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Professor Jakub Gutenbaum
on his 70th birthday

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Soft methods in statistical quality control

by

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Abstract: The paper is devoted to soft methods in statistical quality control. A review of existing tools for dealing with vague data or fuzzy requirements is given. Some new procedures are also proposed.

Keywords: fuzzy sets, sampling plans, control charts, vague data.

1. Introduction

Statistical quality control (SQC) was developed in the early 1920s in the United States. At that time it was applied for quality inspection of items produced in large quantities. Basic ideas of SQC have been developed more or less in parallel with the ideas of statistical testing, and therefore, some notions used in SQC have their counterparts in the theory of statistical tests. Moreover, numerous practical successes of SQC methods have been used as the confirmation of the applicability of the theory of statistical tests.

For many years the methods of SQC have been used in an industrial environment where a frequency interpretation of such terms as, for example, producer's risk and consumer's risk seemed to be rather obvious for the majority of statisticians. Therefore, well defined notions of the theory of statistical tests have been used for the design of the parameters of SQC procedures. However, in the 1950's some specialists in SQC noticed that the procedures of SQC should rather be designed taking into account the economic consequences of their usage. An important stream in the SQC theory developed and resulted in numerous papers on this subject. Main results concerning economic optimality of the acceptance sampling procedures were summarised in the book of Hald (1981). Important results connected with the optimal (from an economic point of view) design of the statistical process control (SPC) were published in the book of von Collani (1989). One of the most general models used for this purpose has been presented in Hryniewicz (1992a).

The economic approach to design of SQC procedures has been heavily criticised by many statisticians. The main opponents of that approach have pointed out serious problems with the precise assessment of involved costs. They have not realised, however, that exactly the same criticism may be applied to the so called "statistical" approach, as there exists a very straightforward relationship between supposedly well known statistical requirements, and unknown costs. Therefore, precise statements about required values of the statistical characteristics of SQC procedures are as well founded as similar statements about economic consequences of the application of these procedures. This may lead to a conclusion that it is advisable to declare the requirements for the statistical characteristics of SQC procedures in a "soft" way.

SQC procedures have been applied mainly for quality control of items produced in production processes. In such a case it is relatively easy to specify whether inspected items are conform or not conform to the stated technical requirements. However, methods of quality control become more frequently applied for quality inspection of such objects as documents, service procedures, etc. In all these cases precise description of quality of inspected items may be rather difficult. We also face the same problem when quality of inspected items is assessed by users. Therefore, there is a need to allow for an imprecise description of the quality of inspected items.

The arguments presented in the preceding paragraphs show rather clearly that there is an urgent need to propose another approach to statistical quality control that takes into account inherently imprecise quality requirements and the possibility of the imprecise description of inspected items. Attempts to fulfil this need are not numerous and theoretical results related to these problems have been published only recently. In this paper we give a rather comprehensive review of existing results. Moreover, we propose some new results which broaden the applicability of classical SQC procedures to such practical situations that cannot be dealt with by the existing statistical procedures.

Statistical procedures of SQC can be roughly divided into two groups: procedures for acceptance sampling, and procedures for statistical process control (SPC). In the second section of this paper we describe the acceptance sampling procedures. First, we briefly recall some basic concepts of classical approach to the acceptance sampling. Then, we show how to design acceptance sampling plans when the requirements for risks involved are relaxed and when both risks and quality requirements are expressed in an imprecise way. Finally, we present the most general case, where not only the requirements are imprecise, but the quality data are imprecise as well. The third section of the paper is devoted to the statistical process control. First, we present the notion of a classical Shewhart control chart and then we suggest two types of control charts that may be used for in the presence of vague data.

2. Acceptance sampling

2.1. Classical sampling plans by attributes

Suppose that a lot of size N has been submitted for inspection. We observe a random sample of n items $X_1, \ldots, X_n$ taken from the lot described as follows

$$X_i = \begin{cases} 0 & \text{if the item is conforming} \\ 1 & \text{if the item is nonconforming} \end{cases}, \quad i = 1, \ldots, n. \tag{1}$$

The observed total number of nonconforming items in the sample is

$$d = \sum_{i=1}^{n} X_i. \tag{2}$$

In the case of the simplest, though the most popular, single sampling plan by attributes, the decision whether to accept or to reject the whole lot depends on the relationship between d and a critical number c, called the acceptance number. More precisely, if $d \leq c$ then we accept the whole lot, otherwise, i.e. if $d > c$ then we reject the lot. Thus, any single acceptance sampling plan by attributes may be completely described by an ordered pair (n, c).

It is worth noticing that a single sampling plan by attributes is equivalent to a hypothesis testing for the critical quality level θ_0. Namely, we consider a statistic d from the hypergeometric distribution, i.e. $d \sim Hy(N, D, n)$, where D denotes the number of nonconforming items in the lot. If the ratio $\frac{n}{N}$ is small (say, $\frac{n}{N} \leq 0.1$) and the sample size N is large enough, then we can use the binomial approximation, i.e. $d \sim Bin(n, \theta)$, where $\theta = \frac{D}{N}$ is the proportion of nonconforming items (also called fraction nonconforming). Thus, our sampling plan is equivalent to testing hypothesis $H : \theta \leq \theta_0$ against the alternative hypothesis $K : \theta > \theta_0$.

Although there are also double, multiple and sequential sampling plans, further on by a sampling plan (or a plan, for short) we will understand only a single acceptance sampling plan by attributes.

The problem of designing a plan is to find such two numbers n and c that certain requirements concerning that plan are fulfilled. Traditionally, we specify four parameters: producer's quality level θ_1, consumer's quality level θ_2, producer's risk δ and consumer's risk β (where $\beta < 1 - \delta$). Then, assuming that binomial sampling is appropriate, the sample size n and the acceptance number c ($c \leq n$) are the solutions of

$$\begin{cases} P(\theta_1) = 1 - \delta \\ P(\theta_2) = \beta, \end{cases} \tag{3}$$

where $P(\theta)$ denotes the probability that a lot of quality θ (i.e. a lot with fraction θ nonconforming) will be accepted.

Usually (3) cannot be realised since both n and c must be integers and a desired plan does not exist. Therefore, following requirements are adopted instead of (3):

$$\begin{cases} P(\theta_1) \geq 1 - \delta \\ P(\theta_2) \leq \beta. \end{cases} \tag{4}$$

There are often many plans (n, c) which satisfy (4). Hence we need a criterion for choosing optimal sampling plan. The most popular optimality criterion is that based on minimizing the sample size. We say that a plan (n^*, c^*) satisfying (4) is $n-$*optimal* if $n^* \leq n$ for all plans (n, c) satisfying (4). The problem of determining optimal sampling plans was considered by Guenther (1973), Hald (1967, 1977, 1981), Jeach (1980), and Stephens (1978).

For more details concerning acceptance sampling we refer the reader to Schilling (1982).

2.2. Sampling plans with relaxed risks

In practice it is not essential that risks and quality levels be exact - it suffices that they are close, in some sense, to the desired values. Thus we face two important problems:

(a) how to describe these relaxed conditions on risks and quality levels?

(b) how to design the optimal plan for relaxed risks and quality levels?

Single sampling plans by attributes with relaxed risks were first discussed by Ohta and Ichihashi (1988). They have reformulated the plan design problem as a fuzzy mathematical programming one. Ohta and Ichihashi considered generalized conditions (3)

$$\begin{cases} P(\theta_1) \cong 1 - \delta \\ P(\theta_2) \cong \beta, \end{cases} \tag{5}$$

where the symbol $\cong$ stands for an approximate relation treated as fuzzy equality associated with fuzzy numbers corresponding to expressions of the type: "about δ", "approximately between δ_1 and δ_2", etc.

Let $\mu_A(\delta)$ and $\mu_B(\beta)$ describe a grade of satisfaction with a sampling plan for actual producer's risk δ and actual consumer's risk β, respectively. Thus, more precisely, we can consider two fuzzy sets A and B with membership functions $\mu_A, \mu_B : [0,1] \to [0,1]$, respectively, for modeling risks, where 1 represents full satisfaction while 0 corresponds to complete lack of satisfaction. Examples of producer's risk are given in Fig. 1. Function given in Fig. 1a describes risk corresponding to the statement: "about δ", while function given in Fig. 1b describes risk corresponding to the statement: "rather smaller than δ but surely not greater than δ_2" or "approximately smaller than δ". Note that the

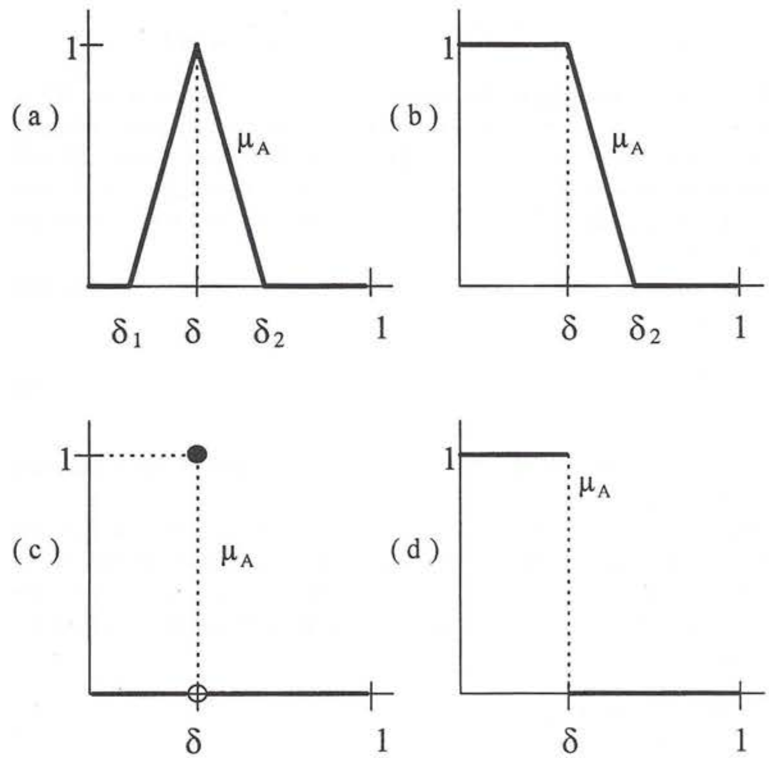

Figure 1. Examples of membership functions corresponding to producers' risk

function given in Fig. 1c corresponds to the traditional model (3): producer's risk is equal to δ, while the function given in Fig. 1d corresponds to the model (4): the risk is smaller than δ.

Now the problem of designing an optimal sampling plan satisfying (5) reduces to finding such (n, c) that maximize

$$\min\left\{\mu_A(\delta(n,c)), \mu_B(\beta(n,c))\right\}. \tag{6}$$

Recently, Kanagawa and Ohta (1990) considered generalized conditions (4)

$$\begin{cases} P(\theta_1) \tilde{\geq} 1-\delta \\ P(\theta_2) \tilde{\leq} \beta, \end{cases} \tag{7}$$

where the symbols $\tilde{\geq}$ and $\tilde{\leq}$ stand for approximate inequalities treated as fuzzy inequalities associated with fuzzy numbers corresponding to expressions of the type: "rather greater than $1-\delta$", "slightly less than β", etc. (see Fig. 1b).

2.3. Sampling plans with relaxed risks and quality levels

Various economic and technological factors must be taken into account while defining θ_1, θ_2, δ and β, which makes it difficult for producers and consumers to uniquely specify these four main factors. Determination involves careful and complex negotiations between producers and consumers, especially for θ_1 and θ_2. Thus, to relax the rigidity of the conventional design, let us also consider relaxed quality levels.

In the case of relaxed both risks and quality levels we have to consider following conditions:

$$\begin{cases} P(\sim\theta_1)\widetilde{\geq}1-\delta \\ P(\sim\theta_2)\widetilde{\leq}\beta, \end{cases} \tag{8}$$

where $P(\sim\theta)$ denotes the probability that a lot of relaxed quality close (in some sense) to θ will be accepted.

For designing plans satisfying conditions given above Tamaki, Kanagawa and Ohta (1991) tried to apply the modal theory of Dubois and Prade (1983). Using the possibility and necessity measures they transform problem (8) into a following fuzzy mathematical programming problem with modal constraints: minimize n subject to

$$\begin{cases} \Im_1\{1-P(\theta_1)\leq\delta\}\geq\nu_1 \\ \Im_2\{P(\theta_2)\leq\beta\}\geq\nu_2 \end{cases} \tag{9}$$

where $\Im_1$ and $\Im_2$ are modal procedures on producer's and consumer's side, respectively, while ν_1 and ν_2 are lower limits on these measures. Unfortunately, it seems that this method involves too much subjectivity connected with the choice of the modal measure (necessity, possibility, etc.) and determination of additional limits for these measures. Therefore, Grzegorzewski (1998c, 1999a) proposed a different method for designing plans with relaxed both risks and quality levels based on Arnold's approach to fuzzy hypothesis testing (Arnold, 1996).

Suppose we have two functions $\lambda_1, \lambda_2 : [0,1]\to[0,1]$, where $\lambda_1(\theta_1)$ describes grade of satisfaction with a sampling plan for actual producer's quality level θ_1 while $\lambda_2(\theta_2)$ is a grade of dissatisfaction with a plan for actual consumer's quality level θ_2. Of course, we can also consider the membership function $\widetilde{\lambda}_2$ corresponding to consumer's satisfaction with given quality level, where $\widetilde{\lambda}_2(\theta) = 1-\lambda_2(\theta)$. According to Arnold's generalized probabilities of type I and type II errors, respectively, we get following conditions for the plan with fuzzy quality levels:

$$\delta(n,c) = \sup_{\theta\in\Theta_1}\{(\lambda_1(\theta)-\lambda_2(\theta))(1-P(\theta))\}\leq\delta \tag{10}$$

$$\beta(n,c) = \sup_{\theta\in\Theta_2}\{(\lambda_2(\theta)-\lambda_1(\theta))P(\theta)\}\leq\beta, \tag{11}$$

where $\Theta_1 = \{\theta \in [0,1] : \lambda_1(\theta) > \lambda_2(\theta)\}$ and $\Theta_2 = \{\theta \in [0,1] : \lambda_1(\theta) < \lambda_2(\theta)\}$.

Thus for the general case of a plan with relaxed both risks and quality levels we can rewrite conditions (8) as

$$\begin{cases} \delta(n,c) \tilde{\lesssim} \delta \\ \beta(n,c) \tilde{\lesssim} \beta. \end{cases} \tag{12}$$

Since we may get many plans satisfying our requirements we need an optimality criterion for choosing the best plan. We suggest to use for that purpose the following function

$$S(n,c) = \min \{\mu_A(\delta(n,c)), \mu_B(\beta(n,c)), \xi(n)\}, \tag{13}$$

where $\xi : N \to [0,1]$. Here, as before, $\mu_A(\delta)$ and $\mu_B(\beta)$ describe grade of satisfaction with a sampling plan (n,c) for producer's risk $\delta(n,c)$ and consumer's risk $\beta(n,c)$, while $\xi(n)$ describes a grade of satisfaction with a sample size n. This function, called *satisfaction function*, seems to be more appropriate than (6) because it takes into account not only risks but sample size as well. Such a function was also used by Kanagawa and Ohta (1990). Hence, from now on our optimality criterion is based on maximizing satisfaction function $S(n,c)$. We say that a plan (n^*, c^*) satisfying (12) is $MS-$*optimal* if $S(n^*, c^*) \geq S(n,c)$ for all plans (n,c) satisfying (12).

Theoretically, one may choose any functions μ_A, μ_B and ξ. However, in practice, these functions should fulfill a few regularity conditions. For example, it is clear that both producer and consumer would be satisfied with a plan with no risk and their grades of satisfaction would decrease if their risks δ and β increased. On the contrary, the grade of producer's satisfaction $\lambda_1(\theta)$ would decrease while the grade of consumer's dissatisfaction $\lambda_2(\theta)$ would increase as θ increases. The grade of satisfaction with a sample size would decrease as n increases. Moreover, there exists such an integer n_0 that the inspection with a sample of size greater than n_0 would be unprofitable.

It can be shown that under quite general and natural assumptions on functions μ_A, μ_B, λ_1, λ_2 and ξ the $MS-$optimal sampling plan exists (see Grzegorzewski, 1998c, 1999a). In his papers Grzegorzewski also published a detailed algorithm for designing $MS-$optimal single-sampling plan by attributes.

2.4. Sampling plans for vague data and requirements

In the previous sections we have considered only those generalisations of single sampling plans where such requirements for the probabilistic characteristics of the sampling plans as producer's risk δ, consumer's risk β, producer's risk quality θ_1, and consumer's risk quality θ_2, have been relaxed in comparison to similar requirements formulated in a classical setting of the statistical quality control. It could be interesting, however, to go a step further, and to allow quality data to be expressed in an imprecise way. Such a generalisation might have been especially useful in cases of quality data concerning fully qualitative quality characteristics. In the case of such data we often face difficulties with the univocal qualification of inspected items as "conforming" or "nonconforming". This difficulty arises also from difficulties with a precise definition of quality requirements based on an imprecise notion of client's satisfaction. In such cases it seems to be worthwhile to consider inspected items not only as "good" or "bad", but also as "almost good", "quite good", "not so bad", etc.

It seems quite natural that in the case of imprecise qualification of the quality of inspected items there is also no need to define quality requirements in a precise way. Sampling plans in a such general setting have been considered so far only in Hryniewicz (1992b,1994). In Hryniewicz (1994) it is assumed that the quality of each inspected item is described by a family of fuzzy subsets of a set $\{0,1\}$, with the membership function defined as $\mu_0/0 + \mu_1/1$, where $0 \leq \mu_0, \mu_1 \leq 1$, such that $\max\{\mu_0, \mu_1\} = 1$. When an inspected item "in general, fulfils quality requirements" we represent the result of its quality assessment as a fuzzy set with the membership function $1/0 + \mu_1/1$. When this item surely fulfils the requirements its quality is expressed as $1/0 + 0/1$. On the other hand, if an inspected item "in general, does not fulfil quality requirements" we represent the result of its quality assessment as a fuzzy set with the membership function $\mu_0/0 + 1/1$. A completely nonconforming or defective item is described by a fuzzy set with the membership function $0/0+1/1$. It is worthwhile to note that this definition of quality is fully equivalent with the quality assessment by a single number $a \in [0,1]$. In such a case a membership function can be expressed as

$$\mu(x) = \begin{cases} \min[1, 2(1-a)], & x = 0 \\ \min[1, 2a], & x = 1 \end{cases} . \tag{14}$$

Let us assume that in the result of a quality inspection of n items in $n_1$cases the quality of inspected items is characterised by a fuzzy set of the type $\mu_{0,i}/0 + 1/1$, $i = 1, \ldots, n_1$, and in $n_2 = n - n_1$cases by a fuzzy set of the type $1/0 + \mu_{1,j}/1$, $j = 1, \ldots, n_2$. Without loss of generality we may assume that these items are ordered in such a way that $0 \leq \mu_{0,1} \leq \mu_{0,2} \leq \ldots \leq \mu_{0,n_1} \leq 1$, and $1 > \mu_{1,1} \geq \mu_{1,2} \geq \ldots \geq \mu_{1,n_2} \geq 0$.

For the so defined results of inspection the total number of inspected nonconforming items cannot be expressed as an integer number (as in a classical setting) but is described as a fuzzy set. To find its membership function we treat the test result for each inspected item as a pseudo-fuzzy number with a membership function given by (14). Then we apply the rules for the addition of fuzzy numbers arriving at the following formula for the membership function

$$\begin{aligned} \tilde{d} = \mu_{0,1}/0 + \mu_{0,2}/1 + \ldots + \mu_{0,n_1}/(n_1 - 1) + 1/n_1 + \\ + \mu_{1,1}/(n_1 + 1) + \ldots + \mu_{1,n_2}/(n_1 + n_2). \end{aligned} \tag{15}$$

It is rather clear that the calculation of a fuzzy total number of nonconforming items $\tilde{d}$ according to (15) is quite easy. In practical cases for the majority of inspected items we have either $\mu_{0,i} = 0$ or $\mu_{1,j} = 0$, so it is always possible to represent $\tilde{d}$ in a more compact way (e.g. in a computer code).

Let us assume now that not only quality data are expressed in an imprecise way, but probabilistic characteristics (requirements) of a sampling plan, namely $\theta_1, \theta_2, \delta$ and β, as well. To simplify calculations assume that these requirements are expressed as the following fuzzy sets:

$$\begin{aligned} \theta_1 &= 1/p_{1,0} + v_{1,1}/p_{1,1} + \ldots + v_{1,m_1}/p_{1,m_1} + 0/p_{1,m_1+1}, \\ \theta_2 &= 1/p_{2,0} + v_{2,1}/p_{2,1} + \ldots + v_{2,m_2}/p_{2,m_2} + 0/p_{2,m_2+1}, \\ \delta &= 1/p_{3,0} + v_{3,1}/p_{3,1} + \ldots + v_{3,m_3}/p_{3,m_3} + 0/p_{3,m_3+1}, \\ \beta &= 1/p_{4,0} + v_{4,1}/p_{4,1} + \ldots + v_{4,m_4}/p_{4,m_4} + 0/p_{4,m_4+1}, \end{aligned} \tag{16}$$

where $0 < p_{i,0} < \ldots < p_{i,m_i+1} < 1$, $1 > v_{i,1} > \ldots > v_{i,m_i} > 0$, $m_i \in \mathcal{N}$, $i \in \{1,2,3,4\}$. To find a single sampling plan, i.e. a sample number n, and an acceptance criterion (a critical number c) we propose to use an appropriate modification of a procedure proposed by Hald (1981) for the design of single sampling plans with a sufficiently large sample size n.

Let us introduce the following quantities:

$$\begin{aligned} q_1 &= 1 - \theta_1, \\ q_2 &= 1 - \theta_2, \\ k_1 &= -0.5 + (1/6)(q_1 - \theta_1)\left(u_{1-\delta}^2 - 1\right), \\ k_2 &= -0.5 + (1/6)(q_2 - \theta_2)\left(u_{\beta}^2 - 1\right), \end{aligned} \tag{17}$$

where u_γ is a quantile of γ order from the standard normal distribution. For precisely defined quality requirements Hald (1981) proposed to find a sample size n as the solution of the following equation

$$n(\theta_2 - \theta_1) - \sqrt{n}\left(u_{1-\beta}\sqrt{\theta_2 q_2} + u_{1-\delta}\sqrt{\theta_1 q_1}\right) + (k_2 - k_1) = 0 \tag{18}$$

rounded to the nearest integer. The critical number of nonconforming items in a sample c is calculated as

$$c = (\theta_1 + \theta_2)/2 + \sqrt{n}\left(u_{1-\delta}\sqrt{\theta_1 q_1} - u_{1-\beta}\sqrt{\theta_2 q_2}\right)/2 + (k_1 + k_2)/2 \tag{19}$$

rounded to the nearest integer.

In case of imprecise quality requirements the values of $\theta_1, \theta_2, \delta$ and β in (18) and (19) should be changed into their fuzzy equivalents. However, it does not seem to be appropriate to propose a fuzzy sample size n, i.e. to allow a certain possibility of varying its value. Therefore, we propose to calculate n from (18) using $p_{1,0}, p_{2,0}, p_{3,0}$ and $p_{4,0}$ instead of $\theta_1, \theta_2, \delta$ and β, respectively. Taking into account that by taking $p_{1,0}, p_{2,0}, p_{3,0}$ and $p_{4,0}$ instead of $\theta_1, \theta_2, \delta$ and β we formulate a more severe quality requirements, the computed sample size n is surely not too small.

The fuzzy critical number $\tilde{c}$ can be calculated by "fuzzyfying" the expression (19). Using the well known extension principle, and the definition of fuzzy addition and fuzzy multiplication (see e.g. Klir and Yuan, 1995, or Zimmermann, 1985, for reference) we obtain a fuzzy set $\tilde{c} = \mu_0/c_0^r + \ldots + \mu_M/c_M^r$, where $c_0^r, \ldots, c_M^r$ are certain real numbers. In the next step we round the values of $c_0^r, \ldots, c_M^r$ to their nearest integer values arriving at the set of integer values $\{c_0, \ldots, c_m\}$. Now we define the value of the membership function for each number from $\{c_0, \ldots, c_m\}$ as the maximum of the values of the membership function for all those numbers from the set $\{c_0^r, \ldots, c_M^r\}$ that have been rounded to this integer number. As the result of applying this procedure we obtain the fuzzy critical number of nonconforming items in a sample of n items as the following fuzzy set:

$$\tilde{c} = \mu_0/c_0 + \mu_1/(c_0 + 1) + \ldots + \mu_m/(c_0 + m). \tag{20}$$

In case of a sampling plan with a fuzzy critical (acceptance) number $\tilde{c}$ and a fuzzy number of nonconforming items $\tilde{d}$, given by (20) and (15), respectively, an appropriate decision has to be made by a comparison of these two fuzzy sets. In case of precisely defined quality requirements and precise quality data we reject the hypothesis about good quality of inspected items when the inequality $d > c$ holds. However, in the case of fuzzy sets the relation $\tilde{d} > \tilde{c}$ is not univocally defined. Therefore, in order to apply the proposed procedure we have to define precisely how we understand this relation. There are many methods proposed for comparison and ranking of fuzzy sets which may be used for this purpose. An extensive simulation experiment with the goal of finding a method which seems to be appropriate in the area of quality and reliability tests has been described in Hryniewicz (1992b). From among many compared methods, the comparison based on the *NSD* index by Dubois–Prade (1983) seems to be the best, as in this case the fraction of obviously wrong decisions observed in the simulation experiment was the smallest one. Let us recall that for any fuzzy numbers A and B with membership functions μ_A and μ_B, respectively, we can evaluate the degree of necessity to which the relation $A > B$ is fulfilled

$$Ness(A > B) = 1 - \sup_{x,y:x \leq y} \min\{\mu_A(x), \mu_B(y)\}. \tag{21}$$

If we apply this approach in practice we have to calculate the value of the *NSD* index for the relation $\tilde{d} > \tilde{c}$. When this value is large enough (in most practical situations it has only to be greater than zero) we decide to reject the hypothesis about good quality of inspected items. Otherwise, the data do not give sufficient evidence for the rejection of the hypothesis.

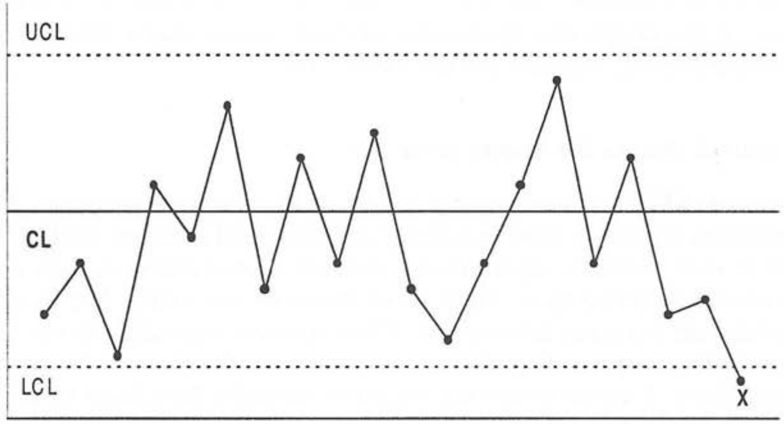

Figure 2. A typical Shewhart control chart

3. Statistical process control

3.1. Shewhart control charts

Statistical process control (*SPC*) is a collection of methods for achieving continuous improvement in quality. This objective is accomplished by a continuous monitoring of the process under study in order to quickly detect the occurrence of assignable causes and undertake the necessary corrective actions. Although many *SPC* procedures have been elaborated, Shewhart control charts are still the most popular and widely used *SPC* tools.

Let us consider a so called $\overline{x}$ control chart for monitoring the process level. The chart contains three lines: a center line (*CL*) corresponding to the process level and two other horizontal lines, called the upper control limit (*UCL*) and the lower control limit (*LCL*), respectively. When applying this chart one draws samples of fixed size n at specified time points, to then compute an arithmetical mean of each sample and plot it as a point on the chart. As long as the points lie within the control limits the process is assumed to be in control. However, if a point is outside of the control limits (i.e. below *LCL* or above *UCL*) we are forced to assume that the process is no longer under control. One will immediately intervene in the process in order to find disturbance causes and undertake corrective actions to eliminate them. A typical control chart is shown in Fig. 2.

Besides the $\overline{x}$ chart there are other charts for monitoring the process level, charts for monitoring the spread of the process (like R or s charts), the chart for the proportion of nonconforming units (p chart), the chart for the number of nonconformities per physical unit (u chart), etc. There are also more sophisticated control charts which take into account not only present sample outcome but the preceding samples, like *CUSUM* charts or *EWMA* charts. A detailed description of the design and application of these control charts can be found in Montgomery(1991), Western Electric (1956), etc.

3.2. Control charts for vague data

The traditional *SPC* tools were constructed for exact data (real numbers). However, sometimes we are not able to obtain exact numerical data but we deal with imprecise or even linguistic data. To use classical control charts in such situations we should compress these vague observations to exact data, but by doing this we often lose too much information. Thus, it seems reasonable to use fuzzy sets for modelling vague or linguistic data and then to design control charts for these fuzzy data. Control charts for linguistic variables have been developed by Wang and Raz (1988, 1990), Raz and Wang (1990), and Kanagawa, Tamaki and Ohta (1993). Then, Höppner (1994) and Höppner and Wolff (1995) proposed a fuzzy-Shewhart control chart for monitoring the process level. Their charts are designed for very particular cases and have many drawbacks (see Grzegorzewski, 1997b), and so cannot be recommended for applications. In this paper we present a fuzzy control chart due to Grzegorzewski (1997c) and propose a new fuzzy control chart based on the necessity index.

3.2.1. Fuzzy control charts for monitoring the process level

It is known that there exists a correspondence between control charts and significance tests. Thus it seems natural to use a general method for constructing fuzzy tests for fuzzy data to design control charts for fuzzy observations. This method was proposed by Grzegorzewski (1997a, 1998b). Control charts for monitoring the process level designed using this method, called *fuzzy control charts*, were also suggested by Grzegorzewski (1997c).

Suppose that the process under consideration is normally distributed. Let us first assume that we know the parameters of the process (i.e. its mean m_0 and standard deviation σ) when the process is thought to be in control. In such a case the traditional Shewhart $\overline{x}$ control chart given by lines

$$\begin{aligned} UCL &= m_0 + u_{1-\delta/2}\tfrac{\sigma}{\sqrt{n}}, \\ CL &= m_0, \\ LCL &= m_0 - u_{1-\delta/2}\tfrac{\sigma}{\sqrt{n}}, \end{aligned} \tag{22}$$

is equivalent to the following test ϕ for the two-sided hypothesis testing problem $H : m = m_0$ against $K : m \neq m_0$

$$\phi(V_1, \ldots, V_n) = \begin{cases} 0, & \text{if } m_0 - u_{1-\delta/2}\frac{\sigma}{\sqrt{n}} < \overline{V} < m_0 + u_{1-\delta/2}\frac{\sigma}{\sqrt{n}}, \\ 1, & \text{otherwise}, \end{cases} \tag{23}$$

where $V_1, \ldots, V_n$ denote a random sample, $u_{1-\delta/2}$ is the $100(1-\delta/2)$ percentile of the standard normal distribution and δ is a significance level (traditionally $\delta = 0,0027$ is accepted).

If we have no crisp observations, but fuzzy data $X_1, \ldots, X_n$, where each $X_i \in \mathcal{FN}(\mathcal{R})$ (i.e. X_i is a fuzzy number), then we can test our hypothesis H using a fuzzy test $\varphi : (\mathcal{FN}(\mathcal{R}))^n \to \mathcal{F}(\{0,1\})$ with the following $\alpha-$cuts

$$\varphi_\alpha(X_1, \ldots, X_n) = \begin{cases} \{0\}, & \text{if } m_0 \in (\Pi_\alpha \setminus (\neg\Pi)_\alpha), \\ \{1\}, & \text{if } m_0 \in ((\neg\Pi)_\alpha \setminus \Pi_\alpha), \\ \{0,1\}, & \text{if } m_0 \in (\Pi_\alpha \cap (\neg\Pi)_\alpha), \\ \emptyset, & \text{if } m_0 \notin (\Pi_\alpha \cup (\neg\Pi)_\alpha), \end{cases} \tag{24}$$

where

$$\Pi_\alpha = \left(\frac{1}{n} \sum_{i=1}^{n} (X_i)_\alpha^L - u_{1-\delta/2} \frac{\sigma}{\sqrt{n}}, \frac{1}{n} \sum_{i=1}^{n} (X_i)_\alpha^U + u_{1-\delta/2} \frac{\sigma}{\sqrt{n}} \right) \tag{25}$$

and $X_\alpha^L = \inf\{x \in \mathcal{R} : \mu_X(x) \geq \alpha\}$, $X_\alpha^U = \sup\{x \in \mathcal{R} : \mu_X(x) \geq \alpha\}$. After simple calculations we get

$$\varphi(X_1, \ldots, X_n) = \begin{cases} 1/0 + 0/1, & \text{if } m_0 \in \Pi_{\alpha=1}, \\ 0/0 + 1/1, & \text{if } m_0 \notin \Pi_{\alpha=0}, \\ \mu_\Pi(m_0)/0 + (1 - \mu_\Pi(m_0))/1, & \text{otherwise.} \end{cases} \tag{26}$$

In situation with crisp data we accept hypothesis H if the test statistic (i.e. sample average $\overline{V}$) belongs to the acceptance region and reject H otherwise. It is easily seen that this reasoning is also true for the traditional $\overline{x}$ control chart, since we consider the process in control if the average remains between control lines (which correspond to the acceptance region limits) and that the process is out of control otherwise.

In a fuzzy situation the average is no longer a real number but a fuzzy number, thus we cannot plot it as a point on a chart. Moreover, a fuzzy test leads to fuzzy decisions. We may get $1/0 + 0/1$ which indicates that we should accept H, or $0/0+1/1$ which means that H should be rejected, but we may also get $\mu_0/0 + (1-\mu_0)/1$, where $\mu_0 \in (0,1)$, which can be interpreted as a degree of conviction that we should accept (μ_0) or reject $(1-\mu_0)$ the hypothesis H, respectively. Thus, unfortunately, we cannot simply generalize the traditional control chart into a fuzzy one. In particular, we have to give up control lines in the traditional form.

Grzegorzewski (1997c) suggested to use two complementary graphs to visualize the process inspection based on fuzzy data. On the first graph we plot the center line, corresponding to the mean of the process when it can be thought to be in control (i.e. m_0) and intervals I which symbolize the fuzzy set Π, where $\Pi = \Pi(X_1, \ldots, X_n)$. Each interval I corresponds to $\Pi_{\alpha=0}$. Moreover, we also mark $\Pi_{\alpha=1}$ on I. Thus, each interval I is represented by four points, so we

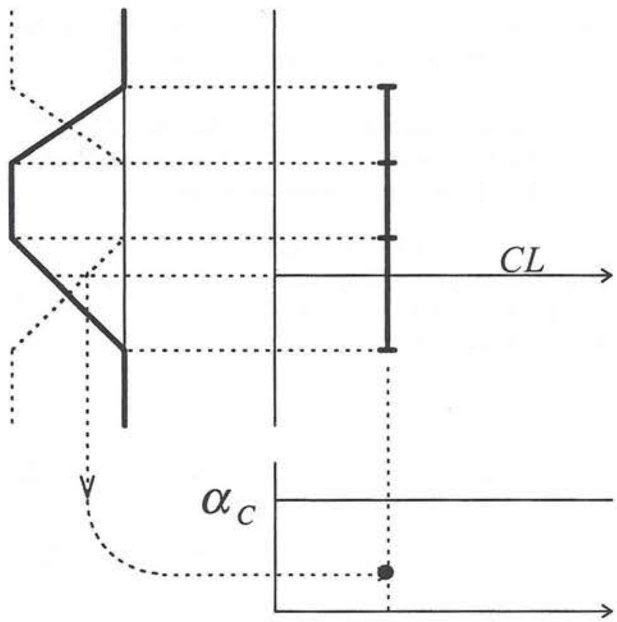

Figure 3. Construction of a fuzzy control chart

can denote it as follows $I = I(\inf \Pi_{\alpha=0}, \inf \Pi_{\alpha=1}, \sup \Pi_{\alpha=1}, \sup \Pi_{\alpha=0})$. This construction is shown in Fig. 3.

On the second graph we plot values which can be interpreted as a degree of conviction that the process is out of control. Precisely, we plot values $\mu_1(X_1, \ldots, X_n) = 1 - \mu_\Pi(m_0)$ on the graph with at least two lines: $\mu_1 = 0$, called the *zero line*, corresponding to the state of control and, so called, *critical line*, $\mu_1 = \alpha_C \in (0, 1]$.

If the sample point $\mu_1(X_1, \ldots, X_n)$ falls above the critical line it is interpreted that the process is out of control. We may also mark an additional line, the so called *warning line*, $\mu_1 = \alpha_W \in (0, \alpha_C)$. If the sample result $\mu_1(X_1, \ldots, X_n)$ falls between α_W and α_C, there is a suspicion of a disturbance. If the next result lies below α_W then this suspicion is considered to be disproved. Otherwise, if a few successive points fall between α_W and α_C one will regard the suspicion to be justified and take corrective action in order to bring the process under control (one can see the correspondence between this approach and warning lines in traditional Shewhart control charts). A typical fuzzy control chart is shown in Fig. 4.

Thus, the fuzzy $\overline{x}$ control chart is designed using the following parameters: the center line CL, the zero line, the critical line α_C (and possibly, the warning line α_W), and a formula for determining intervals I (which depend on the mathematical model and a given significance level δ). In particular, if the process

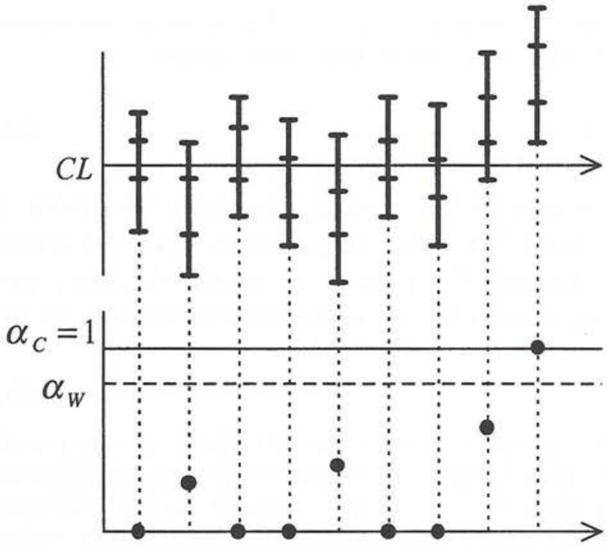

Figure 4. A typical fuzzy control chart

is normally distributed with known mean m_0 and known standard deviation σ then we get

$$CL = m_0, \tag{27}$$

$$\begin{aligned} I = \Bigg(& \frac{1}{n}\sum_{i=1}^{n}(X_i)^L_{\alpha=0} - u_{1-\delta/2}\frac{1}{\sqrt{n}}\sigma, \frac{1}{n}\sum_{i=1}^{n}(X_i)^L_{\alpha=1} - u_{1-\delta/2}\frac{1}{\sqrt{n}}\sigma, \\ & \frac{1}{n}\sum_{i=1}^{n}(X_i)^U_{\alpha=1} + u_{1-\delta/2}\frac{1}{\sqrt{n}}\sigma, \frac{1}{n}\sum_{i=1}^{n}(X_i)^U_{\alpha=0} + u_{1-\delta/2}\frac{1}{\sqrt{n}}\sigma \Bigg). \end{aligned} \tag{28}$$

If the true standard deviation is not known, then instead of (28) we use the following formula to determine I

$$\begin{aligned} I = \Bigg(& \frac{1}{n}\sum_{i=1}^{n}(X_i)^L_{\alpha=0} - t^{[n-1]}_{1-\delta/2}\frac{1}{\sqrt{n}}S_{\alpha=0}, \frac{1}{n}\sum_{i=1}^{n}(X_i)^L_{\alpha=1} - t^{[n-1]}_{1-\delta/2}\frac{1}{\sqrt{n}}S_{\alpha=1}, \\ & \frac{1}{n}\sum_{i=1}^{n}(X_i)^U_{\alpha=1} + t^{[n-1]}_{1-\delta/2}\frac{1}{\sqrt{n}}S_{\alpha=1}, \frac{1}{n}\sum_{i=1}^{n}(X_i)^U_{\alpha=0} + t^{[n-1]}_{1-\delta/2}\frac{1}{\sqrt{n}}S_{\alpha=0} \Bigg), \end{aligned} \tag{29}$$

where $t^{[n-1]}_{1-\delta/2}$ is the $100(1-\frac{\delta}{2})$-th percentile of the $t-$distribution with $n-1$ degrees of freedom and S_α is given by the formula (see Kruse and Meyer, 1987)

$$\begin{aligned} (S_\alpha)^2 &= (S_\alpha(X_1,\ldots,X_n))^2 = \\ &= \frac{1}{n-1}\max\Big\{\textstyle\sum_{i\in A}\left((X_i)^L_\alpha\right)^2 + \sum_{i\in\{1,\ldots,n\}\setminus A}\left((X_i)^U_\alpha\right)^2 - \\ &-\frac{1}{n}\Big[\textstyle\sum_{i\in A}(X_i)^L_\alpha + \sum_{i\in\{1,\ldots,n\}\setminus A}(X_i)^U_\alpha\Big]^2 \Big| A\subseteq\{1,\ldots,n\}\Big\}. \end{aligned} \tag{30}$$

In practice, we very often do not know the true value of the process mean m_0. Then, we can estimate it by the grand average of the means

$$\overline{\overline{X}} = \frac{1}{k}\sum_{j=1}^{k}(\frac{1}{n}\sum_{i=1}^{n}X_{ij}) \tag{31}$$

of the undisturbed prerun k samples (i.e. samples taken when the process is thought to be in control). Since $\overline{\overline{X}}$ is also a fuzzy number, while CL should be a crisp one, we have to defuzzify $\overline{\overline{X}}$ (otherwise we can use the fuzzy chart described in 3.2.2). Thus instead of (27) we get the following formula for the center line

$$CL = REP(\overline{\overline{X}}), \tag{32}$$

where $REP : \mathcal{FN}(\mathcal{R}) \rightarrow \mathcal{R}$ is an operator which converts a fuzzy set into a crisp number that can be considered as its representative value. We may use different operators, such as the fuzzy mode, the $\alpha-$cut fuzzy midrange, the fuzzy median, the fuzzy average, the expected value of the fuzzy number (see Heilpern, 1992) or the weighted average of the fuzzy number (see Grzegorzewski, 1998a).

3.2.2. Control charts based on *NSD* index

As it was stated above, we very often do not know the true value of the process mean m_0 and we have to estimate it. Since fuzzy data are used for the estimation, the estimate is also a fuzzy number. Since $\overline{\overline{X}}$ is a fuzzy number, while CL should be a crisp one, we have to defuzzify $\overline{\overline{X}}$. Otherwise, we can use another fuzzy chart, described below. This chart should be based on a statistical test for verifying fuzzy hypotheses with vague data. Such a test was suggested by Grzegorzewski (1999b).

The first problem in testing fuzzy hypotheses with vague data is that expressions like "$H : M \geq M_0$" or "$K : M < M_0$", where $M_0 \in \mathcal{FN}(\mathcal{R})$, have no sense, since the family of all fuzzy numbers $\mathcal{FN}(\mathcal{R})$ is not linearly ordered and such common notions are meaningless until the relation is defined. Thus we have to introduce a partial ordering in $\mathcal{FN}(\mathcal{R})$ which would be both natural and simple to be useful in hypotheses testing. In his test Grzegorzewski utilized the well known necessity index of strict dominance (*NSD*) due to Dubois and Prade (1983). The definition of that index is given in equation (21). Dubois and Prade proposed also the possibility index of strict dominance and other indices. However, we decided to use the *NSD* index because of its natural interpretation and effectiveness in solving real-life problems (see, e.g. Hryniewicz, 1992b, 1994). In his paper Grzegorzewski showed how to construct statistical tests for the one-sided and two-sided hypotheses formulated with the help of *NSD*. For the details we refer the reader to Grzegorzewski (1999b).

Since the $\overline{x}$ control chart is based on the two-sided test for the mean, a suitable test on the significance level δ for hypothesis $H : \Lambda(m_0) = \Lambda_0$ against one-sided alternative $K : Ness(\Lambda(m_0) \neq \Lambda_0) \geq \xi$, where ξ is a fixed number ($\xi \in [0,1]$) and $\Lambda(m_0) \in \mathcal{FN}(\mathcal{R})$ denotes a fuzzy perception of the true mean m, is

$$\phi(X_1,\ldots,X_n) = \begin{cases} 1 & \text{if } \; (\Lambda_0)^U_{1-\xi} < \Pi^L_{1-\xi} \quad \text{or} \quad (\Lambda_0)^L_{1-\xi} > \Pi^U_{1-\xi} \\ 0 & \text{otherwise,} \end{cases} \tag{33}$$

where $(\Lambda_0)^U_{1-\xi}$ and $(\Lambda_0)^L_{1-\xi}$ denote the lower and the upper bound of the $1-\xi$–cut of the fuzzy number Λ_0, respectively,

$$\begin{aligned}\Pi^L_{1-\xi} = \Pi^L_{1-\xi}(X_1,\ldots,X_n;\tfrac{\delta}{2}) = \inf\{t \in \mathcal{R} : \forall i \in \{1,\ldots,n\} \\ \exists x_i \in (X_i)_{1-\xi} \quad \text{such that} \quad \pi_1(x_1,\ldots,x_n) \leq t\},\end{aligned} \tag{34}$$

$$\begin{aligned}\Pi^U_{1-\xi} = \Pi^U_{1-\xi}(X_1,\ldots,X_n;\tfrac{\delta}{2}) = \sup\{t \in \mathcal{R} : \forall i \in \{1,\ldots,n\} \\ \exists x_i \in (X_i)_{1-\xi} \quad \text{such that} \quad \pi_2(x_1,\ldots,x_n) \geq t\},\end{aligned} \tag{35}$$

and where $[\pi_1, \pi_2]$ is two-sided confidence interval for the mean on the confidence level $1-\delta$.

One can rewrite equation (33) as follows

$$\varphi'(X_1,\ldots,X_n) = \begin{cases} 1, & \text{if} \begin{cases} (\overline{X})^U_{1-\xi} < (\Lambda_0)^L_{1-\xi} - \zeta \text{ or} \\ (\overline{X})^L_{1-\xi} > (\Lambda_0)^U_{1-\xi} + \zeta, \end{cases} \\ 0, & \text{otherwise,} \end{cases} \tag{36}$$

where ζ is a constant depending on a sample size n, confidence level $1-\delta$ and whether the true variance of the process is known.

Therefore, by the analogy to classical $\overline{x}$ control chart, the control lines of the new chart are

$$\begin{cases} LCL = (\Lambda_0)^L_{1-\xi} - \zeta, \\ UCL = (\Lambda_0)^U_{1-\xi} + \zeta. \end{cases} \tag{37}$$

However, now, instead of the center line CL, we have a *center area* CA, where

$$CA = [(\Lambda_0)^L_{1-\xi}, (\Lambda_0)^U_{1-\xi}]. \tag{38}$$

The inspection with our new chart looks as follows: At the beginning one chooses a significance level δ and a required value of the necessity index ξ. Then he draws a sample $X_1,\ldots,X_n$ of a fixed size n at specified time points, computes the arithmetical mean $\overline{X}$, determines interval I corresponding to $(1-\xi)$th cut of $\overline{X}$, i.e.

$$I = [(\overline{X})^L_{1-\xi}, (\overline{X})^U_{1-\xi}], \tag{39}$$

and plots it on the chart. If the whole interval lies outside the control limits (i.e. below LCL or above UCL) it is interpreted so that the process is no longer in control. If the interval intersects one of the control limits it is a warning. An example of the inspection with this control chart is given in Fig. 5.

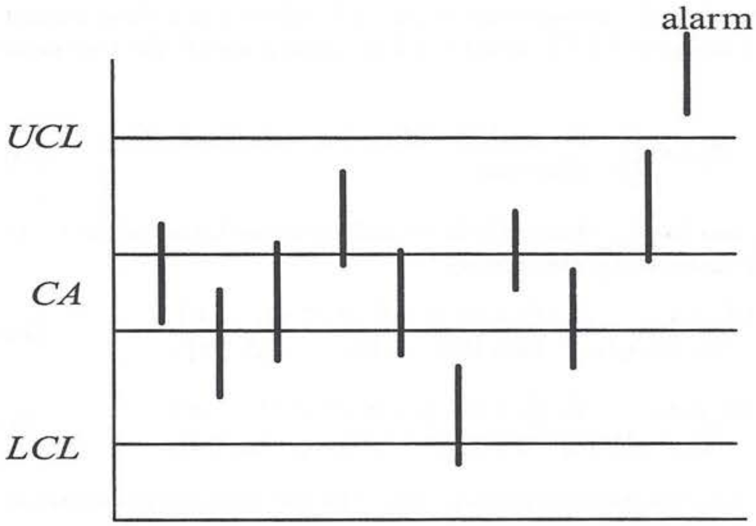

Figure 5. A control chart based on NSD index

In practice, we always estimate Λ_0 by the grand average of the means. Then, assuming that the process is normally distributed with unknown standard deviation we get the following formulas for the chart

$$\begin{cases} LCL = (\overline{\overline{X}})^L_{1-\xi} - u_{1-\delta/2}\frac{1}{\sqrt{n}}(\overline{\overline{S}})^U_{1-\xi}, \\ UCL = (\overline{\overline{X}})^U_{1-\xi} + u_{1-\delta/2}\frac{1}{\sqrt{n}}(\overline{\overline{S}})^U_{1-\xi}, \end{cases} \tag{40}$$

$$CA = [(\overline{\overline{X}})^L_{1-\xi}, (\overline{\overline{X}})^U_{1-\xi}], \tag{41}$$

where

$$\begin{matrix} (\overline{\overline{X}})^L_{1-\xi} = \frac{1}{kn}\sum_{j=1}^k \sum_{j=1}^n (X_{ij})^L_{1-\xi}, \\ (\overline{\overline{X}})^U_{1-\xi} = \frac{1}{kn}\sum_{j=1}^k \sum_{j=1}^n (X_{ij})^U_{1-\xi}, \end{matrix} \tag{42}$$

$$(\overline{\overline{S}})^U_{1-\xi} = \frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})}\sqrt{\frac{n-1}{2}}\frac{1}{k}\sum_{j=1}^{k}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left((X_{ij})^U_{1-\xi} - (\overline{X}_j)^U_{1-\xi}\right)^2}, \tag{43}$$

and Γ is the gamma function (see Höppner, 1994, or Höppner, Wolff, 1995).

It is worth noting that using the *NSD* index we can construct not only the $\overline{x}$−type control charts, but the *EMWA* charts or charts for parameters other than the ones of location.

4. Conclusions

Statistical procedures described in this paper may be used in many practical cases where we have to deal with imprecise information described in a fuzzy form. However, only in few cases the proposed fuzzy statistical procedures may be used without special problems. The level of usefulness of those statistical procedures varies in different areas of applications. It seems that, as of now, the applications in the area of acceptance sampling are more mature, and closer to real applications. The design methods that are used for single sampling plans in the presence of fuzzy risks and fuzzy quality requirements are described precisely and could be directly used in practice. Relaxation of requirements for risks and quality indices seems to be rather natural, and there are no substantial obstacles for the implementation of the proposed methods. The situation looks different, however, when we have to deal with fuzzy data. First of all, despite the fact that everybody agrees that some data may be presented in an imprecise way there is no agreement how to describe the imprecise results of experiments. The solution proposed in Section 2.3 is the simplest possible, and – as a matter of fact – may be considered as a crisp one. If quality data are presented in a really fuzzy form we still need appropriate simple methods to cope with this problem, and further theoretical results are welcome.

In case of the statistical process control (SPC) the situation is even more difficult. SPC methods, such as control charts, are widely used in practice because they are extremely simple. Moreover, even for users who do not understand the statistical background of these procedures, it is relatively simple to understand their practical meaning. Unfortunately, this is not true in the case of fuzzy control charts. The description of data is much more complicated than in a crisp case, and the statistical basis for many of these procedures seems not to be very sound. Only these fuzzy control charts that can be described as certain statistical tests have a sufficient theoretical background. Another serious problem stems from the fact that fuzzy control charts are not so easy to use in practice. It seems to us that from among many proposals only fuzzy control charts the ones presented in Subsection 3.2 of this paper have simple and intuitive interpretation.

Fuzzy approach to the problems of statistical quality control has been developed during the last ten years. In comparison with the long history of the traditional SQC it is a rather short period. Therefore, only few important problems have been appropriately addressed. From the analysis of fuzzy statistical procedures, presented in this paper it is rather obvious that much has to be done if we want to implement the proposed procedures in practice. Future work should be directed not only at theoretical problems but at the practical implementation of the procedures proposed as well.

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