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Book review:

Matrix Diagonal Stability in Systems and Computation

by

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Introduction

Defining a Lyapunov function for a given dynamical system is a crucial step in its analysis since it decides of the success or failure in predicting the system behavior. Although the choice of Lyapunov function for a given stable dynamical system is, generally, not unique, computing even one element of the uncountable set of feasible functions can pose difficulties. Therefore, for many classes of systems, the problem of fitting a Lyapunov function to system dynamics is still an active research area.

The range, where Lyapunov function belongs, is a subset of some function space. It is, however, rather impractical to develop algorithms to search the infinite dimensional function space to obtain the suitable Lyapunov function. Most design methodologies applicable to practical problems are parametric ones, based on confining the range, within which a Lyapunov function is sought, to a finite dimensional space of parameters. An important Lyapunov function candidate is the quadratic form whose fit to system dynamics reduces to the choice of elements of the matrix of parameters. By using a quadratic form one can state both the sufficient and necessary conditions of stability for any linear system. It is also applicable to many nonlinear systems leading to sufficient stability conditions. Many techniques of stability analysis use Lyapunov function defined as a sum of a quadratic form and additional terms, often defined as integrals of system nonlinearities along the system trajectories. Such approaches offer a better fit to system dynamics and less restrictive stability conditions. Other techniques of Lyapunov function construction based on quadratic forms are also available. Recently, a methodology of using quadratic Lyapunov functions with additional constraints on their structure was developed called LMI's (Linear Matrix Inequalities) approach.

For multidimensional systems, the parametric fit of the Lyapunov function candidate to the demands of system stability is realized by applying a variety of multivariable algebraic equations or conditions, like solving vector algebraic Lyapunov or Riccati equations, estimating ranges of singular values of appropriate matrices or solving LMI's.

Topic of the book and its motivation

The book concerns application, to stability problems, of a certain class of functions, called diagonal-type Lyapunov functions. By diagonal-type functions (cf. definitions on page 91 of the book) the authors mean separable functions of state coordinates. The simplest example of a diagonal-type function is the quadratic form with diagonal matrix of parameters, in other words, a weighted sum of squares (x_i^2) of components x_i of state vector x with no cross products $x_i x_j$, $i \neq j$. The authors give the overview of stability conditions obtained with the use of diagonal-type Lyapunov functions. A motivation to studying such, quite a narrow, class of functions is at least twofold. First motivation is related to numerical aspects of solving algebraic problems associated with the parametric fit of the Lyapunov function. With increasing size of vector of system variables and, more generally, increasing system complexity, algebraic conditions of stability may become difficult to solve. It is often reasonable to confine the range of search to a smaller space and thereby to reduce the number of parameters to be computed and increase the size of the problem tractable by the method. The second motivation stems from the robustness issues. Systems of significant complexity have many parameters to be determined, and often at least a part of the parameters are uncertain, or can change in time in a rather unpredictable way. It is also possible that system structure is uncertain, or time variable. Conditions on system parameters obtained by defining a diagonal-type Lyapunov functions and imposing limits of some type on systems uncertainties or faults, guarantee system's robust stability against variable parameters and structure.

One more reason for paying interest to diagonal stability is, as the authors point out, that it happens surprisingly often that diagonal-type Lyapunov function is the best one applicable to the problem. Several examples are shown to illustrate this observation, like the problem of robust stability of a mechanical system (page 5 of the book).

The above arguments are supported by an extensive literature devoted to the diagonal-type Lyapunov functions and a wide range of areas of their possible applications. The authors reference over 400 literature items, mostly papers published during the last decade. Possible applications of the diagonal-type Lyapunov functions, highlighted in the book, include stability of models of neural networks, nonlinear electric circuits, models of interacting pupulations and trophic chains, models of multiprocessor systems of asynchronous computations and models of interconnected and variable structure systems.

Organization and contents of the book

Chapter 1 is an introduction, which gives the reader motivation to the study and overviews the material presented in the book.

The rest of the book can be divided into two parts. The first part includes Chapters 2 and 3. This part has theoretical character and contains mathematical results on some algebraic characteristics of matrices and some general properties and types of models of dynamical systems. The second part includes Chapters 4–6, which contain examples of applications of results collected in Chapters 2–3.

Chapter 2 contains algebraic results concerning classification of square matrices into several types, defined by special structure of matrix parameters, and showing properties of matrices, which belong to these types. This chapter serves as a basis of notions and definitions used in the subsequent parts of the book. The introduced types of matrices are related to the application of the diagonal-type Lyapunov functions to systems stability. In many cases the class of matrices defines the interconnection structure of the associated dynamical system. The book most often refers to the class of diagonally stable matrices. These are system matrices of linear continuous-time systems, which have the additional property of existence of a diagonal quadratic form that establishes system stability. The authors give various algebraic conditions for diagonal stability of quadratic matrices. Several other classes of matrices are also defined, related to aspects of diagonal stability on dynamical systems, and relations between introduced classes of matrices are stated in the form of theorems and lemmas. The case of discrete-time systems is also covered with appropriate definitions of matrix classes. The discrete counterpart of diagonal stability is named Schur diagonal stability. In this chapter an important issue of persistence of diagonal stability under perturbations (Frobenius and bounded entrywise) is also discussed. Finally, numerical tests for diagonal stability of matrices are developed. The most general approach uses LMI formulation to verify diagonal stability (diagonal Schur stability) of a matrix and to find a diagonal solution of the associated Lyapunov (Schur) matrix algebraic equation.

Chapter 3 covers mathematical models of dynamical systems, admitting diagonal-type Lyapunov functions. This chapter contains basic stability theorems developed in the book. The mathematical models introduced contain functions assumed to belong to special classes. So, the chapter begins with defining classes of nonlinear functions: sector nonlinearities and positive infinite sector nonlinearities. In each of the classes, additionally, two types of nonlinearities are distinguished: time invariant and time dependent. Basing on the introduced classes of functions, a definition of a Persidskiĭ-type system is given (page 92). This, continuous-time system contains time invariant, positive infinite sector nonlinearities, and is additionally described by a matrix of parameters that define how nonlinear functions are combined in system model. The authors state fundamental stability theorem (Theorem 3.2.3) which reads as follows: for global, asymptotic stability of time-invariant Persidskiĭ-type system it is sufficient that nonlinearities belong to positive infinite sector and the system matrix is diagonally stable. This theorem is further shown in the book to have numerous and important applications. The time varying version of Persidskiĭ-type system is then introduced and its stability conditions are stated. The authors also discuss the relation of the class of Persidskiĭ-type systems with the class of Lur'e systems, better known in system theory. Further, several classes of discrete-time systems which can be understood as discrete counterparts of the Persidskiĭ-type system are defined, and the appropriate stability theorems are proven. As a separate section a class of discrete-time interval systems is introduced and results on their Schur diagonal stability are given. The last part of Chapter 3 is devoted to discrete-time models for asynchronous systems. The authors give them a separate treatment because such models of systems are not covered by the previously defined classes of systems. The important property of models of asynchronous systems is that they include multiple and time-variable delays in state. The only assumption is that all delays are bounded by some predefined number d. Results on well posedness and stability of discrete systems with multiple, bounded delays are given. These results are based on diagonal-type Lyapunov functions.

Chapter 4 is the first of three chapters devoted to discussion of practical applications of diagonal stability concepts. In this chapter problems of convergence of numerical iterations are studied. At the beginning, models and problems are presented, related to computer implementation of numerical iterative algorithms. With the recent development of information technology, implementations of numerical algorithms often become parallel or distributed, i.e., computational load is shared by several processing units, while coordination is maintained by information exchange via communication links. Iterative procedures that give solutions to large systems of algebraic, nonlinear or linear, equations are presented. Their block-partitioned versions are good examples of tasks that can be performed in a distributed computer system. First, the authors introduce the model of synchronous iterations and then a more general model of asynchronous iterations. Then, they discuss conditions for diagonal stability of the introduced models which guarantee convergence of iterations. A special case of asynchronous iteration to solve almost linear equations is treated separately. The next part of Chapter 4 covers another interesting class of asynchronous computations, namely parallel asynchronous team algorithms. An example leading to parallel asynchronous team algorithm is the situation where the problem of solving the system of nonlinear algebraic equations can be separated into a number of different subproblems. Each of the subproblems may need a different algorithm. Then, combining of the results of computations is necessary, leading to a team algorithm. A discussion of the conditions sufficient for convergence of this class of algorithms concludes Chapter 4.

Chapter 5 includes results on diagonal stability of models from several fields: neural networks, circuits, digital filters, 2D systems and population dynamics. A common feature is that they are all analyzed with the use of techniques from Chapters 2 and 3. At the beginning, mathematical models of continuous and discrete Hopfield–Tank neural networks are presented, and it is shown that they are variants of Persidskiĭ-type systems. Basing on results from Chapter 3, theorems concerning global stability of their equilibria are stated. The next class of models presented includes RLC circuits. The ladder-type circuits are presented and the diagonal (sign) stability of system matrices of their models is studied. The relationship between diagonal stability and passivity of circuits is discussed. After concluding that the two properties are not equivalent, the authors make an observation that the strong robust stability of the ladder-type circuits is not only a consequence of their passivity, but is also related to diagonal stability of the appropriate system matrices. Further, the authors develop state-space models of digital filters. These models contain sector nonlinearities resulting from overflow in digital operations. These models are also special cases of discrete-time Persidskiĭ-type systems, and variants of diagonal stability theorems are used to establish conditions for their asymptotic stability. The next class under consideration are the discrete-time 2D dynamical systems. The 2D (two-dimensional) systems are governed by discrete equations with two integer indices. The authors introduce Roesser and Fornasini–Marchesini models for 2D dynamical systems. Then they discuss conditions of their diagonal stability. Interestingly, conditions for diagonal stability of 2D systems concern Shur-diagonal stability of appropriate system matrices. The model of discrete 2D systems with sector nonlinearities and shifts in system indices is also introduced and its diagonal stability conditions are developed. The authors highlight, as well, relations between shifted 2D systems and models of asynchronous computations. The last topic presented in Chapter 5 concerns the field of population dynamics models. Variants of Lotka–Volterra models are used to describe trophic chains of predator-prey populations. Models are divided into two groups, with open and closed chains. Linearization technique together with diagonal quadratic forms as Lyapunov function candidates are used to predict local, asymptotic stability of equilibrium points.

Chapter 6 is devoted to interconnected and parameter-uncertain systems. The material is organized separately, and not included as a section of Chapter 5, because results concern not only stability, but also stabilization issues. The chapter begins with discussion of a diagonal stability approach to large-scale, interconnected systems. Some notions from the theory of vector Lyapunov functions are introduced and their relation to diagonal-type Lyapunov functions is discussed. The quasimonotone property of $R^n \to R^n$ functions is defined and the comparison principle that can be applied to analyze stability of an interconnected system is stated. It is pointed out that models of large-scale interconnected systems are special cases of the Persidskiĭ-type system from Chapter 3 and previously derived stability theorems apply. It is shown that the system interconnection structure can be exploited to obtain nonconservative stability results. In particular, a diagonally stable structure of a large-scale system can lead to useful stability conditions. Next, the problem of robust stabilization of interval and parameter-dependent systems by linear state feedback is addressed. This area of application of diagonal stability concepts is of particular interest since it is known that robust stabilization problems can lead to difficult numerical problems. The technique applied is as follows: conditions are formulated for the system matrix of the closed loop plant to belong to the class of diagonally stable matrices. This, on the one hand, guarantees robust stability, and on the other hand makes it easier to solve stability conditions in terms of entries of the gain matrix. Basing on this idea results are developed concerning robust stabilization of interval and interconnected systems. The obtained stability conditions are compared with several references from the literature. Finally, an application of diagonal-type, robust stabilizability techniques to power-frequency control systems is studied.

Conclusions

This book is a valuable monograph covering the topic of applying diagonal stability concepts to models of systems in a wide range of fields. It identifies classes of system admitting diagonal-type stability and puts together related results on algebraic properties of system matrices. Although mostly known in the literature, these results are scattered in many references so the reader benefits from having them organized as a logical and interrelated sequence of facts. The presented range of applications of diagonal-type Lyapunov functions is impressive. It is worth stressing that although in order to go through details of a specific application the reader needs consulting further references, still, the presentation given in the book is sufficient to understand basic properties of the analyzed specific model. The book is written clearly and comprehensively with a lot of attention paid to completeness of the presented material and references to literature. There are "notes and references" sections in every chapter, supplying additional information regarding both theoretical basics and history of many notions and problems.

The book provides a unified perspective on results regarding diagonal stability by classifying models into categories. It also shows ways to obtain new results by exploiting diagonal stability concepts. Several issues can be further studied, like applying the diagonal-type Lyapunov functions to stability of the sliding-mode control systems. Such a research can start from the results on variable-structure systems developed in the book.

The audience to whom the topic of the book may be of interest is wide since the book is rather interdisciplinary. The potential audience contains researchers in many fields: control, stability, nonlinear systems, convergence of numerical algorithms, neural networks, population dynamics, etc. The book will be frequently used as a support in research and a reference in many studies.

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