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Book review:

Basic Ergodic Theory

by

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The second edition, like the first edition of this book, is devoted to basic topics of ergodic theory. In this second edition a section on rank one automorphisms appears in Chapter 7 and a discussion of the ergodic theorem due to Wiener and Wintner is added to Chapter 2. The book is written in a revised form and is based on papers written between 1920 and 1990. The book is a very interesting one and covers a lot of topics of ergodic theory as well as the Glimm-Effros theorem. Theorems are proved in the book in their descriptive, measure theoretic or topological versions. The book is especially recommended for anyone who would like to learn about the descriptive approach to ergodic theory and about measure preserving transformations and flows. The volume consists of the following twelve chapters and contains 95 references.

Chapter 1 ("The Poincaré Recurrence Lemma") is devoted to Borel automorhisms on Borel spaces. In this section a notion of orbit equivalence is introduced and the Poincaré recurrence lemma is proved in three versions: the measure theoretic version, for conservative automorphisms, and in the category version.

Chapter 2 ("Ergodic Theorems of Birkhoff and von Neumann") deals with an ergodic theorem for permutations and some generalisations of it. In this section also an ergodic theorem for almost periodic functions and the classical Birkhoff's ergodic theorem for measure preserving transformations are proved.

Chapter 3 ("Ergodicity") is a brief discussion of ergodicity and provides sufficient and necessary conditions for a measure preserving transformation to be ergodic.

Chapter 4 ("Mixing Conditions and Their Characterizations") deals with mixing and weakly mixing automorphisms.

Chapter 5 ("Bernoulli Shift and Related Concepts") is devoted to measure preserving automorphisms arising from shifts on product spaces. It deals with Bernoulli, Markov and Kolmogorov shifts. Bernoulli shifts provide us with some examples of mixing measure preserving automorphisms.

Chapter 6 ("Discrete Spectrum Theorem") proves the theorem due to Halspectrum and having the same set of eigenvalues. Such automorphisms appears to be metrically isomorphic.

Chapter 7 ("Induced Automorphisms and Related Concepts") introduces a construction called the Kakutani tower and the notion of induced automorphism and the automorphism built under a function. In this section also one rank automorphisms and Chacon's automorphism are considered.

Chapter 8 ("Borel Automorphisms and Polish Homeomorphisms") proves the very useful result due to Ramsay and Mackey.

Chapter 9 ("The Glimm-Effros Theorem") provides a sufficient condition for existence of a continuous probability measure on Borel subsets of a complete separable metric space, which is ergodic with respect to a countable group of Borel automorphisms.

Chapter 10 ("E. Hopf's Theorem") provides a necessary and sufficient condition for a Borel automorphism to admit a finite invariant measure. In this section also a notion of compressibility is introduced.

Chapter 11 ("H. Dye's Theorem") proves a very important result on orbit equivalence stating that any two free ergodic measure preserving automorphisms on a standard probability space are orbit equivalent.

Chapter 12 ("Flows and Their Representations") treats flows built under a function and provides theorems for representations of non-singular free flow and of measure preserving free flow as a flow built under a function.

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