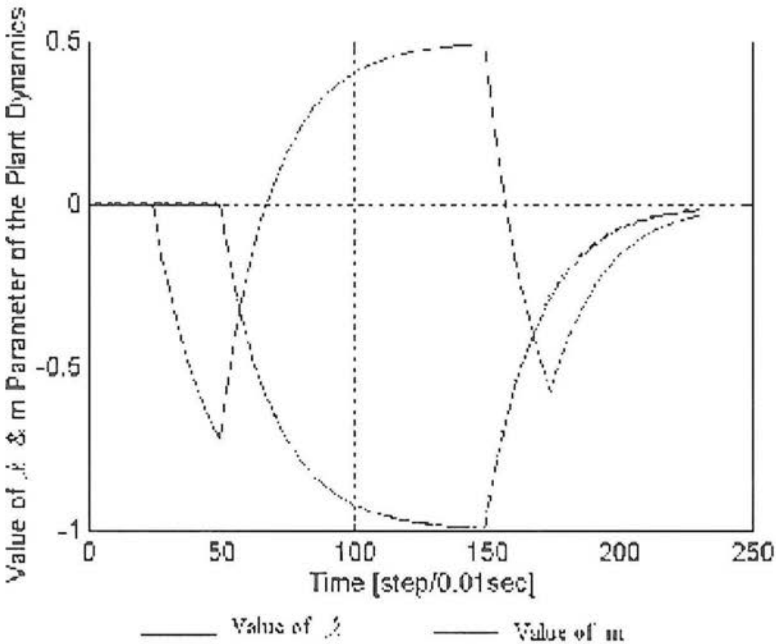

Figure 3. Values of l and m of (21).

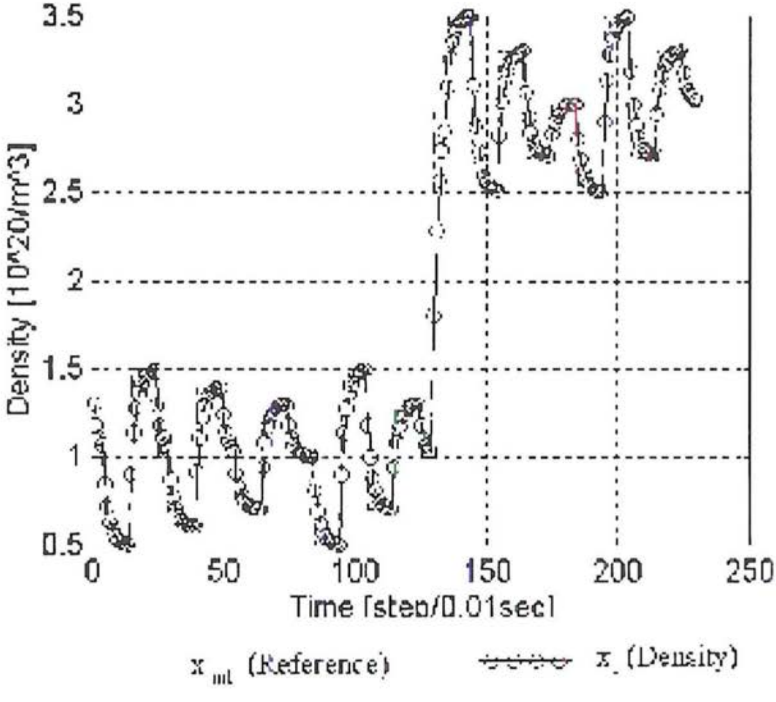

Figure 4. State x_1 and its desired value x_{m1}.

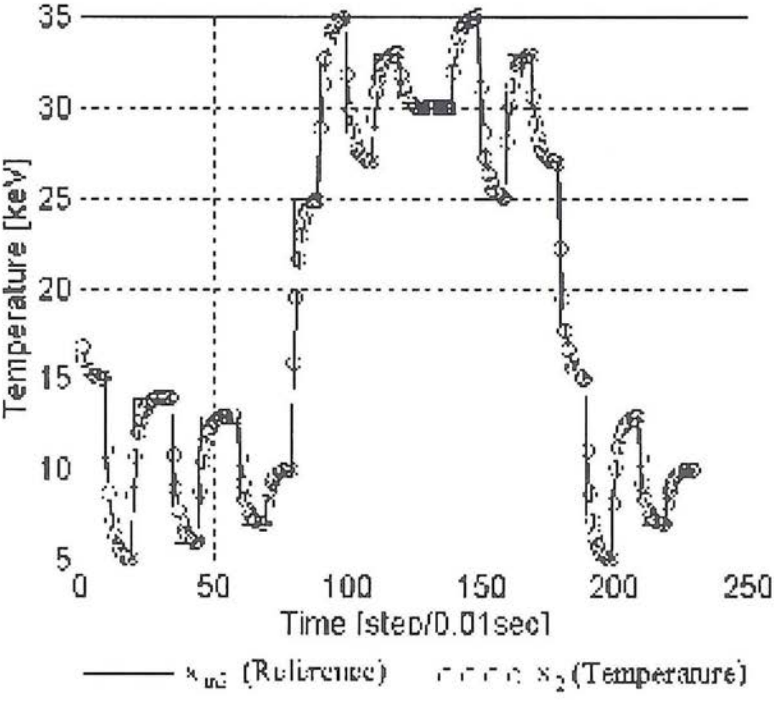

Figure 5. State x_2 and its desired value x_{m2}.

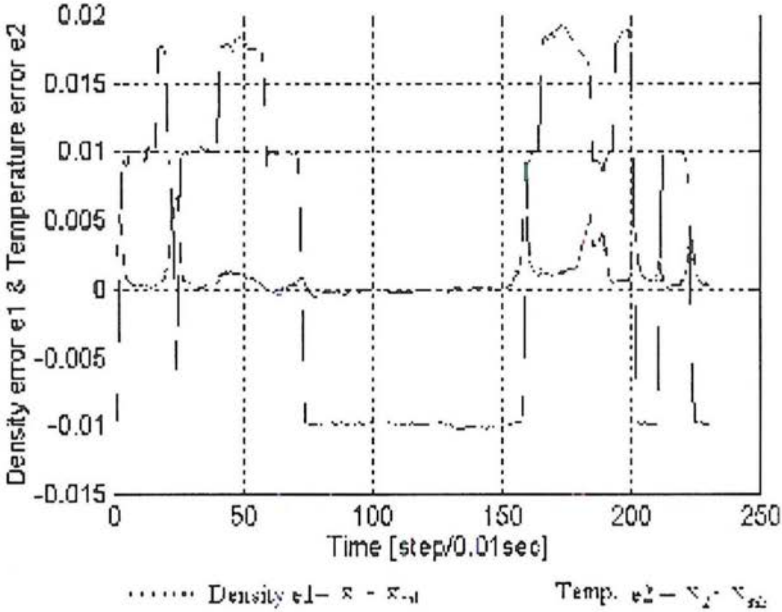

Figure 6. Tracking errors e_1 and e_2.

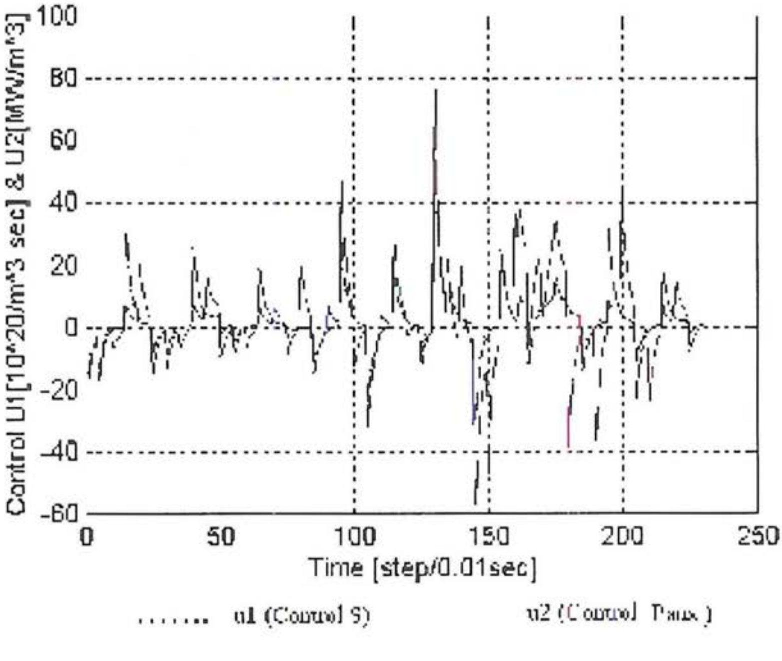

Figure 7. Control inputs u_1 and u_2.

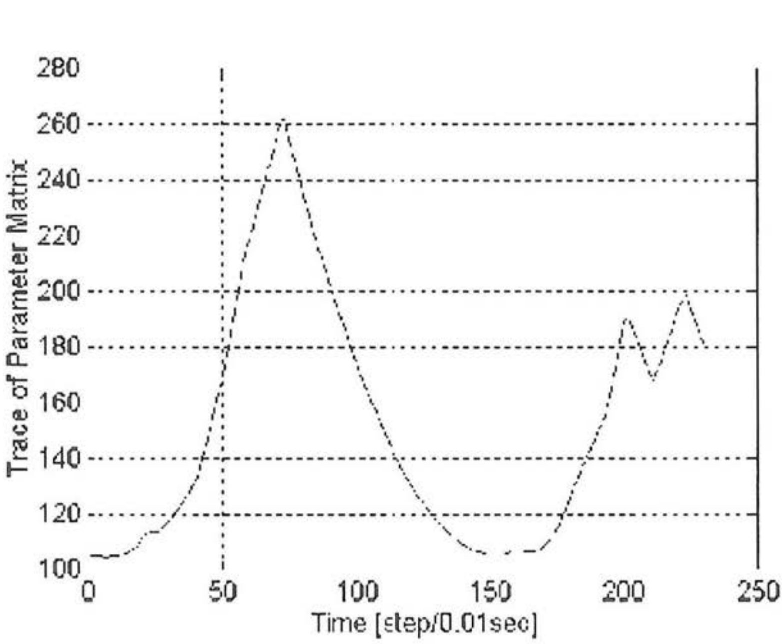

Figure 8. $tr(\Theta\Theta^T)$.

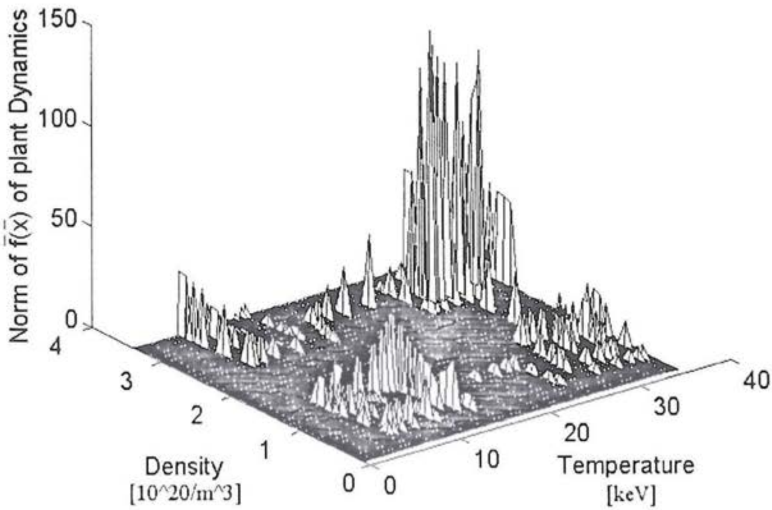

Figure 9. Norm of $\bar{f}(\bar{x})$ of plant dynamics.

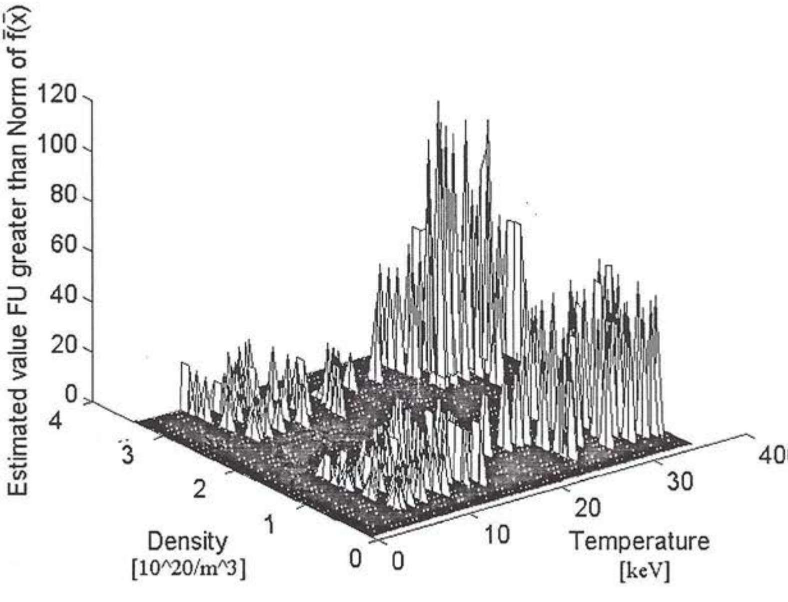

Figure 10. Estimated value of $f^u(\bar{x})$.

nonlinear system globally. The estimated $f^u(\bar{x})$ is good enough for control purposes. But further research is required for the systems which have uncertainty in $g_{ij}(\bar{x})$. For the demonstration of the validity of our proposed method, we apply the proposed controller to the burn control of the Tokamak fusion reactor which has the dynamic equation with state-dependent uncertainty. The simulation result shows that our scheme is effective.

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