Control and Cybernetics
vol. **30** (2001) No. 3

Book review:

Optimal Control: An Introduction

by

Arturo Locatelli

The classical theory of optimal control, which reached the apogee of its development in the late 1950s and early 60s, constituted a major breakthrough in the history of the development of control engineering. It brought a definitive close to the initial period of "joyful creation", when the basic requirement was only the proper operation of the automatic control device, which was, later on, additionally expected to satisfy no more than the condition of stability with an assumed margin. The excessive number of solutions that would meet such liberal requirements made it naturally necessary to seek theoretical procedures to select the one best possible solution. Thanks primarily to the principles of Pontriagin and Bellman, the classical theory of optimal control found an answer to this problem, which is a matter of fundamental importance from both the scientific and practical points of view.

Such a control, of course, being optimal in respect to one particular criterion of quality, cannot satisfy all the practical requirements of advanced engineering. The basic significance of the classical theory of optimal control resulted, however, from the fact that it was the last general problem for which a universal mathematical apparatus could be found. Thus, many of the later achievements of sophisticated tasks in control engineering were based to some extent on the results of the classical theory, whose cases constituted points of departure for further explorations. As a consequence, this theory, though developed in the 1960s, continues to provide the basis for various monographs, which differ primarily in the way the material is presented: the mathematical apparatus here is at the limits of the level of comprehension of most engineers, while the diversity of contemporary applications renders it impossible for them to be fully followed by mathematicians. One such monograph summarizing the area of the classical theory of optimal control, useful both for scientific investigations and educational purposes, is the book *Optimal Control: An Introduction*, by Arturo Locatelli, published by Birkhäuser in 2001.

The book is divided into two parts, where the organizing criterion is not the specific nature of the problems considered, but rather the mathematical apparatus used and the resulting degree of advancement of the tasks taken up.

Part I, then, entitled "Global methods", presents the basic problems of op-

Hamilton–Jacobi Theory. In the first chapter of this part, the foundations of the latter theory are formulated. The second chapter presents the solution of the basic form of the deterministic linear-quadratic problem with finite and infinite horizon, and a series of mathematicized impressions concerning the optimal regulator designed here. The stochastic linear-quadratic problem constitutes the substance matter of the third chapter, where the discussion is broken down into the optimal estimation of a state (the Kalman filter) and the optimal control task. In the fourth and final chapter of this part, the Riccati equation is considered in its differential and algebraic forms; its solution and characteristic features are crucial for the validity of the results achieved earlier.

In the course of Part II, entitled "Variational methods", the fundamental problem is augmented by the advanced tasks in classical optimal control theory, which are resolved in the next two chapters using the Pontriagin Maximum Principle and the second variation methods. The topic of the first chapter is constituted by the various kinds of constraints added to the quadratic control problem, as well as the time-optimal control task, which, surprisingly, appears towards the end. Finally, the second and final chapter is devoted to the second variation methods applied here to sufficient conditions of optimality and perturbations of the optimal control in the event of the uncertainty of an initial state.

The most controversial feature of this clearly very valuable book is its unsuitable title, doubtless advantageous from the commercial point of view, but quite deceptive in respect to both the contents and the means of presentation. In reality this book presents the mathematical theory of classical optimal control with the quadratic performance index, and does so in a very interesting, competent, and comprehensive manner.

Locatelli takes up the mathematical side of the task, both in the contents as described above, and in the form. Accordingly, the arrangement of each of the sections is as follows: the formulation of the problem, followed by a series of definitions, theorems, conclusions, and examples, though with no practical interpretations. There is also no discussion of the evolution that has taken place in the problems of the classical theory during the last 40 years, especially in the area of sophisticated control engineering.

When viewed from the contemporary perspective, the classical optimal control consists of two fundamental problems: quadratic and time-optimal. This book is essentially reduced to the first of these, while time-optimal control appears near the end in such a way that an unwary reader might even treat it as an extended specific case of control with quadratic performance index. From the point of view of contemporary engineering, such an approach gives a false impression.

It should be noted, however, that these remarks would lose their critical force entirely if the title of the book had limited the topic of consideration to the mathematical theory of classical optimal control with quadratic performance

written in a language that is mathematically precise, and yet clear and comprehensible. There is a distinct preference for the modern time approach, but, when necessary, it is supported by the frequency methods. In providing a succinct summary of this admittedly familiar material, the book here reviewed constitutes a valuable source of knowledge, even for persons who are well acquainted with the subject matter, or are involved in the many divisions of contemporary control engineering that are based on it, especially robust control and the H^∞ approach.

This book may be of use for students of applied mathematics who are interested in optimal control (though only after passing a course on the fundamentals of control theory, due to the lack of a preliminary part defining the fundamental terms). It will certainly also be helpful for doctoral students in both applied mathematics and control engineering with theoretical interests, and especially for scientists and lecturers who wish to systematize their command of the subject at a high level of mathematical precision. In this respect, *Optimal Control: An Introduction* by Arturo Locatelli deserves the highest recommendation.

Piotr Kulczycki

A. Locatelli: *Optimal Control: An Introduction.* Birkhäuser Verlag, Basel–Berlin–Boston, 304 pages, 2001. ISBN 3-7643-6408-4. Price: CHF 68.– (hardcover).