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Application of modern portfolio theory to the Russian state bond market[1]

by

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Abstract: The behaviour of the Russian state bond market is analyzed. Attention is mainly paid to short-term fluctuations and efficiency of short-term investments. Analysis of return time series has shown that there exists a significant autocorrelation, and that distribution of random fluctuations is non-Gaussian. It predetermines a choice of forecasting schemes. The most efficient ones appear to be non-linear. The efficiency was checked not only by the traditional statistical indices but by direct numerical experiments where various types of predictors were used as basic elements of decision rules. The decision algorithms have included the solution to the modified optimal portfolio problem where the forecasts were used as expected returns and the covariance matrix was estimated via forecasting errors.

Keywords: portfolio optimization, forecasting, Russian state bond market.

1. Introduction

The Russian state bond market was highly fluctuating and non-stationary. Unlike US Treasury bills, the Russian short-term bonds (GKO) could not be considered as risk-free securities and starting with the very beginning in 1994, this market was primarily used for active speculative games. The speculators redistributed investments between various GKO issues and foreign currency market rates, the corporate stock market and the international financial market. One should also point out some specific difficulties arising from the initial concept of considexing GKO as risky securities.

The complexity of the problem is due to the fact that the data samples on bond trades are short by definition, and therefore any estimation concerning any specific bond issue is not precise. In order to overcome that obstacle, we suggested (Barinov et al., 1997) a scheme for transforming the original series into evolution series (ES). Each ES consists of the segments of series which correspond to different original series. They are united in such a way that the first ES includes the history segments of issues with the longest time of maturity at any t. The second ES includes the segments for issues which are next to the first one in relation to maturity, and so on. Formally, the ES-scheme implementation can be written as follows. Let $f_i(t)$, $t \leq T_i$, define the history of issue i up to maturity T_i. Then, the j-th ES consists of the elements of $f^{(j)}(t)$ such that $f^{(j)}(t) = f_{i_j^*(t)}(t)$, where $i_j^*(t) = \arg\min_t \{T_i - t \mid t \leq T_i,\ i \leq j-1\ \}$. The given period of any evolutionary series is close to that of the market as a whole, which provides an acceptable basis for statistical conclusions. At the same time the forecasting of ES allows one to obtain the forecast of any given issue.

The alternative way is more traditional and consists in grouping the returns of issues into indices, e.g., a group with a time of maturity shorter than a month, from one to three months and so on. It is obvious that in that case the possibility of individual forecasting is lost and hence, the decision rules, formed on the basis of the forecast, can bear only an aggregate character.

Before constructing the forecasting schemes, the preliminary statistical processing was carried out in order to check the validity of the main hypotheses of the classical financial market theory which form the foundation of the Random Walk Model (RWM). Namely, they are the hypotheses of stationarity, normality and non-correlation. The hypotheses have been checked in application to the evolutionary series, designed on the basis of one-day return series for every original issue, i.e.:

$$f_i(t) = r_i(t) = \frac{P_i(t+1) - P_i(t)}{P_i(t)} \approx \ln \frac{P_i(t+1)}{P_i(t)}$$

where $P_i(t)$ is the sale price of issue i on day t.

Statistical analysis has shown that: a) the real market behaviour not can be described using a model of RWM type, b) a short-term forecasting of speculative operations returns is possible, c) the forecasting schemes must take into account the non-stationarity and the non-Gaussian character of return fluctuations. Let us remark that the absence of normality is a natural consequence of a market structure. Gaussian behaviour would take place if there were a lot of independent agents, having approximately the same power at the market. However,

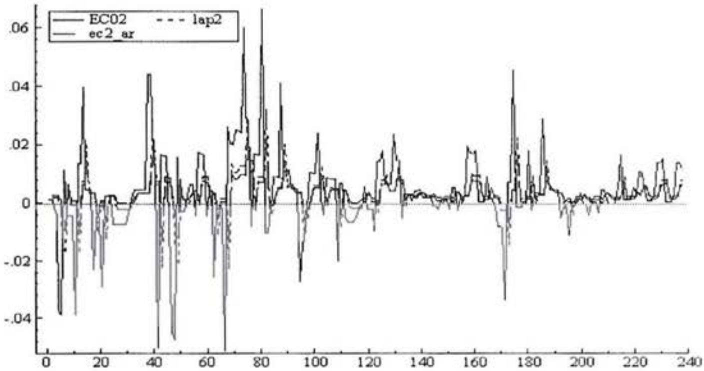

Figure 1.

At first, some schemes of parametric statistics having the form:

$$R_t = q_0 + q_1 R_{t-1} + \varepsilon_t$$

were considered. If the errors ε_t are normally distributed then the traditional Least Squares (LS) estimates for the parameters q_0 and q_1 can be used. However, the strong non-Gaussian character of the fluctuations mentioned above has to be taken into account.

The simplest class of distributions, reflecting the effect of 'fat tails', is the Laplace distribution. For this class the maximum likelihood of estimations is achieved by using the method of least error modulus (LMM). This fact simplifies the parameter estimation. Fig.1 presents graphs of the true values of returns and the estimations $\hat{r}_G$ (predictor by Gaussian errors model), $\hat{r}_L$ (predictor by Laplace model). One can see that $\hat{r}_G$ provides a better tracing of peaks.

There are some other known ways to explain and to take into account the 'fat tails effect'. The method of construction of conditional heteroscedastic (CH) models is the most popular in the world literature on financial series forecasting (Kariya, 1993; Gourieroux, 1997). The main idea of the CH-model is to take into account the dependence of volatility on forecasting errors at the previous time steps and, possibly, on the previous values of volatility itself. Formally, three important classes of CH-models can be distinguished:

- ARCH(q) - the Engle model (Engle, 1982):

$$\sigma_t = S_0 + \sum_{\tau=1}^{q} S_\tau \varepsilon_{t-\tau}^2, \quad S_\tau > 0,$$

- GARCH(q, p) - the Bollerslev model (Bollerslev, 1986):

$$\sigma_t = S_0 + \sum^{q} S_\tau \varepsilon_{t-\tau}^2 + \sum^{p} \gamma_\tau \sigma_{t-\tau} \quad S_\tau, \gamma_\tau > 0,$$

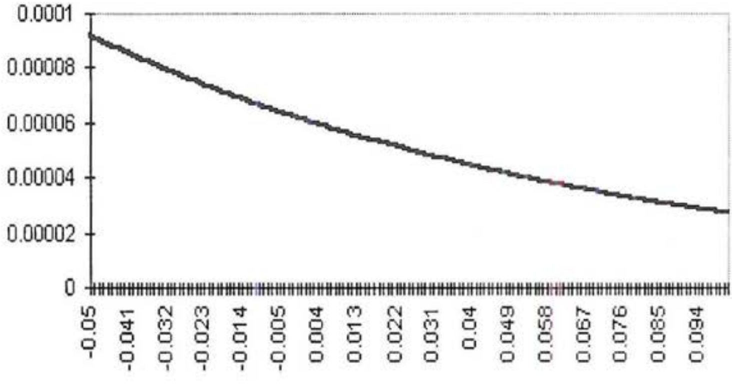

Figure 2.

- EGARCH - the Nelson exponential model (Nelson, 1991):
  $\sigma_t = a\exp(b\varepsilon_{t-1}), \quad a > 0.$

In all cases ε_t is interpreted as a deviation of returns from the expected value. If this deviation is positive, then it is taken as 'good news', otherwise - as 'bad news'. The Nelson model is the only one of the three considered models which takes into account the sign of ε_t and hence, the effect of asymmetry, the significance of which was mentioned above. One should emphasize that any type of CH-model is able to explain the 'fat tails' effect (see Gourieroux, 1997).

Fig.2 presents a so-called *news impact curve* (Nelson, 1991), showing the volatility σ_t as a function of ε_{t-1} for ES5. One can see that 'good news' ($\varepsilon_t > 0$) do not stimulate market volatility. Only 'bad news' have that property: the market 'gets scared' of an unpredictable fall of returns.

	MTV-AR	AR	Laplace	MTV-L	ARCH(1,1)	Nelson
μ	0.00279	0.00211	0.00192	0.00258	0.002558	0.002266
σ	0.00553	0.00495	0.00492	0.00561	0.004977	0.005022

Table 1. Results for various forecasting methods.

Table 1 presents comparative results using various forecasting methods which apply to one of the evolutionary series in 1996. As is clear from the table, the best one of the parametric procedures is the scheme of one-dimensional forecasting by the Laplace predictor. The non-parametric statistics show an absolutely different way of taking the nonlinear effects into account.

Let us assume, as before, that the pairs $\{y_\tau, R_\tau; \tau < t\}$, where y_τ is the value of an information factor and R_τ is the observed value of returns, are known. It is required to obtain an estimation of returns $\hat{R}$ for the next moment of time, using the history and the last information y_t. In the schemes of nonparametric mate $\hat{R}_t$ is formed directly based on initial data. In practice, the simplest and the most important estimates are as follows:

- the Nadaraya-Watson estimation:
$$\hat{R}_t = \frac{\sum_{\tau<t} \mu(|y_t - y_\tau|) R_\tau}{\sum_{\tau<t} \mu(|y_t - y_\tau|)},$$
where $\mu(.)$ is the given decreasing function (potential). The rate of a potential decrease is a parameter of the algorithm.
- the "nearest neighbours" method (NNM), that can be described by the following two-step procedure. First, N values $d_\tau = |y_t - y_\tau|$ are ordered so that
$$d_1 \leq d_2 \leq \ldots \leq d_k \leq \min_{\tau>N} d_\tau,$$
where k is a given number of the "nearest neighbours", which is a parameter of the algorithm. Then,
$$\hat{R}_t = \sum_{i=1}^{k} \varrho_i R_i, \quad \sum_{i=1}^{k} \varrho_i = 1, \ \varrho_i > 0,$$
where ϱ_i are given weights, determined by a hypothesis concerning a local behaviour of R as a function of y.

From the general point of view, it is essential that the NNM approach demonstrates a similarity to the recommendations of the classical technical analysis (chartism). According to them, one must find the pattern(s) of the market prehistory which is close to the current market behaviour and future forecast must be based on further behaviour of that pattern.

k	Weights	$\mu \times 10^{-3}$	$\sigma \times 10^{-3}$
10	Parab.	1.81	4.93
	Triang.	1.59	4.70
	Uniform	2.23	5.21
6	Parab.	0.80	4.65
	Triang.	0.80	4.39
	Uniform	0.86	5.93
5	Parab.	0.76	4.34
	Triang.	0.77	4.30
	Uniform	0.94	5.05
4	Parab.	0.60	4.24
	Triang.	0.69	4.32
	Uniform	0.44	4.19
3	Parab.	0.79	4.55
	Triang.	0.85	4.61
	Uniform	0.69	4.42
2	Parab.	1.01	5.03
	Triang.	1.02	4.94
	Uniform	1.01	5.13

Table 2. Efficiency of nonparametric estimations.

The preliminary testing of efficiency nonparametric estimations of was carried out on the same statistical base as the parametrical estimations testing de-

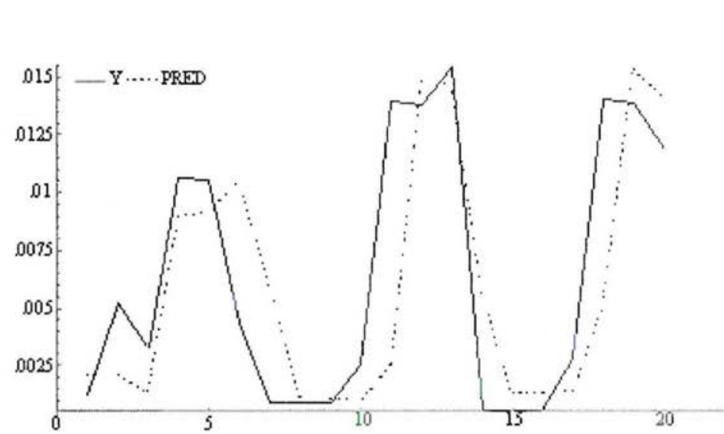

Figure 3.

presents the number of nearest points, the second - variants of approximation weights, the third and the fourth - mean values and standard deviations of forecasting errors. The best parameter choice gives the result of (4.19, 0.44) which is essentially better than the results of parametric forecasting presented above in Table 1. The forecasting efficiency is also illlustrated by Fig.3.

It is easy to see that "the nearest neighbours method" can be interpreted as a generalization of classical technical analysis which is based on the study of similarity of the current market behaviour to the situations observed earlier, represented in the form of charts of prices and sales volumes. If y_t includes only such factors then "the nearest neighbours method" formalizes a procedure of selection of similar situations and forecasting by "similarity".

4. Empirical estimation of the decision rules efficiency

The implementation of the procedures of efficiency estimation is rather labour-consuming and has required the elaboration of a special software package, "Monitoring of the market", which provides the information database and the consistent forming of decision rules and efficiency estimations. The research was made for two periods relating to 1996 and 1997. The set of considered securities was used in the aggregate form:
x_1 – investment in GKO with time of maturity from 1 to 30 days,
x_2 – the same, but with time of maturity from 1 to 3 months,
x_3 – the same, but with time of maturity more than 3 months,
x_4 – investment in currency (USD).
All returns were calculated in rubles, therefore, all kinds of investments were risky.

The original set of factors included:

- the history of yield to maturity (YTM) and secondary trade volumes $Y_{i,\tau}$, $V_{i,\tau}$, $i = 1, 2, 3$;
- the history of Dow Jones Industrial DJ_τ, the corporate stock index of the Russian trade system RTS_τ and its relative increment ΔRTS_τ.

According to the results of correlation and factor analysis, for each period and each forecasted variable the most significant factors were selected. Finally vectors y were formed as shown in Table 3.

Forecasted variable	Factors, 1996	Factors, 1997
$r_{1,t}$	$r_{1,t-7}, r_{4,t-7}$	$r_{1,t-7}, Y_{1,t-1}, RTS_{t-1}$
$r_{2,t}$	$r_{2,t-1}$	$r_{2,t-1}$
$r_{3,t}$	$r_{3,t-1}$	$r_{3,t-1}, r_{4,t-2}, \Delta RTS_{t-1}$
$r_{4,t}$	$r_{4,t-1}, Y_{1,t-1}, Y_{1,t-2}$	$r_{4,t-2}$ $Y_{1,t-1}$

Table 3. The most significant factors.

It is noteworthy that the basic factors' structure changes in time. In 1997 the influence of corporate stock market became apparent. As for the influence of the external market indicators mentioned above, it was essential only during a short period of time. Systematically, that influence was expressed only indirectly through the internal market factors.

From the variety of forecasting algorithms only the following types were considered:

- AR model with an estimation by LSM (predictor GAUSS),
- AR model with an estimation by LMM (predictor LAPLACE),
- Nonparametric rule of 'nearest neighbours' (predictor NNM).

First predictor was considered as a basic one for comparison. Two other ones were selected due to their advantages in forecasting accuracy which were seen during the preliminary tests (see above). Tables 4 and 5 give some idea concerning the comparative empirical estimation results. The percentage portfolio returns calculated in rubles and dollars are denoted by R_p and R_p^{USD}, respectively. The corresponding annual returns are given in parentheses.

Predictor	t_e	λ	R_p: %	R_p^{USD} : %	T_S
GAUSS	80	0.1	81.6 (154.3)	61.8 (112.2)	1
GAUSS	80	1	74.6 (139.1)	55.6 (99.6)	1
GAUSS	80	10	73.3 (136.3)	54.4 (97.3)	1
LAPLACE	100	0.1	83.6 (158.6)	63.6 (115.9)	1
LAPLACE	100	1	82.8 (156.9)	62.9 (114.5)	1
LAPLACE	100	10	77.5 (145.3)	58.1 (104.8)	1
NNM	130	0.1	80.8 (152.5)	61.1 (110.8)	1
NNM	130	1	80.6 (152.0)	60.9 (110.41)	1
NNM	130	10	80.7 (152.3)	61.0 (110.6)	1

Table 4. Results for the period of 30.04.96-20.12.96. Market return: 34.0% (58.1) in rbls., 19.4% (31.9) in USD.

Predictor	t_e	λ	R_p: %	R_p^{USD} : %	T_S
GAUSS	80	0.1	13.6 (30.3)	10.3 (22.5)	16
GAUSS	80	1	13.6 (30.3)	10.3 (22.5)	16
GAUSS	80	10	13.4 (29.8)	10.0 (22.0)	15
LAPLACE	80	0.1	14.0 (31.2)	10.7 (23.4)	7
LAPLACE	80	1	14.0 (31.2)	10.7 (23.4)	7
LAPLACE	80	10	14.0 (31.2)	10.7 (23.4)	7
NNM	150	0.1	11.1 (24.4)	7.9 (17.0)	5
NNM	150	1	11.0 (24.2)	7.8 (16.7)	5
NNM	150	10	10.9 (23.9)	7.7 (16.5)	5

Table 5. Results for the period of 30.06.97-23.12.97. Market return: 6.3% (13.5) in rbls., 3.2% (6.7) in USD.

Table 4 presents the data for 1996, Table 5 - for 1997, including the observed market returns and returns in US dollars. It follows from Tables 4 and 5 that:

- The portfolios significantly overperformed the market during both 1996 and 1997 periods.
- Portfolios' returns expressed in hard currency also appeared to be very high.
- In 1996 the portfolios overperformed the market portfolio since the very first day of management, however, these were not valid for 1997.
- The less the risk aversion, the more the return on portfolio. This effect is strengthened with decline of the forecast errors (i.e. for better predictors).

5. Conclusions

Let us give some qualitative conclusions:

- the application of the decision rules based on the use of statistical predictors and the optimal portfolio theory, allows 'to beat the market' systematically,
- the decision rules based on nonlinear predictors demonstrated some advantages, though the gain in portfolio return is less significant than the gain in forecasting accuracy,
- the higher the forecasting reliability, the more effective the rules with a higher risk level,
- the non-diversified portfolio using currently only one asset, forecasted as the best one under the "nearest neighbours" scheme (i.e. the most risky optimal portfolio) gave satisfactory results in both considered periods.

The last conclusion confirms the judgment stated above, i.e. that there is no absolute contradiction between the recommendations of the classical technical analysis based on the study of similarity of current situations in the market to the situations observed earlier, and the recommendations of the optimal portfolio theory. Of course, it refers to the modified theory which uses forecasting, metric statistics. Moreover, the authors not can insist on the correctness of this conclusion outside the investigated empirical material. One can only assume that it is reasonable for strongly unstable markets, the typical example being that of the Russian state bond market. It is also necessary to have in mind that in the accomplished comparison of various decision rules the transaction costs were not taken into account. However, they play an essential role even if the structure of investment portfolio remains unchanged. Besides, the absence of diversification is understood above in the aggregate sense and does not exclude the diversification within each group of assets.

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