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Book review:

MATRIX DIAGONAL STABILITY IN SYSTEMS AND COMPUTATION

by

Eugenius Kaszkurewicz and Amit Bhaya

Introduction

Defining a Lyapunov function for a given dynamical system is a crucial step in its analysis since it decides of the success or failure in predicting the system behavior. Although the choice of Lyapunov function for a given stable dynamical system is, generally, not unique, computing even one element of the uncountable set of feasible functions can pose difficulties. Therefore, for many classes of systems, the problem of fitting a Lyapunov function to system dynamics is still an active research area.

The range, where Lyapunov function belongs, is a subset of some function space. It is, however, rather impractical to develop algorithms to search the infinite dimensional function space to obtain the suitable Lyapunov function. Most design methodologies applicable to practical problems are parametric ones, based on confining the range, within which a Lyapunov function is sought, to a finite dimensional space of parameters. An important Lyapunov function candidate is the quadratic form whose fit to system dynamics reduces to the choice of elements of the matrix of parameters. By using a quadratic form one can state both the sufficient and necessary conditions of stability for any linear system. It is also applicable to many nonlinear systems leading to sufficient stability conditions. Many techniques of stability analysis use Lyapunov function defined as a sum of a quadratic form and additional terms, often defined as integrals of system nonlinearities along the system trajectories. Such approaches offer a better fit to system dynamics and less restrictive stability conditions. Other techniques of Lyapunov function construction based on quadratic forms are also available. Recently, a methodology of using quadratic Lyapunov functions with additional constraints on their structure was developed called LMI's (Linear Matrix Inequalities) approach.

For multidimensional systems, the parametric fit of the Lyapunov function candidate to the demands of system stability is realized by applying a variety of multivariable algebraic equations or conditions, like solving vector algebraic Lyapunov or Riccati equations, estimating ranges of singular values of appro-

A.M. Kovshov, *Geodesic Parallel Pursuit Strategy in a Simple Motion Pursuit Game on the Sphere*, where a geodesic parallel strategy of pursuit is considered in a pursuit-evasion game in which both pursuer and evader move on a surface of a unit sphere and can constantly change their directions. Main properties of such strategies, including an estimate of the capture time, are described.

R. Lachner, M.H. Breitner, H.J. Pesch, *Real Time Collision Avoidance: Differential Game, Numerical Solutions, and Synthesis of Strategies*, devoted to a problem how a correct driver (evader) avoids a collision with a wrong driver (pursuer) driving the wrong side of the road. An optimal collision avoidance strategy is derived using worst-case approaches based on pursuit-evasion differential games.

W.S. Lim, *Rendezvous-Evasion as a Multistage Game with Observed Actions*, where a zero-sum game in which two agents of a team R can optimally rendezvous while evading an enemy searcher S. The optimality of a special class of strategies for R is established and the value of the game is found.

A.A. Melikyan, J. Shinar, *Identification and Construction of Singular Surfaces in Pursuit-Evasion Games*, considering a method of singular characteristic that can be used for a construction of nonclassical solution of Hamilton–Jacobi–Bellman–Isaacs equations arising in pursuit-evasion games.

T. Raivio, H. Ethamo, *On the Numerical Solution of a Class of Pursuit-Evasion Games*, where a solution of a pursuit-evasion game is decomposed into a sequence of iteratively solved optimal control problems. The convergence of these iterations is illustrated by a number of examples.

The title of the third part suggests that virtually anything may appear in it, since coupling between players is a fundamental feature of game theory. Nevertheless, the papers contained in the third part present a number of new results, in which the coupling appears not only indirectly in the payoff function as in the classical models, but there is also coupling in state constraints. Such games, more realistic than the classical ones, can find new applications. These papers are:

D.A. Carlson, A.B. Haurie, *Inifinite Horizon Dynamic Games with Coupled State Constraints*, in which the state variables of individual players are coupled by convex constraints that are known a priori (as in environmental management). One of the results is the existence of an averaging Nash equilibrium in which the coupled state contraint is satisfied asymptotically.

E. Altman, A. Shwartz, *Constrained Markov Games: Nash Equilibria*, on noncooperative Markov games in which the players' cost must satisfy additional constraints specified a priori, with potential applications in telecommunication and flow control. Sufficient conditions for the existence of stationary equilibria are presented and a link with coupled linear programs is established.

E. Altman, T. Başar, Z. Pan, *Piecewise-Deterministic Differential Games and Dynamic Teams with Hybrid Controls*, considering game with coupled continuous and discrete dynamics: player 1 controls the continuous deterministic dynamics depending also on a state of a Markov process with transition probabilities conthe usual assumptions (zero-sum, linear-quadratic), existence and uniqueness of a saddle point is proven. The paper includes also numerical algorithms.

J. Nakagani, M. Kurano, M. Yasuda, *A Game Variant of the Stopping Problem on Jump Processes with a Monotone Rule*, describing a game with a given stochastic process influencing players' fortune. At every time instant players vote whether to stop the process. Contidions for the existence of Nash equilibria are derived and their characterization is given by integral equations.

The fourth part contains papers whose starting point were extensions and refinements of well-established game theoretic concepts and models. These extension and refinements led to applications. These papers are:

L.A. Petrosjan, *Refinement of the Nash Solution for Games with Perfect Information*, in which preference vectors are introduced for players and it is shown that for these vectors the Nash equilibrium is unique (in the sense of payoffs).

Y. Kimura, Y. Sawasaki, K. Tanaka, *A Perturbation on Two-Person Zero-Sum Games*, showing that the existence of max-inf (min-sup) implies the existence of the saddle point value in a zero-sum game perturbed by constrained functions, as well as the existence of weak optimal solutions for the players.

G.S.R. Murthy, T. Parthasarathy, B. Sriparna, *The linear Complementarity Problem in Static and Dynamic Games*, devoted to interconnections between the classical linear complementarity problem and matrix games and applications of the linear complementarity problem to a certain class of stochastic games.

E. Altman, E.A. Feinberg, A. Shwartz, *Weighted Discounted Stochastic Games with Perfect Information* on stochastic games with payoff being a linear combination of expected total rewards discounted on the infinite time horizon with different discount factors. Properties of optimal policies are established. The existence of an optimal pure Markov strategy that becomes stationary from some time on is also proven.

H.-U. Küenle, *Stochastic Games with Complete Information and Average Cost Criteria*, on two-person nonzero-sum stochastic games with complete information and average cost criterion. The existence of a quasi-stationary deterministic 2ε-equilibrium pair under the assumption that in two related zero-sum games stationary ε-equilibrium pairs exist and a certain ergodicity assumption is satisfied, is shown.

The fifth part is devoted to game theoretic models arising from specific applications. These papers are:

H. David, G. Feichtinger, S. Jørgensen, *Crime and Law Enforcement: A Multistage Game*, modelling the conflict between a potential criminal offender and a law enforcement agency as a two-stage game with imperfect information. Relevant sequential equilibria are identified and their dependence on parameters of the model is analysed.

G.I. Bischi, A. Naimzada, *Global Analysis of a Dynamic Duopoly Game with Bounded Rationality*, studying a discrete time dynamical system representing more complex attractors appear as a certain parameter of the model changes its values.

L.A. Petrosjan, G. Zaccour, *A Multistage Supergame of Downstream Pollution*, modelling the management of a downstream pollution dispute between two neighbouring regions. The set of Nash equilibria is constructed and a time-consistent procedure to share the cooperative outcome between two players is devised.

A. de Palma, *Solution and Stability for a Simple Dynamic Bottleneck Model* —a survey paper on transportation models. Stability of the solutions using adjustment processess is also discussed.

M.K. Sain, C.-H. Won, B.F. Spencer Jr., S.R. Liberty, *Cumulants and Risk-Sensitive Control: A Cost Mean and Variance Theory with Application to Seismic Protection of Structures*, devoted to problem of minimization of variance while meeting a certain mean cost level. Sufficient conditions in the form of Hamilton–Jacobi–Bellman equations are derived. Applications of results to the control of representative civil engineering structures under seismic disturbances is demonstrated.

This book is adressed at readers working in game theory or optimization theory as well as their applications. Nowadays, game theory spans such disciplines as mathematics, economics, electrical and electronic engineering, operations research, computer science, theoretical ecology, environmental science, political science, etc. Therefore there exists a large group of potential readers.

The readers are required to be familiar with mathematical apparatus used in dynamic games, but do not need to be mathematicians.

Perhaps adding a short introduction to each part with description of the problem considered and the basic terminology would make this volume easier to read also for researchers who may not be game-theorists but who look for new interesting problems.

The monograph series, *Annals of the International Society of Dynamic Games*, can be regarded as a journal of international character devoted to dynamic games and their applications. Since there is no such a journal, a collection of papers like *Advances in Dynamic Games and Applications* is really needed. Readers who want to be familiar with the latest trends in the field can find them in *Annals of ISDG*. This volume, although most of the papers it contains were presented at the 7th International Symposium on Dynamic Games and Application, has also such a character. It consists of new results in dynamic games and their applications, spanning quite a wide range.

Agnieszka Wiszniewska-Matyszkiel

J.A. Filar, V. Gaitsgory, K. Mizukami: *Advances in Dynamic Games and Applications*. Birkhäuser Verlag, Basel–Berlin–Boston, 480 pages, 2000, ISBN 0-8176-4002-0. Price: DEM 218,– (hardcover)