Control and Cybernetics

vol. **30** (2001) No. 1

Book review:

QUALITATIVE THEORY OF HYBRID DYNAMICAL SYSTEMS

by

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A hybrid dynamical system (HDS) is the one that involves continuous and discrete dynamics interacting with each other. Relay, switch, or hysteresis in a simple control system cause it to be considered as hybrid. Other common examples include gear box, air-conditioning, and disk drive. A computer controlled system is hybrid if a logical discrete-event decision-making algorithm operates on the supervisory level. In software area, timed automaton is common example of the HDS since it consists of a set of integrators (clocks) coupled with finite-state automaton. Timed automata model communication protocols with timing constraints.

Properties of HDS have been studied for about fifty years, particularly in Soviet literature (e.g. Filippov, 1988; Krasnosel'skiĭ and Pokrovskiĭ, 1989; Tsypkin, 1984). However, a number of new results have been reported during the last decade motivated mainly by manufacturing systems, computer and communication networks, and traffic control (Brockett, 1993; Caines and Wei, 1998; Perkins and Kumar, 1989; Ramadge and Wonham, 1989). They deal with mathematical modeling, heuristic algorithms and stability criteria, gradually creating the HDS as an interdisciplinary field between control theory, computer science and applied mathematics. So far, however, no systematic theory of the HDS has been developed.

The book *Qualitative Theory of Hybrid Dynamical Systems* by A.S. Matveev and A.V. Savkin (St. Petersburg, Russia, and Perth, Australia, respectively) partially fills this gap. "Partially" follows from the fact that the book presents the Authors' own results, not covering the entire field. The HDS being considered in the volume is a switched flow network consisting of a number of interconnected buffers. Tavernini's (1987) differential automaton is employed as general model of such network. By beginning with two contrasting examples, three buffers either being emptied or filled by a single server, the Authors demonstrate that typical HDS can exhibit either predicable limit cycle or chaotic irregular behavior, inacceptable in practice.

Main results of the book are concerned with specification of conditions for periodic hybrid dynamical systems, i.e. such that have two basic properties:

(i) there exists a finite number of limit cycles, The periodic HDS can be viewed as a counterpart of the globally stable system in control theory. The Authors also develop a decentralized control policy that under some assumptions guarantees predictable periodic behavior of the closed-loop HDS.

The book is organized as follows. Chapter 1, whoch is a introduction to HDS, presents the two contrasting examples, specifies the goal of the book and its contents. Other examples and qualitative analysis explaining ideas underlying the HDS are included in Chapter 2. Linear differential automaton is employed as general description, with periodic automata given particular attention. General theory of differential automata is developed in Chapter 3. It is shown that under some mild assumptions the discrete-event dynamics of any differential automaton can be reduced to a finite-state automaton. Limit cycles are studied using Brouwer fixed-point theorem. An analogue of Poincaré–Bendixon theorem for two-dimensional HDS is developed in Chapter 4. This gives necessary and sufficient conditions for the system to exhibit non-chaotic behavior. Limit cycles in HDS with constant derivatives are studied in Chapter 5. Conditions for existence and stability are established and regions of attraction are given. Chapter 6 illustrates the previous results by means of models from manufacturing and queueing networks. Regions of attraction of limit cycles are computed. Chapter 7 shows that any switched single server flow network from a class considered exhibits a globally periodic behavior, i.e. there exists a unique limit cycle that attracts all trajectories of the system. Regularization by means of feedback is studied in Chapter 8. A closed-loop HDS is called regular if its trajectories are bounded and average behavior is specified by periodic finite-state timed automaton. Decentralized control policy that guarantees regularizability is proposed. It can be implemented in distributed, real-time fashion at the system servers. Some open problems are listed in Chapter 9. They include switched server systems, processor sharing model, stabilizability, chaotic switched flow networks, nonlinear automata.

To summarize, hybrid dynamical systems are one of directions along which control gradually evolves. A few years ago a question was even posed whether it would become a main direction (Habbard, 1995). In this context, the book by Matveev and Savkin, despite the fact that it is strongly based on the Authors' own results, is certainly a valuable contribution. One must keep in mind their broad background created by many years of Russian advancements in the field. The book will be of interest for researchers in control theory, theoretical computer science and applied mathematics. Those working in applications may find it somewhat too abstract. Examples presented are primarily of theoretical value.

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A.S. Matveev, A.V. Savkin: *Qualitative Theory of Hybrid Dynamical Systems.* Birkhäuser Verlag, Basel–Berlin–Boston, 360 pages, 2000. ISBN 0-8176-4141-6. Price: DEM 148.– (hardcover).