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Time and electric energy losses minimization in speed control of induction motors

by

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Abstract: The paper presents a method for solving the problem of the simultaneous time and electric energy losses minimization during the frequency speed control of induction motors with the electromagnetic transients. To solve this vector optimization problem, the optimization index in the form of linear combination of the scalar indexes is assumed and the Pontryagin maximum principle is used. Examples of solution for two cases of optimal frequency starting of two induction motors are presented.

Keywords: optimal control, induction motors

1. Introduction

The AC induction motors are nowadays the most widely used actuators in industrial applications, due to their advantages, in comparison with DC motors, such as: simplicity of design, high reliability, ruggedness, low cost, minimum maintenance, low power weight ratio, small size, small weight, small rotor inertia, maximum speed capability, efficiency etc.

The advances in power electronic technology and the recent developments in microprocessor technology have made various variable-speed induction motor drive systems available. These systems work in real time and are designed on the basis of the optimal control theory, adaptive control theory, using the classical PI controller etc.

The induction motor has a nonlinear and highly interacting multivariable control structure, whereas the separately excited DC motor has a decoupled control structure with independent control of flux and torque.

In recent years a control technique termed "field-oriented control" introduced by Blaschke (1972) has been used for transforming the dynamic structure of the AC induction motor into a separately excited compensated DC motor (see Murphy, Turnbull, 1988). Generally, the field-oriented control consists in nonlinear state space change of coordinates and nonlinear state feedback. On the basis of the field-oriented control, the theory of nonlinear feedback control and the methods of high-performance controller design have been developed.

The field-oriented control has the following disadvantages:

- the rotor speed is only asymptotically decoupled from the rotor flux (see Hu, Dawson, Qu, 1994)
- the instabilities may occur in certain conditions during field-oriented control (see Salama, Holmes, 1992)
- the asymptotic decoupling in the control of rotor speed and flux amplitude is obtained only in steady state, i.e., when the flux amplitude is kept constant (see Marino, Peresada, Valigi, 1993).

Field-oriented control requires sophisticated signal processing and complex coordinate transformation in real time (see Murphy, Turnbull, 1988).

Digital control with fast microprocessors or a multitransputer system (see Asher, Summer, 1990) made the low cost construction and the realization of the field-oriented control calculations in real time possible.

Advances in digital and power electronics and microprocessors and substantial increase in processor speed, have also made the digital implement of control strategies based on other principles such as, for example, optimal control theory, possible. In this paper a theoretical solution of the rotor speed optimal frequency control which minimizes both the control time and the electrical energy losses in rotor and stator windings is presented.

The most efficient and effective method of induction motor speed control is frequency control. This method consists in the variation of the frequency and amplitude of the voltage or current feeding the motor. Application of the proper control laws makes it possible to control motor speed with or without limitation of the stator current, while optimizing some criterion such as, for example, minimization of the control time, minimization of the electric energy losses in the stator and rotor windings etc.

The optimization problems of induction motor speed control system mentioned above, have a single optimization index and belong to the class of scalar dynamic optimization problems because their optimization index is a scalar functional.

Sometimes it is necessary to minimize or maximize two or more scalar optimization indexes simultaneously. For example, for induction motor speed frequency control system, it may be necessary to minimize simultaneously the control time and the electric energy losses in the stator and rotor windings during the control and, additionally, it may be necessary to limit the stator current to a given value. The optimization problem so formulated, belongs to vector optimization problems because the optimization index, having two or more components, is a vector functional.

The solution of the scalar or vector optimization problem depends, to a great extent, on the complexity of the mathematical model of the control object and on the analytical method selected for the problem solution.

In the induction motor case the complexity of the mathematical model depends, among others, on whether the electromagnetic transients are neglected or not in defining of the model. By neglecting the electromagnetic transients, it is possible to obtain the closed-loop optimal control in the form of mathematical model of the optimal inertialess controller.

In this paper, the problem of vector optimization of induction motor speed frequency control system, which minimizes both the time and the electric energy losses in the stator and rotor windings while limiting the stator current, using the mathematical model of the induction motor taking into account electromagnetic transients, is solved. The Pontryagin maximum principle is used to solve this problem.

2. Induction motor mathematical model

The following assumptions are made for an induction motor:

1. Induction motor and its sinusoidal supply voltage and current are symmetric
2. Resistances and inductances are invariant
3. Saturation of magnetic material does not occur
4. Magnetic losses can be neglected

Besides, it is assumed that the induction motor is supplied from a frequency current converter which produces the current given, for a two-phase equivalent motor, by the following relationships (the list of notations used is given in the Appendix):

$$\left.\begin{aligned} i_{1d} &= i_1 \cos\omega t \\ i_{1q} &= i_1 \sin\omega t \end{aligned}\right\} \tag{1}$$

Under these assumptions, the mathematical model of the double-phase equivalent induction motor on the $d-q$ axes, associated with the stator (fixed stator reference frame), may be described by the following equations (see Krause, 1987):

$$\left.\begin{aligned} \frac{d\psi_{2d}}{dt} &= -a\psi_{2d} + bi_1 \cos\omega t - \omega_r\psi_{2q} \\ \frac{d\psi_{2q}}{dt} &= -a\psi_{2q} + bi_1 \sin\omega t + \omega_r\psi_{2d} \\ \frac{d\omega_r}{dt} &= ci_1(\psi_{2d}\sin\omega t - \psi_{2q}\cos\omega t) - \frac{pM_o}{J} \end{aligned}\right\} \tag{2}$$

On the basis of Kovac, Rac (1963) the relationship that describes the amplitude of the magnetization current can be found:

$$i_o = \frac{1}{X_o + X_2^{\prime}}\sqrt{(X_2^{\prime} i_1 \cos\omega t + \omega_n\psi_{2d})^2 + (X_2^{\prime} i_1 \sin\omega t + \omega_n\psi_{2q})^2} \tag{3}$$

3. Optimization index

In this paper, for minimization of the control time and electric energy losses in the stator and rotor windings a vector optimization criterion is used, which is a linear combination of two scalar optimization indexes: for minimizing the electric energy losses only and for minimizing the control time only. The following relationship describes, in a general way, this vector optimization criterion:

$$\left.\begin{array}{c} Q_3 = (1-s)Q_1 + sQ_2 \\ 0 < s < 1 \end{array}\right\} \tag{4}$$

The optimization index (4) belongs to vector optimization index class, which is based on the compromise given by Salukwadze (1975).

The scalar indexes Q_1 and Q_2 are given by the following relationships:

$$Q_1 = \int_0^{t_r} dt \tag{5}$$

$$Q_2 = \frac{3}{2}\int_0^{t_r} (i_1^2 R_1 + i_2'^2 R_2')dt \tag{6}$$

Using well-known equations which describe the dependencies between internal variables of induction motors (see Krause, 1987), the electrical energy losses in the stator and rotor windings (6) may be given as follows:

$$Q_2 = \frac{3}{2}R_2' \int_0^{t_r} \left[(g\psi_{2d} - ei_1 \cos\omega t)^2 + (g\psi_{2q} - ei_1 \sin\omega t)^2\right] dt \tag{7}$$

Therefore, the global optimization index (4) may take the form of:

$$\begin{aligned} Q_3 = & \int_0^{t_r} [(1-s) + \frac{3}{2} i_1^2 R_1 s]dt + \\ & + \frac{3}{2}R_2' s \int_0^{t_r} \left[(g\psi_{2d} - ei_1 \cos\omega t)^2 + (g\psi_{2q} - ei_1 \sin\omega t)^2\right] dt \end{aligned} \tag{8}$$

4. Problem solution

We will find the open-loop optimal control, i.e. we will find the amplitude and frequency of the stator current as functions of time,

$$\left.\begin{array}{l} i_1 = i_1(t) \\ \omega = \omega(t) \end{array}\right\} \tag{9}$$

minimizing the optimization index (8). Besides, we wish to limit the stator current amplitude during the control:

$$i_1 \leq i_1^o \tag{10}$$

To solve this problem we apply the mathematical method of the Pontryagin maximum principle (see Athans, Falb, 1969).

Considering (2) and (8) the hamiltonian takes the following form:

$$\begin{aligned} H \;=\; & -(1-s) - s\frac{3}{2}R_1 i_1^2 - s\frac{3R_2'}{2}\left[(g\psi_{2d} - ei_1\cos\omega t)^2\right. \\ & \left.-(g\psi_{2q} - ei_1\sin\omega t)^2\right] + V_1(-a\psi_{2d} + bi_1\cos\omega t - \omega_r\psi_{2q}) \\ & +V_2(-a\psi_{2q} + bi_1\sin\omega t + \omega_r\psi_{2d}) + \\ & +V_3 ci_1(\psi_{2d}\sin\omega t - \psi_{2q}\cos\omega t) - V_3\frac{p}{J}M_o \end{aligned} \tag{11}$$

where V_1, V_2, V_3 are the conjugate variables which fulfil the following conjugated equations:

$$\left.\begin{array}{c} \frac{dV_1}{dt} = sm_1\psi_{2d} - sli_1\cos\omega t + V_1 a - V_2\omega_r - V_3 ci_1\sin\omega t \\ \frac{dV_2}{dt} = sm_1\psi_{2q} - sli_1\sin\omega t + V_2 a + V_1\omega_r + V_3 ci_1\cos\omega t \\ \frac{dV_3}{dt} = V_1\psi_{2q} - V_2\psi_{2d} + \frac{p}{J}\frac{\partial M_o}{\partial\omega_r} \end{array}\right\} \tag{12}$$

The optimal control is to maximize the hamiltonian (11) and therefore to satisfy the following equations:

$$\frac{\partial H}{\partial i_1} = 0; \quad \frac{\partial H}{\partial \omega} = 0 \tag{13}$$

Solving the equations (13) for the hamiltonian described by (10) yields:

$$\begin{aligned} i_{1opts} \;=\; & n(\psi_{2d}\cos\omega t + \psi_{2q}\sin\omega t) + \frac{r}{s}(V_1\cos\omega t + V_2\sin\omega t) + \\ & +\frac{s_1}{s}V_3(\psi_{2d}\sin\omega t - \psi_{2q}\cos\omega t) \end{aligned} \tag{14}$$

$$\left.\begin{array}{l} \sin\omega t = \frac{sl\psi_{2q} + V_3 c\psi_{2d} + V_2 b}{\sqrt{(-sl\psi_{2d} - V_1 b + V_3 c\psi_{2q})^2 + (sl\psi_{2q} + V_2 b + V_3 c\psi_{2d})^2}} \\ \cos\omega t = \frac{sl\psi_{2d} - V_3 c\psi_{2q} + V_1 b}{\sqrt{(-sl\psi_{2d} - V_1 b + V_3 c\psi_{2q})^2 + (sl\psi_{2q} + V_2 b + V_3 c\psi_{2d})^2}} \end{array}\right\} \tag{15}$$

The amplitude $i_{1opts}$of the stator current described by (14) guarantees that the hamiltonian has the maximum value, but without the condition given by (10). When the limitation (10) is taken into consideration, the stator current amplitude should satisfy the following equations:

$$\left.\begin{array}{lll} i_{1opt} = i_{1opts} & \text{for} & i_{1opts} \leq i_1^o \\ i_{1opt} = i_1^o & \text{for} & i_{1opts} > i_1^o \end{array}\right\} \tag{16}$$

The optimal control described by (14),(15) and (16) depends on the conjugate variables V_1, V_2, V_3 and the state variables ψ_{2d}, ψ_{2q}, and thus if one wants to arrive at the form (9), one needs to solve the state equations (2) and conjugate equations (12) together.

To solve the canonical equations (2) and (12) we need to know the initial conditions of the state and conjugate variables and the type of the load. In this case the initial values of state variables are known. For example, if the motor starts, all initial values of state variables are null. Also, the final value of the motor speed ω_r is known. This value is the desired value for speed control and the rated value for the starting.

The initial values of the conjugate variables V_1, V_2, V_3 are not known. From the transversability conditions (see Athans, Falb, 1969) it is possible to prove that the final values of the conjugate variables V_1, V_2 are null.

To solve the canonical equations, it is necessary to solve the two-point boundary value problem, which consists in finding initial values of the conjugate variables V_1, V_2, V_3, knowing the initial values of the state variables $\psi_{2d}, \psi_{2q}, \omega_r$, the final values of the conjugate variables V_1, V_2 and the final value of the state variable ω_r.

5. Voltage control

When the frequency converter used for the induction motor supply is a voltage converter, it is necessary to find the frequency and amplitude variations with time of the induction motor supply voltage, to minimize given optimization index.

Knowing the optimal control in the form of the stator current amplitude and frequency variations with time, we can calculate the corresponding stator voltage frequency and amplitude variations with time. These relationships are (see Krause, 1987, and Schreiner, Gildebrand, 1973):

- Phase voltage amplitude:

$$\left.\begin{array}{c} u_{1d} = hi_1 \cos\omega t + k\frac{di_1}{dt}\cos\omega t + ki_1\frac{d(\cos\omega t)}{dt} - \frac{l}{3}\psi_{2d} - e\omega_r\psi_{2q} \\ u_{1q} = hi_1 \sin\omega t + k\frac{di_1}{dt}\sin\omega t + ki_1\frac{d(\sin\omega t)}{dt} - \frac{l}{3}\psi_{2q} - e\omega_r\psi_{2d} \\ u = \sqrt{u_{1d}^2 + u_{1q}^2} \end{array}\right\} \tag{17}$$

- Angular frequency of the phase current:

$$\omega = \cos\omega t\frac{d(\sin\omega t)}{dt} - \sin\omega t\frac{d(\cos\omega t)}{dt} \tag{18}$$

6. Simulation examples

To illustrate the solution method of the time and electric energy losses minimization during speed control of induction motors two numerical examples based on the digital simulation are presented. In these examples the idle starting

($M_o = 0$) of an induction motor is presented and the stator current limitation value i_1^o, which guarantees the motor operating in the linear part of the magnetization curve, is selected.

In order to determine in the approximate way the permissible maximum value of the stator current amplitude for which the saturation of the magnetic material of the motor does not happen during the optimal starting, the approach of the magnetization curve by a linear piecewise approximation is accepted in which the saturation does not happen until the amplitude of the magnetization current arrives at the value determined by the following relationship (see Sandler, Sarbatov, 1966):

$$i_{o\,\max} = \frac{u_m}{\sqrt{R_1^2 + (X_o + X_1)^2}} \tag{19}$$

This value corresponds to the ideal synchronous revolving movement of the induction motor (when in the circuit of the rotor no current flows).

For the idle starting, optimal from the point of view of the simultaneous minimization of the electric energy losses in the stator and rotor windings and the starting time, it is possible to demostrate (see Kawecki, 1980) that, for the permissible value of the stator current amplitude fulfilling the following condition:

$$i_1^o \leq i_{o\,\max} \sqrt{\frac{R_2' X_o^2 + 2R_1 (X_o + X_2')^2}{(R_1 + R_2') X_o^2 + 2R_1 X_2' (X_o + X_2')}} = i_{11} \tag{20}$$

the amplitude of the magnetization current fulfils the following condition:

$$i_o \leq i_{o\,\max} \tag{21}$$

To solve the two-point boundary value problem, the parametric optimization algorithm derived by Kawecki, Niewierowicz (1991), based on the Gauss-Seidel method, is used. In Fig. 1 the flow-diagram of this algorithm is presented.

The optimization index (8) is also used as parametric optimization criterion during the exploration of the initial values of the conjugate variables. The use of (8) as parametric optimization criterion is possible, because if the control is optimal, it must minimize (or maximize) the optimization index. Then we need to search only for the initial conditions of the conjugate variables for which, applying the optimal control, we obtain the minimum (or maximum) value of the optimization index.

To compare the vector optimal control with the scalar optimal controls, the latter ones were also found by using the method presented above: to minimize the time only ($s = 0$, we obtain the maximum of the hamiltonian for $i_1 = i_1^o$, because, in this case, the hamiltonian is a linear function of i_1 (see Kawecki, Niewierowicz, 1992) and to minimize the energy losses in the stator and rotor windings only ($s = 1$) (see Kawecki, Niewierowicz, 1996).

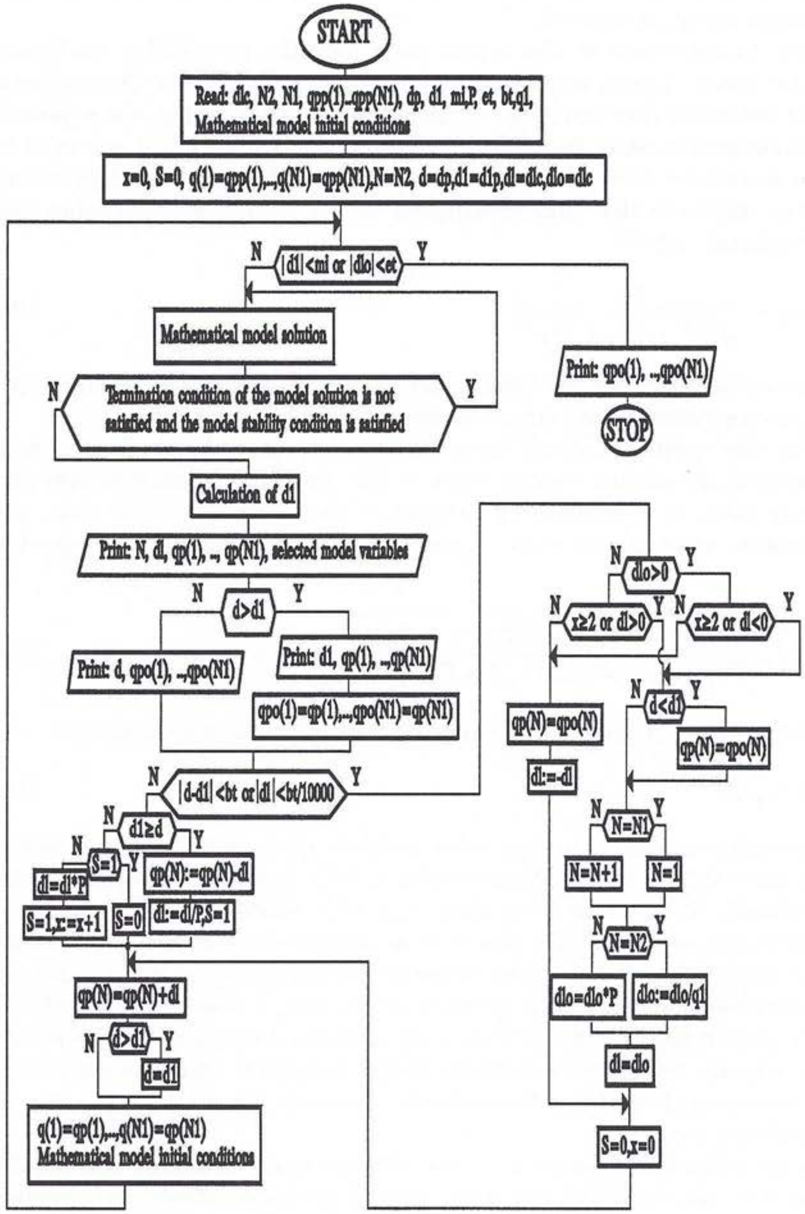

Figure 1. Parametric optimization algorithm

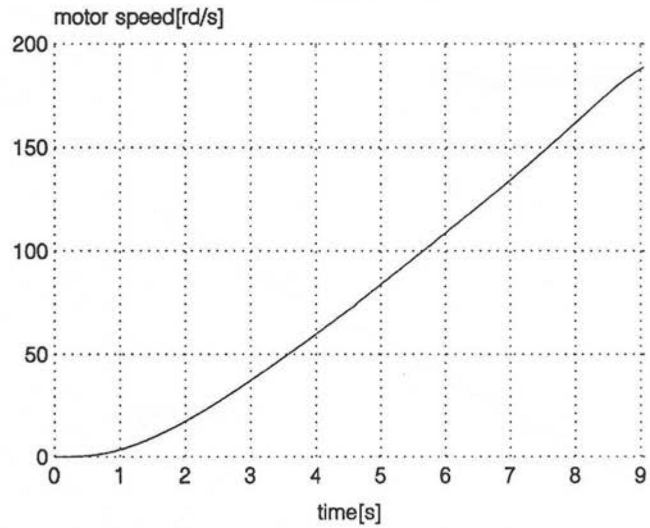

Figure 2. Motor sped (2250 h.p. motor)

In the first example, we find the optimal control which minimizes the index (8) for $s = 0.1$ and $i_1^o = 177.7[A]$ during the idle start of a 2250 h.p. induction motor, whose parameters are as follows:

$$\begin{aligned} u_m &= 1877.94[V],\ m = 3,\ p = 2,\ \omega_n = 377[rd/s] \\ R_1 &= 0.029[\Omega], R_2^{\prime} = 0.022[\Omega], X_1 = X_2^{\prime} = 0.226[\Omega] \\ X_o &= 13.04[\Omega], J = 63.87[kgm^2] \\ i_{o\max} &= 141.56[A], i_{11} = 177.76[A] \end{aligned}$$

The following initial values of the conjugate variables were obtained:

$$V_1(0) = 52.2831, V_2(0) = 66.21841, V_3(0) = 5.832544$$

The relevant variation curves are presented in Figs. 2 - 7.
Fig.8 presents the curves of the relative variables.
We obtain the following values of the variables during the motor starting:

$$t_r = 9.06[s], i_{1m} = 177.7[A], Q_{2a} = 16160.137[J]$$

During the starting (Fig.7) the amplitude of the magnetization current attains the maximum value $i_{om} = 133.8[A]$ and the limit stays below the value $i_{o\max} = 141.56[A]$ for which the magnetic material of the motor begins to enter the saturation.

Using the scalar optimal controls for idle starting of the same motor we obtain:

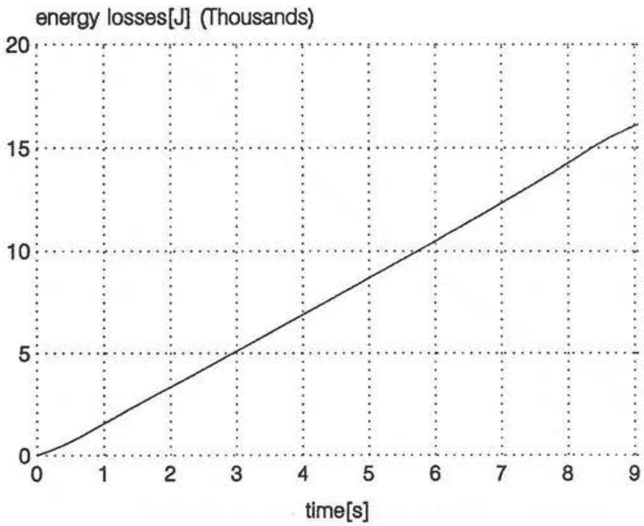

Figure 3. Electric energy losses (2250 h.p. motor)

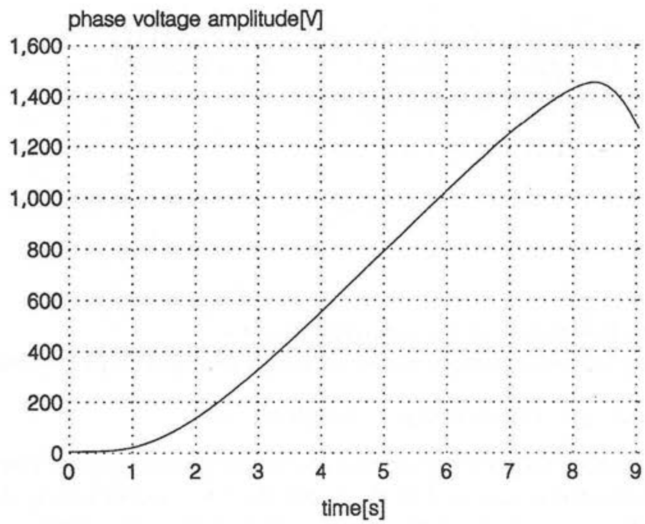

Figure 4. Phase voltage amplitude (2250 h.p. motor)

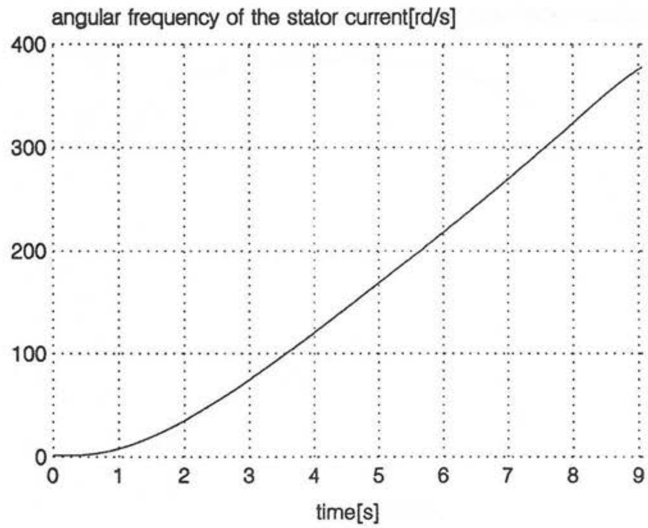

Figure 5. Stator current angular frequency (2250 h.p. motor)

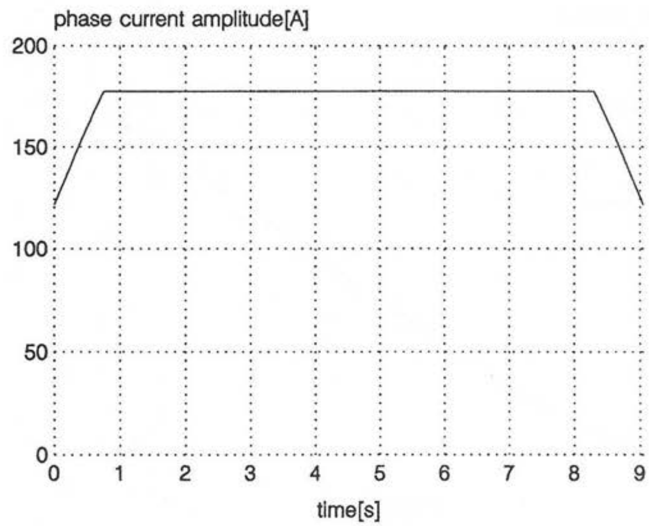

Figure 6. Phase current amplitude (2250 h.p. motor)

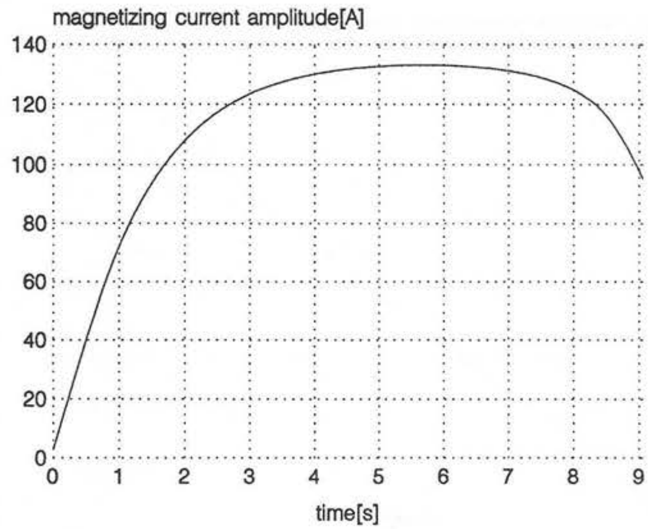

Figure 7. Magnetization current amplitude (2250 h.p. motor)

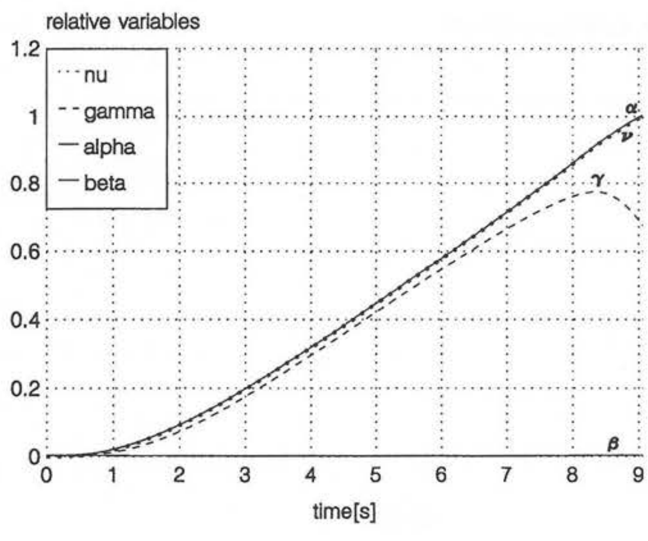

Figure 8. Relative variables (2250 h.p. motor)

- For the time optimal control:

  $t_r = 8.7[s], i_{1m} = 177.7[A], Q_{2a} = 16979.48[J]$

- For the optimal control to minimize the electric energy losses in the stator and rotor windings:

  $t_r = 230[s], i_{1m} = 42.37[A], Q_{2a} = 13581.3[J]$

Comparing the results obtained for the three cases mentioned above, we may conclude that, for the optimal control which minimizes both the time and electric energy losses during starting we obtain:

- The starting time is 4.1% longer than the control time obtained for the time optimal starting and 2439% shorter than the control time obtained for the control minimizing the electric energy losses only.
- The electric energy losses are 19% greater than the electric energy losses obtained with the control which minimizes this energy losses only and 5.1% less than the electric energy losses obtained with the time optimal control.
- During the starting period (Fig.6) the stator current amplitude reaches its limit value. For the time optimal control case this amplitude is equal to the limit value of i_1^o during the whole time of starting. During starting, which minimizes the electric energy losses only, the stator current amplitude is less than the limit value i_1^o during the whole control time interval.

The results obtained for other values of the coefficient s are:
For $s = 0.2$:

$$t_r = 18.8[s],\ i_{1m} = 157.68[A],\ Q_{2a} = 14220.02[J]$$

For $s = 0.5$:

$$t_r = 87.7[s],\ i_{1m} = 77.04[A],\ Q_{2a} = 13771.2[J]$$

In the second example, we find the optimal control which minimizes the index (8) for $s = 0.5$ and $i_1^o = 7.3[A]$ during the idle starting of a 3 h.p. induction motor, whose parameters are as follows:

$$\begin{aligned} u_m &= 179.6[V],\ m = 3,\ p = 2,\ \omega_n = 377[rd/s] \\ R_1 &= 0.435[\Omega],\ R_2' = 0.816[\Omega],\ X_1 = X_2' = 0.754[\Omega] \\ X_o &= 26.13[\Omega],\ J = 0.089[kgm^2] \\ i_{o\max} &= 6.68[A],\ i_{11} = 7.38[A] \end{aligned}$$

The following initial values of the conjugate variables were obtained:

$$V_1(0) = -4.894553, V_2(0) = 3.829396, V_3(0) = 0.3006305$$

The relevant variation curves are presented in Figs. 9 – 14.

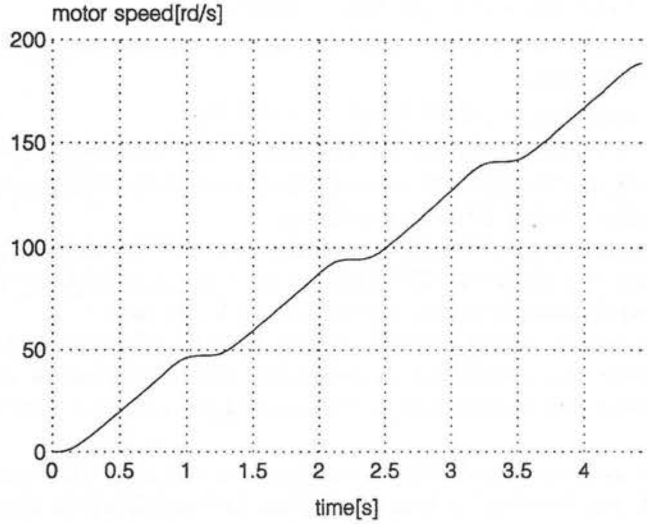

Figure 9. Motor speed (3 h.p. motor)

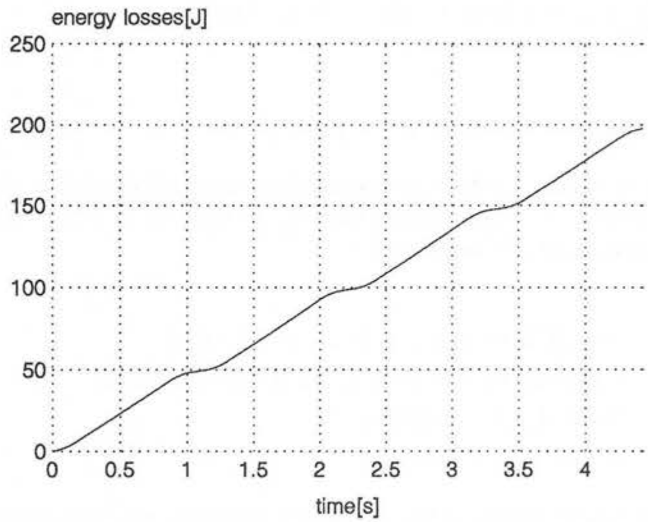

Figure 10. Electric energy losses (3 h.p. motor)

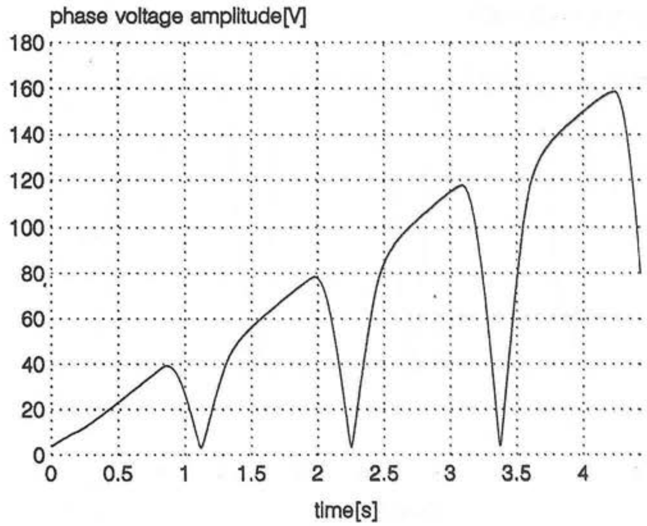

Figure 11. Phase voltage amplitude (3 h.p. motor)

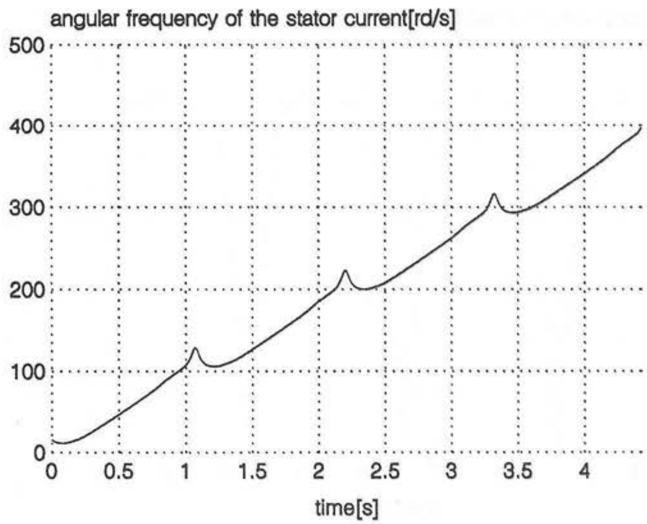

Figure 12. Stator current angular frequency (3 h.p. motor)

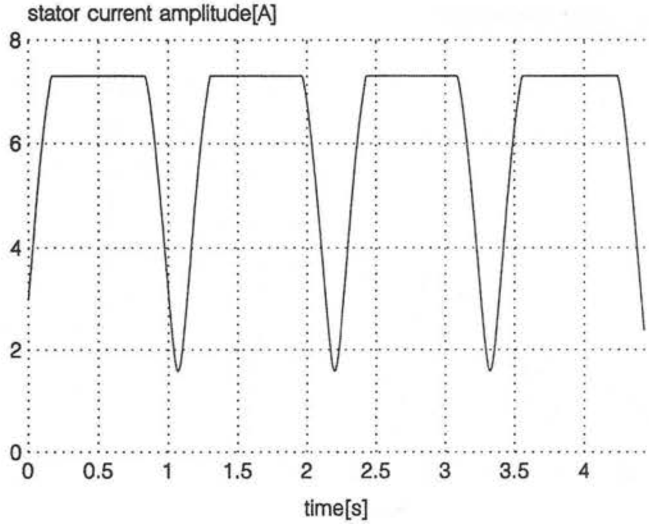

Figure 13. Stator current amplitude (3 h.p. motor)

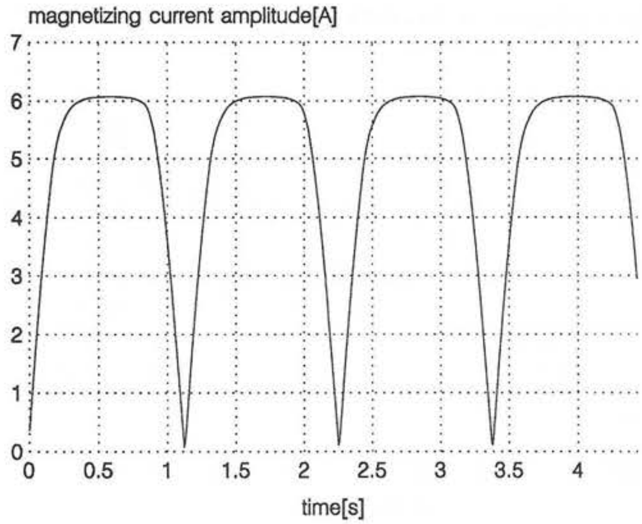

Figure 14. Magnetization current amplitude (3 h.p. motor)

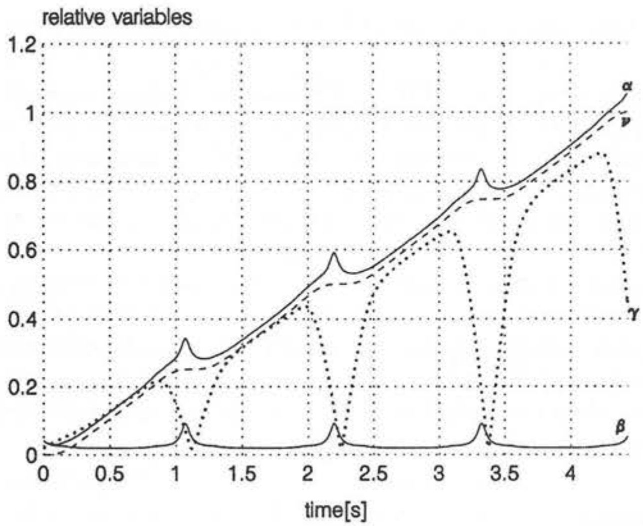

Figure 15. Relative variables (3 h.p. motor)

Fig.15 presents the curves of the relative variables.

We obtain the following values of the variables during the motor starting:

$$t_r = 4.41[s],\ i_{1m} = 7.3[A],\ Q_{2a} = 197.83[J]$$

During the starting (Fig.14) the amplitude of the magnetization current attains the maximum value $i_{om} = 6.08[A]$ and the limit stays below the value $i_{o\,\max} = 6.68[A]$ for which the magnetic material of the motor begins to enter the saturation.

The results obtained for the scalar optimal control are:

- For the time optimal control:

  $$t_r = 3.47[s],\ i_{1m} = 7.3[A],\ Q_{2a} = 242.2[J]$$

- For the optimal control to minimize the electric energy losses in the stator and rotor windings:

  $$t_r = 193[s],\ i_{1m} = 1.5[A],\ Q_{2a} = 180.42[J]$$

Comparing the results obtained for the three cases mentioned above, we may conclude that, for the optimal control which minimizes both the time and the electric energy losses during starting we obtain:

- The starting time is 27.1% longer than the control time obtained for the time optimal starting and 4276% shorter than the control time obtained for the control, which minimizes the electric energy losses only.
- The electric energy losses are 9.65% greater than the electric energy losses obtained with the control which minimizes the energy losses only and

22.4% less than the electric energy losses obtained with the time optimal control.

- During the starting period (Fig.13) the stator current amplitude reaches periodically its limit value. For the time optimal control case this amplitude is equal to the limit value of i_1^o during the whole time of starting. During starting, which minimizes the electric energy losses only, the stator current amplitude is less than the limit value i_1^o during the whole control time interval.
- Comparing the results obtained in this example with the ones obtained in the first example, we may conclude that the shape of the optimal control curves depend significantly on the relation between the electromagnetic time constants and the mechanical time constants. The respective curves are periodic when the inductances and resistances are relatively large in comparison with the rotor inertial torque.

7. Conclusions

Based on the obtained results we may conclude that:

- It is possible to find the analytical description of the optimal frequency control which minimizes both the electric energy losses and the time, during speed control of induction motors (relationships (14), (15) and (16)), using the induction motor mathematical model, which takes into account electromagnetic transient.
- The derived form of control may be termed implicit, and this is because it is describing the control as function of state and conjugate variables in a relatively complex way.
- To obtain the explicit description of the control (the amplitude and frequency variations of the motor supply voltage or current as a function of time) it is necessary to solve the two-point boundary value problem at a given motor load.
- In the present stage of the microprocessor speed development, the results obtained in this paper may be applied to the induction motor starting in the open-loop control system, generating previously the voltage or current amplitude and frequency control curves by a computer.
- It is possible, practically to consider that the resistances and inductances of the stator and rotor windings are constant during the induction motor starting, then the assumption 2 adopted when formulating the mathematical model of the induction motor is practically satisfied.
- The stator current limitation (10) is introduced not only for the frequency convertor protection, but also for the rotor acceleration limitation during the rotor speed control. The excessive rotor accelerations can destroy the rotor ball bearings. This may be the case when the control approaches the time-optimal control (small values of the s coefficient in the optimization index (8)). Through adequate selection of the stator current limitation

value (10) it can also be guaranteed that the induction motor will work during the speed control without the magnetic material saturation.

- The simulation results based on the optimal control described in this paper may be used for evaluation of other practical control systems. For that purpose it is sufficient to compare the results of two control system simulations: the optimal and the evaluated.

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Appendix: List of symbols

$$a = \frac{R_2' \omega_n}{X_o + X_2'}$$

$$b = \frac{X_o R_2'}{X_o + X_2'}$$

bt – small number determining the calculation exactitude of the parametric algorithm optimization

$$c = \frac{3}{2} p^2 \frac{X_o}{X_o + X_2'} \frac{1}{J}$$

$d1$ – parametric optimization index value in the present iterative step of the parametric optimization algorithm

d – parametric optimization index value in the preceding iterative step of the parametric optimization algorithm

$d1p$, dp – $d1$ and d values, respectively, for the initiation of the parametric optimization algorithm calculation

dl – parameter increment value in the present iterative step of the parametric optimization algorithm

dlo – parameter increment value beginning a cycle of the parameter change in the parametric optimization algorithm

dlc – parameter increment value beginning the parametric optimization algorithm execution

et – small number determining the calculation exactitude of the parametric optimization algorithm

$$e = \frac{X_o}{X_o + X_2'}$$

$$g = \frac{\omega_n}{X_o + X_2'}$$

$$h = \frac{R_1 (X_o + X_2')^2 + R_2' X_o^2}{(X_o + X_2')^2}$$

H – hamiltonian
i_1 – phase stator current amplitude
i_{1d}, i_{1q} – stator current components on the d-q axes
i_2' – phase rotor current amplitude related to the stator circuit
i_{1opts} – value of the amplitude i_1 for which the hamiltonian attains the maximum value without limitation of the stator current
i_{11} – admissible value of the stator current amplitude i_1^o for which the amplitude of the magnetization current i_o does not exceed to value $i_{o\max}$ during the optimal control of the motor speed
i_o – magnetization current amplitude
i_{om} – maximum value of the amplitude i_o during the motor speed control
$i_{o\max}$ – maximum value of the magnetization current amplitude for which the motor still operates in the linear part of its magnetization curve
i_{opt} – value of the amplitude i_1 for which the hamiltonian obtains the maximum value taking into consideration the limitation i_1^o
i_1^o – maximum admissible value of the amplitude i_1
i_{1m} – maximum value of the amplitude i_1 during the motor speed control
J – inertial torque of the rotor

$$k = \frac{(X_o + X_2')(X_o + X_1) - X_o^2}{(X_0 + X_2')\omega_n}$$

$$l = \frac{3R_2'X_o\omega_n}{(X_0 + X_2')^2}$$

m – number of the phases of the motor

$$m_1 = \frac{3R_2'\omega_n^2}{(X_0 + X_2')^2}$$

mi – small number determining the calculation exactitude of the parametric optimization algorithm
M_o - load torque

$$n = \frac{R_2'X_o\omega_n}{R_1(X_o + X_2')^2 + R_2'X_o^2}$$

N – the currently changed parameter number in the parametric optimization algorithm
$N1$ – number of the parameters for optimization in the parametric optimization algorithm
$N2$ – number of the first parameter meant for change at the start of the calculations in the parametric optimization algorithm
p – number of pairs of poles

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SCHREINER, R.T., GILDEBRAND, A.D. (1973) Frequency time-optimal speed control of induction motors in the closed loop system (in Russian). *Energia, Electrichestvo*, **10**.

Appendix: List of symbols

$$a = \frac{R'_2 \omega_n}{X_o + X'_2}$$

$$b = \frac{X_o R'_2}{X_o + X'_2}$$

bt – small number determining the calculation exactitude of the parametric algorithm optimization

$$c = \frac{3}{2} p^2 \frac{X_o}{X_o + X'_2} \frac{1}{J}$$

$d1$ – parametric optimization index value in the present iterative step of the parametric optimization algorithm

d – parametric optimization index value in the preceding iterative step of the parametric optimization algorithm

$d1p$, dp – $d1$ and d values, respectively, for the initiation of the parametric optimization algorithm calculation

dl – parameter increment value in the present iterative step of the parametric optimization algorithm

dlo – parameter increment value beginning a cycle of the parameter change in the parametric optimization algorithm

dlc – parameter increment value beginning the parametric optimization algorithm execution

et – small number determining the calculation exactitude of the parametric optimization algorithm

$$e = \frac{X_o}{X_o + X'_2}$$

$$g = \frac{\omega_n}{X_o + X'_2}$$

$$h = \frac{R_1 (X_o + X'_2)^2 + R'_2 X_o^2}{(X_o + X'_2)^2}$$

ψ_{2d}, ψ_{2q} – rotor magnetic flux linkage components on the $d-q$ axes
ω – angular frequency of the stator current
ω_n – nominal angular frequency of the stator current
ω_r – angular speed of the rotor

P – an integer number (for example $P = 2$), dividend used in the parametric optimization algorithm in order to decrease the parameter increment during a cycle of the parameter changes

$q1$ – integer number greater than P^{N1}, divisor used in the parametric optimization algorithm for the decrease of the parameter increment, finishing a cycle of the parameter changes

$q(N)$ – initial value of the N-number parameter in the mathematical model used in the parametric optimization algorithm

$qp(N)$ – initial value of the N-number parameter beginning the iterative cycle of the parameter changes in the parametric optimization algorithm

$qpo(N)$ – optimal value of the N-number parameter

$qpp(N)$ – initial value of the N-number parameter beginning the parametric optimization algorithm execution

$Q_1 = t_r$ – optimization index for the control time minimization only

Q_2 – electric energy losses in the stator and rotor windings

Q_{2a} – Q_2 value when the control is finished

Q_3 – vector optimization index

$$r = \frac{1}{3}\frac{X_o R_2'(X_o + X_2')}{R_1(X_o + X_2')^2 + R_2' X_o^2}$$

R_1, R_2' – resistances of the stator winding and of the rotor winding related to stator circuit, respectively

s – constant coefficient in the vector optimization index

$$s_1 = \frac{1}{2}\frac{p^2}{J}\frac{X_o(X_o + X_2')}{R_1(X_o + X_2')^2 + R_2' X_o^2}$$

S – parametric optimization algorithm flag

t – time

t_r – speed control time

u – phase voltage amplitude

u_{1d}, u_{1q} – stator voltage components on the $d - q$ axes

u_m – nominal supply voltage amplitude

V_1, V_2, V_3 – conjugate variables

x – parametric optimization algorithm flag

X_1, X_2' – dissipation reactances of one phase of the stator winding and one phase of the rotor winding, related to the stator circuit, in a two-phase equivalent motor, calculated for the nominal frequency of the stator current, respectively

X_o – magnetizing reactance of two-phase equivalent motor calculated for nominal frequency of the stator current

$\alpha = \frac{\omega}{\omega_n}$ – relative angular frequency of the stator current

$\gamma = \frac{u}{u_m}$ – relative amplitude of the phase voltage

$\beta = \frac{\omega - p\omega_r}{\omega_n}$ – relative slip

$\nu = \frac{p\omega_r}{\omega_n}$ – relative angular frequency of the rotor