Numerical simulations of a portfolio selection model with information cost

by

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Abstract: Results of numerical simulations of a portfolio selection model with information cost are presented. Simulations are based on the real data from the Warsaw Stock Exchange. The results show that buying the information according to the presented model may lead to an essential reduction of investment risk depending on the required level of return. If the return level is not too high, reduction of risk really takes place.

Keywords: portfolio, purchase of information, numerical simulations.

1. Introduction

The mathematical portfolio selection theory, developed since the 1950s, despite its remarkable achievements, neglected the aspect of cost of acquiring information necessary to take a correct decision as to how to allocate capital. There have been very few papers, e.g. Artstein, Wets (1993, 1994, 1995), Bricker, DeBruine (1993), where cost of information and its influence on investor's decisions was considered. Proposals forwarded in Banek (1999A, 1999B) were attempts to take into account the neglected cost of information. Results of further investigations on the problem of the information purchase were presented in Banek, Kowalik (1999), and Banek, Kowalik, Kozłowski (1999). In particular, the paper by Banek, Kowalik, Kozłowski (1999), proposing an extension of the classical model of Roy (1952) with the aspect of information purchase became the theoretical basis to this paper, showing an attempt to find an answer to the following question: does the possibility of information purchase (according to the model from Banek, Kowalik, Kozłowski, 1999) lead to selection of a less risky portfolio than that composed according to the classical model if the real-world data are involved? Numerical simulations, for which the real data from the Warsaw Stock Exchange were used, show that this can happen. The results obtained show clearly that there is a need of creating a new kind of financial

2.   Mathematical model

The mathematical model considered in the paper was presented by the authors for the first time in Banek, Kowalik, Kozłowski (1999). Some basic notations and assumptions from that paper are recalled.

We consider selecting a portfolio of n assets. Let x be a vector of investments, j — an n-element vector of 1's, M — total capital, $z > 0$ — a minimal level of return, required by the investor. Let $c(t)$, $0 \leq t \leq M$ denote the amount of money spent on purchase of information.

A random vector ξ_t represents a distribution of future unknown returns on assets whose parameters are estimated by analysts in time of t working hours. We assume that the investor knows *a priori* that the vector ξ_0 has a normal distribution with the parameters (estimated in any way): m — a vector of expected values (returns) and Q — a covariance matrix. The job done by analysts consists in farther estimations of the mean vector m and the covariance matrix Q. As a result of their studies they produce $m(t), Q(t)$.

We assume unlimited short-selling on all the assets. Nevertheless, the amount $c(t)$ cannot be negative because short-selling of information is impossible. It cannot exceed M either, otherwise all the money invested in assets and a part spent on information would come from short-selling.

Under assumptions from Banek, Kowalik, Kozłowski (1999) we can formulate the following stochastic model of the decision problem of the investor: given $z > 0$, M, m, $Q(t)$, find (t_{opt}, x_{opt}) — a solution to the following stochastic programming problem

$$\max_{\substack{t+\langle x,j \rangle = M \\ 0 \leq t \leq M}} P(\langle x, \xi_t \rangle \geq z + M). \tag{1}$$

From now on, we will consider a case $c(t) = t$ for reasons of computational simplicity. By Lemma 1 (from Appendix in Banek, Kowalik, Kozłowski, 1999) Problem (1) reduces to the following non-linear programming problem

$$\max_{\substack{t+\langle x,j \rangle = M \\ 0 \leq t \leq M}} \frac{\langle x, m \rangle - z - M}{\sqrt{x^{\mathrm{T}} Q(t) x}} \tag{2}$$

In Section 4 we also retain all the notations used in Banek, Kowalik, Kozłowski (1999).

3.   Data and their processing

In calculations the authors used historical data (stock prices) from the Warsaw Stock Exchange available publicly on the Internet on the server yogi.ippt.gov.pl belonging to the Institute of Fundamental Technical Research in Warsaw.

The data used are prices from the Stock Exchange sessions between 1st Jan-

The prices, available as plain text files, were imported to the spreadsheet (Microsoft Excel 5.0). Vectors of average values and covariance matrices were estimated. In order to solve the main problem formulated in Theorem 1 of Banek, Kowalik, Kozłowski (1999), i.e. to find an optimal portfolio, Mathematica 2.2 was used.

4.   Obtained results

Portfolios composed of shares of well performing companies were considered.

EXAMPLE 1. A portfolio of 4 companies: 3 banks, namely, Bank Handlowy, Kredyt Bank PBI and Powszechny Bank Kredytowy, and a garment factory Vistula. The average return vector was estimated to be $m = (1.23, 1.22, 1.12, 1.12)$ and the covariance matrix to be

$$Q = \begin{pmatrix} 0.025 & 0.016 & 0.022 & 0.014 \\ 0.016 & 0.020 & 0.0195 & 0.015 \\ 0.022 & 0.0195 & 0.036 & 0.019 \\ 0.014 & 0.015 & 0.019 & 0.018 \end{pmatrix}.$$

A decision situation of a fictional investor willing to invest a capital of 1000 Polish zlotys in shares of the above companies is considered. The investor has a possibility to buy information according to the above model and wants to obtain a return of $z = 150$ zlotys. By Theorem 1 (see Banek, Kowalik, Kozłowski, 1999) $t_{opt} = 133.016$ (i.e. the money spent on purchase of information) for $z = 150$. The optimal portfolio will then be

$$x_{opt} = (119.279, 289.942, 279.246, 178.517).$$

If the investor buys information for 133.016 Polish zlotys at $z = 150$ and creates the portfolio as above, the maximal value of

$$\frac{\langle x_{opt}, m \rangle - z - M}{\sqrt{x_{opt}^{\mathrm{T}} Q(t_{opt}) x_{opt}}} = 8.95635$$

is obtained, so the corresponding probability is practically equal to one.

If the investor does not buy information, he/she selects the following portfolio according to the classical model of Roy (1952):

$$x = (94.0794, 116.588, 448.194, 341.138).$$

Then

$$\frac{\langle x, m \rangle - z - M}{\sqrt{x^{\mathrm{T}} Q(0) x}} = 0.653335$$

EXAMPLE 2. The investor has a capital of 1000 Polish zlotys to be invested in six companies: PBR (a bank), Żywiec (a brewery), Elektrim (electric industry and trade holding), Orbis (travel agency and hotels), Warta (an insurance company), Okocim (a brewery). The vector of average returns is then $m = (1.64, 1.23, 1.23, 1.52, 1.24, 1, 19)$ and the covariance matrix is

$$Q = \begin{pmatrix} 0.53 & -0.04 & 0.02 & 0.27 & 0.02 & -0.05 \\ -0.04 & 0.05 & 0.01 & -0.04 & 0.01 & 0.02 \\ 0.02 & 0.01 & 0.03 & 0.02 & 0.02 & 0.01 \\ 0.27 & -0.04 & 0.02 & 0.23 & 0.01 & -0.03 \\ 0.02 & 0.01 & 0.02 & 0.01 & 0.02 & 0.02 \\ -0.05 & 0.02 & 0.01 & -0.03 & 0.02 & 0.02 \end{pmatrix} .$$

The results are shown in Table 1.

	$z = 180$	$z = 210$	$z = 240$
$\dfrac{t}{\bar{t}}$	−6.424, 113.37	−4.51582, 89.1045	−1.2053, 63.4337
P	(0, 113.37)	(0, 89.1045)	(0, 63.4337)
h_1 h_2 h_3	−1829.36, 845.846, 983.518	−1859.8, 864.404, 995.397	−1890.06, 882.315, 1007.75
h_0 t_{opt}	983.518; 16.482	995.397; 4.603	1000; 0
Portfolio vector (with purchase of information)	412.19, 125.251, −27.1701, 383.373, −32.9214, 122.795	359.041, 295.797, −149.142, 487.497, −182.418, 184.622	320.00, 2826.67, −1624.76, 2300.95, −1996.19, −826.667
$\dfrac{\langle x, m \rangle - z - M}{\sqrt{x^{\mathrm{T}} Q(t) x}}$	2.372	1.356	0.7024
Probability (with purchase of information)	0.99	0.91	0.759
Portfolio vector (without purchase of information)	74.3851, 315.937, −170.566, 339.932, −220.356, 660.668	97.7199, 554.47, −308.722, 526.24, −389.07, 519.363	320.00, 2826.67, −1624.76, 2300.95, −1996.19, −826.667
Probability (without purchase of information)	0.826	0.782	0.759

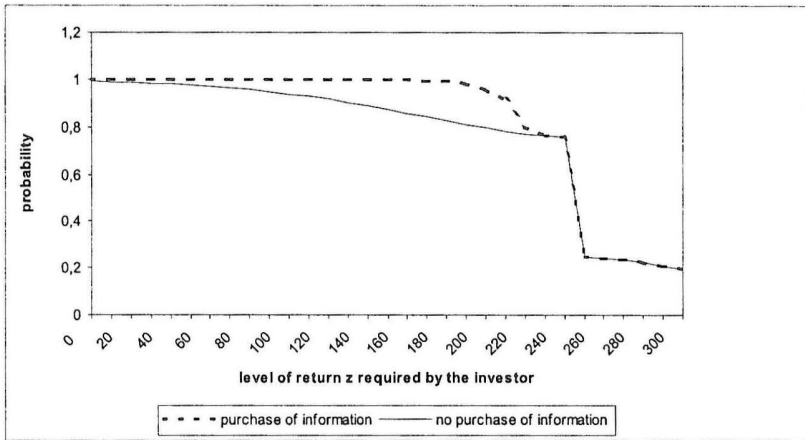Table 1. Some detailed results of optimal selection of portfolios for Example 2

Figure 1. Probabilities of exceeding the level of return z for portfolios from Example 2 — the six companies — with and without purchase of information

In order to make some kind of synthesis, we used portfolios composed of the above six companies to plot a graph (Fig. 1) showing dependence of probabilities associated with portfolios in both the classical and modified model upon the required level of return z. Fig. 1 shows the probabilities of exceeding the required level of return z as a function of z. The function was plotted for values $0 \leq z \leq 310$ with the step of 10. The dashed line represents the function with purchase of information (PI), whereas the continuous line represents the function for the standard model, i.e. with no purchase of information (NPI). Obviously, both functions are decreasing, but they differ significantly. They both start at the value practically equal to 1 for $z = 0$. However, the PI function remains almost constant up to z equal about 180 and starts to decrease strongly at $z = 200$. At z between 230 and 240 the two functions start to be identical because for such high levels of required return the purchase of information does enter in the optimal portfolio as too costly.

There are a few conclusions. It is justified to say that for $z < 180$ purchase of information would give an almost perfect immunisation to risk. Because the NPI function decreases faster than the PI function for those z, purchase of information gives relatively bigger and bigger reduction of risk as z increases. At z equal to about 190 the difference in values of the NPI and PI functions reaches its maximum of about 0.17. This means that for such z purchase of information is the most efficient because it gives the largest relative reduction of risk. Beyond 190 the relative efficiency of purchase of information decreases quickly to zero.

The results obtained suggest that it is possible to eliminate risk on risky investment almost completely by purchasing relevant information under the

1. Unlimited short-selling
2. Normality of the distribution of the random vector of returns
3. The specific form of the $Q(t)$ matrix implied by the Fisher information
   (see Banek, Kowalik, Kozłowski, 1999, for details).

It is worth stressing that none of these assumptions is satisfied in 100% in the real world.

5.   Final remarks

The results obtained show the purposefulness of existence of an information seller based on the model described in Banek, Kowalik, Kozłowski (1999). Purchase of information might make it possible to take a better (less risky) investment decision for a given level of return. Usage of the real-world data from the Warsaw Stock Exchange in the numerical simulations does not imply of course that the real decisions taken by investors would be such as the results suggest. It would be caused by the fact that the information purchase (if it had been available in the period from which the data were taken) would change the state of the market and the resulting data.

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