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Book review:

Compartmental Modelling with Networks

by

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This book arose from the courses in mathematical modelling given by the authors. The compartmental models were used there to exemplify an easily conceivable transition from verbal statements to mathematical models. That is why this book differs considerably in composition from other books in the area, like those by Jaquez (1998), Anderson (1983), or Godfrey (1983).

Thus, after preliminaries, the book starts with Part I where directed graphs are discussed. This is one of natural representations of the compartmental models. The discussion extends into six chapters and includes, besides some basic definitions and preliminary examples, the problems of isomorphic graphs, connected graphs, multigraphs, spanning trees, tournaments, planar graphs and matrices related to graphs.

Part II, consisting of five chapters, deals with the Markov chains and their relation to directed graphs, specifically stochastic graphs. The problems discussed there include classification of Markov chains and some of their special cases: regular and absorbing Markov chains.

Part III opens the subject of compartmental models with the chapter where presentation of basic concepts and discussion of one- and two-compartment models is included. It is followed by six chapters with examples of applications of compartmental modelling in different fields. One group of examples consists of models of epidemics, with recovered individuals confering immunity, becoming susceptible, or only some confering immunity and some becoming susceptible. A group of population models comprise the Leslie age-classes matrix model, predator-prey or host-parasite models, and one- and multispecie fisheries models. A group of ecosystem models include a model of concentration of dissolved oxygen in streams and lakes, a model of the forest ecosystem, and the food webs. As examples of the drug kinetics bilirubin metabolism, lead kinetics and HIV dynamics models are analysed.

Finally, Part IV deals with the theory of compartmental models: properties of compartmental matrices, analytical solutions, identifiability, parameter estimation, complexity and stability. This part differs considerably from other books in the area both because it contains material not very extensively discussed in the other books and because some subjects are not mentioned, see

The Appendix contains some mathematical prerequisities covering matrix operations and analysis as well as solution of the systems of differential equations.

To some extent, the book has a "lecture notes" character. It covers subjects taught during courses, with stress on the modelling side. The well prepared two parts on graphs and Markov chains give a good introduction to these areas for those who are not much familiar with them. The choice of topics covers the basic facts from theory and allows using these parts of the book, and particularly the first one, for an independent introductory study.

Also the third part of the book — with examples of applications of compartmental models — can be interesting for an application oriented reader who would like to learn how to use the compartmental modelling and analysis. Usually, books on compartmental models concentrate on theory and on some chosen applications, mainly in drug kinetics or ecosystem modelling. This book tries to show the larger universality of the compartmental models, as applied in many fields, including, besides those mentioned already, also economy, population, genetics and decision making. And this certainly does not exhaust the list of possible applications of the compartmental models. One could mention at least many technical applications.

Probably because of the didactic purposes, the problems posed in the book are often more of an academic type. The analysis is sometimes limited to simpler cases. The parameter estimation chapter gives only a rough idea of the methods and problems encountered there. The identifiability chapter concentrates mainly on the Laplace transform method and lacks presentation of other methods, like the exhaustive modelling method, the Taylor series expansion method, the Markov parameter matrix method or the modal matrix method. Quite surprising, the indistinguishability problem is not considered at all, although in this very problem graph theoretical analysis has been earlier applied in the proofs of some results.

Yet, the book is a valuable source of information on compartmental modelling, particularly to newcomers to the area. Also persons who have been working in this area for a longer time will find there many interesting results, and specifically those connections and advantages arising from applying graph theory to analysis of compartmental models. This direction of analysis may prove efficient, as shown by the up to date results, and may lead to further developments in understanding the properties of this kind of models and ways to get the results more meaningful than those which can be obtained now. The area of compartmental analysis found already its place in the routine work in some disciplines, let us mention pharmacokinetics. Further developments can therefore help in its broader applications.

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