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Critical price of sulphur dioxide emission allowances

by

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Abstract: The paper presents a decision-making model of environmental modernization in a power plant. The goal of the model is to analyse strategies implemented by power producers trying to comply with environmental regulations. Detailed analysis is related to the sulphur dioxide emission. Tradable emission allowances might be a new economic tool in fighting environmental problems in Poland. The model applies the real options approach. The approach is appropriate to model investments that are irreversible and an investor can afford to wait rather than invest immediately. The model assumes price uncertainty. The investment rule takes the form of a critical value optimal for immediate investment. The critical value of the project is calculated using dynamic programming. The project value follows a geometric Brownian motion. The results presented demonstrate that it is possible to calculate a critical price of emission allowances.

Keywords: emission of sulphur dioxide, tradable emission allowances, decision-making investment model.

1. Introduction

Sulphur dioxide along with dust and nitrogen oxides are among the most significant ingredients found in the emissions from fossil fuelled power plants. This paper concentrates on the issues surrounding the SO_2 emissions because it is one of the most important environmental impacts of energy generation in Poland. In order to comply with the requirements of the international conventions and agreements, e.g. the *II Sulphur Protocol*, serious efforts in creation of the future emission level regulations have to be undertaken in Poland.

Under the regulations of the Polish Ministry for Environmental Protection, Natural Resources and Forestry of 1990 and 1998, power plants in Poland are restricted in their emissions of SO_2. The Polish system of environmental regulations, relying on the permitted emission levels, fees and fines for plants that violate the target emission levels has positively impacted the situation leading to a reduction in emissions. Regulations concerning the SO_2 emission restricpollution emissions. Polish regulations of restricted levels of emission (*Ministry of Environmental...*, 1990, 1998) are constructed in the way that gives power plants enough time to adjust to new requirements by utilizing technical and economic methods. The plants can comply with the restrictions in two different ways. They can pay fines for excess emissions and continue to emit at current high levels. Alternatively, they can avoid the cost of paying the fines by sufficiently reducing their SO_2 emissions. The latter can be accomplished in a few different ways. One possibility is to modernize the power plant and switch from high-sulphur to low-sulphur coal. Another possibility is to invest in scrubbers. Each alternative involves costs that can unpredictably change over time. The influence of the costs of the environmental modernization on the critical value of the project is investigated in the paper. The model described in Section 3 and 4 uses some coefficients of investment efficiency (see Section 2). The additional and possible, but relatively new way to comply with the regulation could be the trade in the SO_2 emission allowances in Poland. Before implementing the trade in allowances, the researchers and decision makers will have to answer a lot of questions and to investigate the possible impacts of this new economic tool. The presented model could help to predict how the emission trading prices may be influenced by costs of reducing SO_2 emissions. Some results of the model, obtained from the analysis of a selected modernization case, are presented in Section 5.

2. Economic measure of investment efficiency

The following are the measures most commonly used in assessing economic efficiency of an investment: Net Present Value (*NPV*), Profitability Index (*PI*), Internal Rate of Return (*IRR*) and Modified Internal Rate of Return (*MIRR*), Break Even Point (*BEP*), Simple and Discounted Pay Back (*SPB* and *DPB*).

In the paper, the *NPV* coefficient is applied to evaluate efficiency of power plants modernization and environmental investments. The *NPV* coefficient is equal to discounted cash flows, R_t (called Present Value, V), reduced by discounted investment cost, I, and increased by the residual value in the last year of the lifetime, L_T:

$$NPV = \sum_{t=1}^{T} \frac{R_t}{(1+r)^t} - I + \frac{L_T}{(1+r)^T} \qquad (1)$$

where

r—discount rate.

The investment cost in the investment period, N_b, is described by the formula:

$$I = \sum^{t=0} \frac{I_t}{} . \qquad (2)$$

Taking into consideration the classical investment rule based on the net present value (NPV) the investment should be made as long as $NPV > 0$. In case of continuous-time cash flows, the discounted net present value is described by the equation:

$$NPV = V - I = \int_0^T R(t)e^{-rt}\, dt - I. \tag{3}$$

In equation (3), the residual value, usually relatively small, is neglected.

In the long-time analysis, the NPV method of efficiency evaluation is always convergent with a global financial gain of a firm, e.g. maximization of revenues, whereas IRR is only a relative measure of revenues.

3. A model of irreversible environmental investment under uncertainty

The aim of the model is to analyse certain strategies implemented by power producers trying to comply with environment regulations (detailed analysis is related to SO_2 emission). The analysis makes it possible to test some impacts of applying the trade of SO_2 emission allowances in Poland. To test that new economic tool, we assume that a power plant is permitted to emit only e_{dop} milligrams of SO_2 per cubic meter of fumes from the combusted fuel. A power plant emitting below the permitted level may sell the emission allowances (in this case the difference between the higher permitted level and the actual lower emitted level) using the possible price c_p PLN (Polish new zloty) per tonne. A power plant emitting over the permitted level can buy the emission allowances or has to pay a fine. Of course, a power plant may reduce its SO_2 emission by investing in scrubbers or shifting the fuel to meet the required level.

We assume price uncertainty in the decision-making model. The price of allowances, c_p [PLN/Mg], the difference between prices of better fuel and fuel with high contents of sulphur, c_w [PLN/Mg], and the operating cost of the scrubber, c_e [PLN/kWh], that depends on sorbent price and operating materials, are stochastic variables. The presented model is based on the Herbelot's model (Dixit and Pindyck, 1994, pp. 405–412, and Herbelot, 1992).

It is possible to assume that the variables behave according to the geometric Brownian motion that is a special case of Ito's process. Let us note that the Wiener process, also called a Brownian motion, can be used as a building block to model an extremely broad range of variables that vary continuously (or almost continuously) and stochastically through time. The Ito process is the generalization of the simple Brownian motion with drift. For detailed explanations, see, e.g., Dixit and Pindyck (1994). Thus,

$$\begin{aligned} dc_p &= \alpha_p c_p dt + \sigma_p c_p dz_p \\ dc_w &= \alpha_w c_w dt + \sigma_w c_w dz_w \end{aligned} \tag{4}$$

where
α—coefficient of price drift,
σ—standard deviation,
dz—the increment of a Wiener process in continuous time, $dz = \xi_t\sqrt{dt}$,
ξ_t—a normally distributed random variable with a mean of zero and a standard deviation of 1.
The correlation of stochastic variables, z, is described by formula (5):

$$\begin{aligned}\varepsilon(dz_p dz_w) &= \rho_w dt\\ \varepsilon(dz_p dz_e) &= \rho_e dt\end{aligned} \tag{5}$$

where
ε—the expectation,
ρ—correlation coefficient.
We denote:

$$\begin{aligned}\delta_p &\equiv \mu_p - \alpha_p\\ \delta_w &\equiv \mu_w - \alpha_w\\ \delta_e &\equiv \mu_e - \alpha_e\end{aligned} \tag{6}$$

where
μ—risk adjusted expected returns.

Following the analysis from Sowiński (1998), concerning the possibility of switching of coal, from high-sulphur to low-sulphur, we assume that a decision about an investment in a power plant would be made at time T in the period $0 \leq T \leq N_e$. Then, the expected present value V^w of the flows of the cost in the period from T to N_e, assuming the possibility of trade in allowances, is:

$$\begin{aligned}V^w = &\int_T^{N_e} v_{sp} B_t (e_m - e_z) 10^{-9} c_{pT} e^{-\delta_p(t-T)}\, dt\\ &- \int_T^{N_e} B_t c_{wT} e^{-\delta_w(t-T)}\, dt - \int_T^{n_e} \Delta k_{zm} w_{op} B_t e^{-r(t-T)}\, dt\end{aligned} \tag{7}$$

where
v_{sp}—capacity of fumes from the combusted coal, [m^3/Mg],
w_{op}—heat rate of coal, [GJ/Mg],
B_t—consumption of the coal in the year t, [Mg], $\dfrac{1}{w_{op}} q P_i c_{ft} T_y 10^{-3}$,
q—coefficient of energy in coal burnt to produce electricity, [kJ/kWh],
P_i—installed capacity, [MW],
c_{ft}—capacity factor in year t,
T_y—hours per year, $T_y = 8760$ [h],
e_m, e_z—emission coefficient of high-sulphur and low-sulphur coal, [mg/m^3],
Δk_{zm}—increase in operating costs caused by switching to low-sulphur coal, [PLN/GJ],

A good estimation of the capacity of the fumes from the combusted coal, v_{sp} [m^3/Mg], is provided by the empirical formula developed in Juda and Chróściel (1974):

$$v_{sp} = 1375 + 227w_{op} + (\lambda - 1)(500 + 242w_{op}) \tag{8}$$

where

λ—excess air, usually $\lambda = 1{,}3 \div 2{,}0$, depending on technology (type of boiler).

The first component on the right hand side of formula (7) describes the discounted cash flow of the avoided cost of emission allowances to the required level and some income from selling of allowances, earned owing to low emission level due to burning of low-sulphur coal. The second component of cash flows represents the price difference between low and high-sulphur coal. The last component means a possible increase of operating costs in a power plant as a result of coal switching. A power plant has also to take into consideration some modernization costs, K_m (generally, modernization of burning devices, if necessary).

Otherwise, if a power plant decides to install scrubbers in year T, then the expected present value V^s of the flows of cost in the period from T to N_e is:

$$V^s = \int_T^{N_e} v_{sp} B_t \eta e_m 10^{-9} c_{pT} e^{-\delta_p (t-T)}\, dt - \int_T^{N_e} c_{eT} A_t e^{-\delta_e (t-T)}\, dt \tag{9}$$

where

η—efficiency of scrubbers,

A_t—production of electricity, $A_t P_i c_{ft} T_y 10^3$ [kWh].

The first component on the right hand side of the formula (9) describes discounted cash flow of an avoided cost of the emission allowances and some income from selling of allowances. The second component of cash flows represents increase of operating costs in a power plant as a result of installation of scrubbers. In this case a power plant has to invest in scrubbers and the modernization costs is denoted as K_i.

The net present value of a project, NPV, is equal to the present value diminished by the discounted cost of modernization or investment, K_m or K_i, (discounted to the base T-year, the first year of the beginning of operation after the modernization or investment). So:

$$NPV^w = V^w(c_{pt}, c_{wt}) - K_m \tag{10}$$

and

$$NPV^s = V^s(c_{pt}, c_{et}) - K_i. \tag{11}$$

It is possible to calculate a critical price value of emission allowances, c_p^*, for each investment strategy using the condition that the net present value NPV is

4. Application of the McDonald and Siegel model

The presented decision-making model is one of the most basic continuous-time models of the irreversible investment (Dixit and Pindyck, 1994, pp. 135–142). In the model developed by McDonald and Siegel (1986), a firm should decide when to invest in a project. The investment cost, I, is known but the project value, V, follows a geometric Brownian motion. A goal of the McDonald and Siegel's model is to evaluate a critical value of the project, V^*, which implies that immediate investment is recommended. The value V evolves according to the following equation:

$$dV = \alpha V dt + \sigma V dz. \tag{12}$$

$F(V)$ denotes the value of an investment option. This is a maximum value of the expected net present value of the investment and the maximization is subject to equation (12) for V:

$$F(V) = \max \varepsilon\{(V_T - I)e^{-rT}\} \tag{13}$$

where
ε—the expectation,
T—the future time that the investment is made,
r—the discount rate.
To receive a meaningful solution it is necessary to assume that $\alpha < r$, so:

$$\delta = r - \alpha > 0. \tag{14}$$

If we consider uncertainty, we assume $\sigma > 0$. We have to determine the critical value of the project V^*, optimal for immediate investment. One of the possible methods of finding a solution is dynamic programming. From the Bellman's equation we obtain:

$$\rho F dt = \varepsilon(dF) \tag{15}$$

(this form of the Bellman's equation is valid in the continuation region (values of V for which it is not optimal to invest), see Dixit and Pindyck, 1994, Chapter 4). It means that in a time interval dt the total returned cost of the investment described by the left hand side of the equation (15) is equal to an expected rate of capital appreciation. From the Ito's Lemma:

$$dF = F'(V)dV + \tfrac{1}{2}F''(V)(dV)^2 \tag{16}$$

where
$F' = \dfrac{dF}{dV}$, $F'' = \dfrac{d^2F}{dV^2}$.
Upon inserting equation (12) into (16) and assuming $\varepsilon(dz) = 0$ we obtain:

The Bellman's equation (15) can be written in the form of:

$$\tfrac{1}{2}\sigma^2 V^2 F''(V) + \alpha V F'(V) - rF = 0 \tag{18}$$

or

$$\tfrac{1}{2}\sigma^2 V^2 F''(V) + (r-\delta) V F'(V) - rF = 0. \tag{19}$$

Additionally, $F(V)$ must fulfil the boundary conditions:

$$F(0) = 0 \tag{20}$$

$$F(V^*) = V^* - I \tag{21}$$

$$F'(V^*) = 1. \tag{22}$$

Condition (20) arises from the observation that if $V = 0$, it will stay at zero and the option to invest will be of no value. Formula (21) is the value-matching condition and condition (22) is the "smooth-pasting" condition. For more information and useful interpretation of conditions (20)–(22) see Dixit and Pindyck (1994), pp. 140–144.

To satisfy equation (20) the solution is provided as:

$$F(V) = AV^{\beta_1} \tag{23}$$

A constant A and a critical value V^* are calculated by substituting equation (23) into (21) and (22). We then obtain:

$$V^* = \frac{\beta_1}{\beta_1 - 1} I \tag{24}$$

and

$$A = \frac{(V^* - I)}{(V^*)^{\beta_1}} = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1} I^{\beta_1 - 1}}. \tag{25}$$

By substituting (23) into (19) we get the expression:

$$\tfrac{1}{2}\sigma^2 \beta(\beta - 1) + (r - \delta)\beta - r = 0. \tag{26}$$

The two roots of the equation (26) are:

$$\beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} \tag{27}$$

$$\beta_2 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} - \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}}. \tag{28}$$

The values of the roots comply with the inequalities $\beta_1 > 1$ and $\beta_2 < 0$. The general solution to the equation (19) is:

$$F(V) = A_1 V^{\beta_1} + A_2 V^{\beta^2} \tag{29}$$

where

A_1, A_2—constants.

The boundary conditions (20)–(22) imply $A_2 = 0$ leaving the solution in the

5. Analysis of influence of environmental modernization in a power plant on critical price of SO_2 emission allowances—an example

The analysis concentrates on a power unit that emits more SO_2 than the permitted level. To make this example more concrete consider a power unit with the capacity $P_i = 200$ MW that emits $e_m = 3300\ \mathrm{mg/m^3}$, while the permitted level of SO_2 emission is $e_{dop} = 2350\ \mathrm{mg/m^3}$. The owner of the power unit needs to buy the emission allowances or has to reduce SO_2 emission. In the present example we restrict the possible strategies to the installation of scrubbers. The model presented in Section 3 takes into account two different ways of reduction in emissions: fuel shifting and investing in scrubbers. One could easily calculate the critical value of the project of fuel shifting using a method similar to the one presented in the example. The operating period of the scrubbers is $N_e = 15$ years. The capacity factor is equal to about 0.5 (it means that the duration of using installed capacity is $T_i = c_{ft}T_y = 4500h/a$). The coefficient of energy in coal burnt to produce electricity, q, is equal to 10800 kJ/kWh and the heat rate of the coal, w_{op}, is 20000 kJ/kg. For the scrubbers, the efficiency is $\eta = 0.9$ and the operating cost is $c_e = 0.008$ PLN per kilowatt-hour. The investment cost of scrubbers, c_i, is equal to 0.2 million PLN per megawatt for 200 megawatts of capacity P_i. The total investment, K_i, is 40 million PLN.

We assume that (i) the scrubbers can be installed instantaneously, (ii) the coefficients of the price drift are $\alpha_p = \alpha_e = 0.07$, (iii) the discount rate r is 0.12, (iv) the coefficients are $\delta_p = \delta_e = 0.05$, (v) the standard deviations are $\sigma_p = 0.15$ and $\sigma_e = 0.10$, and (vi) the correlation coefficient is $\rho_e = 0.8$.

The price of SO_2 emission allowances and the operating cost of scrubbers are described by the equations:

$$\begin{aligned} dc_p &= 0.07c_p dt + 0.15c_p dz \\ dc_e &= 0.07c_e dt + 0.10c_e dz. \end{aligned} \tag{30}$$

From formula (9) we can calculate the expectation of cash flows in PLN million:

$$\varepsilon\{V^s\} \approx 0.01c_{pt} - 900c_{et} = (0.01c_{p0} - 900c_{e0})e^{0.07t}. \tag{31}$$

Using the expression describing the standard deviation of a random variable which is a sum of two variables:

$$\sigma_V = \sigma(V^s) = \sigma(w_p c_p + w_e c_e) = \sqrt{w_p^2\sigma_p^2 + w_e^2\sigma_e^2 + 2w_p w_e \rho_e \sigma_p \sigma_e} \tag{32}$$

it is possible to estimate the standard deviation of a random variable V^s. The estimated value of σ_V is equal to 0.20. The value of the standard deviation is the uncertainty measure of the project. One of the possible ways to decrease uncertainty is to wait with the decision to invest. That possibility is taken

A critical value of the project, which indicates that it is optimal to invest immediately, is determined by formula (24). By substituting in (24) the value $\beta_1 = 1.5$ calculated from (27) we determine a critical value of the project (in PLN million):

$$V^* = \frac{1.5}{1.5 - 1} K_i \approx 3.0 c_i P_i = 3.0 \cdot 40 = 120. \tag{33}$$

The expectation of the net present value of cash flows is specified by formula (23). After substituting sample values for variables we calculate:

$$F(V^*) = 0.0609 V^{1.5} = 0.0609 \cdot 120^{1.5} \approx 80. \tag{34}$$

We obtain the same result from equation (21):

$$F(V^*) = V^* - K_i = 120 - 40 = 80. \tag{35}$$

Using equation (3) we are able to calculate the critical value of the cash flow in the year in which the investment is made:

$$V^* = \int_0^{15} V_0^* e^{-0.05t}\, dt = 20(1 - e^{-0.05 \cdot 15} V_0^* \approx 10.55 V_0^*. \tag{36}$$

Then, the critical value in the first year is $V_0^* \approx 11.4$ million PLN. If expected revenues in the first year are greater than 11.4 million PLN the investment should be realized at once. To secure this value of revenues the critical price of emission allowances should fulfil the following equation (additionally, we assume that the operating cost is $c_{e0} = 0.008$ PLN/kWh):

$$0.01 c_{p0}^* - 7.2 = V_0^*. \tag{37}$$

Then, $c_{p0}^* \approx 1850$ PLN/Mg.

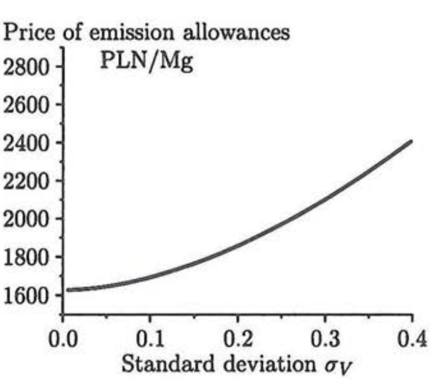

Figure 1. Critical price of emission allowances (example: power unit of 200 MW,

The increase of price uncertainty implies the increase of revenue uncertainty and, as a result, implies also the increase of the critical value (see Fig. 1). For example, assuming that the value of the standard deviation is $\sigma_V = 0.30$, it is possible to calculate the value of the coefficient $\beta_1 = 1.37$, the critical value of the project $V^* = 148.1$ million PLN, the critical value of the investment in the first year $V_0^* \approx 14$ million PLN, and the critical price of emission allowances $c_{p0}^* \approx 2107$ PLN/Mg.

6. Conclusions

This investment decision-making model under uncertainty is appropriate to model investments that are irreversible and an investor can afford to wait rather than invest immediately. Managers are able to postpone the investment in order to gather more information about trends in the capital and energy market.

Detailed analysis considers the emission reduction of sulphur dioxide and the different ways to comply with the regulations, e.g. the option to switch fuels from high-sulphur to low-sulphur coal and the option to install scrubbers. The possibility of purchasing tradable emission allowances from other power plants is tested as a new economic tool that enables to mitigate environmental problems in Poland.

The model assumes price uncertainty. The uncertainty of the future cash flows is a very important element of the decision-making analysis of the investment. Increase of the uncertainty of future income measured by the rising value of σ, implies some increase of the critical value of the project V^*. The investment rule takes the form of a critical value, being the threshold of optimality for immediate investment.

The results demonstrate that it is possible to calculate a critical price of emission allowances c_p^*. It is necessary to take into consideration the fact that the emission allowances are a market commodity and their price will oscillate. The price should be such that both selling and buying firms will profit. Otherwise, a potential buyer chooses to invest in modernization rather then in purchase of allowances. So, the equilibrium price of allowances will depend on critical price values of various technologies of emission reduction and the possibilities of applications of those technologies.

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