Book review:

NONLINEAR AND ROBUST CONTROL OF PDE SYSTEMS

by

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The book deals with control of nonlinear partial differential equation (PDE) systems describing the transport-reaction industrial processes. These processes are characterized by the coupling of chemical reaction with significant convection diffusion and dispersion phenomena. The examples of appearance of these processes include, in particular, plug-flow and packed-bed reactors used to produce specialty chemicals, the Czochralski crystallization of high-purity crystals, the chemical vapor deposition of thin films for microelectronic manufacturing, or the solidification of liquid solution coatings for photographic films.

The traditional approach for controlling transport-reaction processes is based on a simplyfying assumption that the manipulated (i.e., control) and controlled variables are spatially uniform. This assumption allows for use of spatial discretization of the original PDE system. Resulting from this discretization an ordinary differential equation (ODE) system is a base for the design of a controller. This approach has certain disadvantages. The observability and controllability properties of the system may depend not only on the location of sensors and actuators but also on the discretization method, the location and the number of discretization points. Neglecting the infinite-dimensional nature of the original system may lead to erroneous conclusions concerning the stability properties of the open-loop and the closed-loop systems as well as to the worsening of the controller perfomance and to unacceptable control quality. Taking into account the above considerations the author develops the general nonlinear and robust control methods for hyperbolic and parabolic PDE systems that directly account for their spatially distributed nature. The author considers spatially one-dimensional quasi-linear first order hyperbolic and parabolic PDE systems. These systems are assumed to posses unique and sufficiently regular solutions. Moreover the manipulated inputs, the controlled and the measured outputs are assumed to be bounded. By employing geometric as well as Lyapunov control methods the nonlinear and robust controllers for hyperbolic and parabolic PDE systems in fixed spatial domain as well as for parabolic PDE systems in time dependent spatial domain are synthesized. These controllers are using a finite number of measurment sensors and control actuators to achive sta-

of the model uncertainty. The proposed controllers are applied to numerous convection-reaction and diffusion-reaction processes. The comparison of the proposed nonlinear and robust control methods with other traditional control methods as well as the discussions of practical implementation issues are provided.

The contents of the book is as follows: Chapter 2 deals with feedback control problems for spatially one-dimensional systems described by quasi-linear first-order hyperbolic PDEs. These systems describe transport-reaction processes in which the diffusive and dispersive phenomena are neglible compared to the connective phenomena. Chemical processes modelled by such systems include, among others, plug-flow reactors, fixed-bed reactors and heat exchangers. The considered quasi-linear hyperbolic PDE systems are assumed to posses a solution. The synthesis of the output feedback controller is based on the concept of the characteristic index. This index is defined as the lowest order time derivative of the output variable explicitly dependent on the manipulated variable. The notion of the characteristic index is used for synthesis of a state feedback controller that induces output tracking in the closed-loop system. The equation defining the characteristic index implies the form of the equation describing the input-output response for the closed-loop system depending on adjustable, parameters, which can be chosen to guarantee input-output stability and enforce the desired performance specifications in the closed-loop system. A notion of zero-output constraint dynamics for first order hyperbolic PDEs is introduced and used to formulate conditions that guarantee the stability of the closed-loop systems. Next, the output feedback controllers are synthesized through combination of the appriopiate distributed state observers and the developed state feedback controllers. Controller implementation issues are discussed. Application of the control method developed is illustrated through a nonisothermal plug-flow reactor example modelled by a system of three quasi-linear hyperbolic PDEs. Numerical results are provided. For the sake of comparison of the traditional approach and the proposed approach to control of the reactor, the author solved this control problem by designing an input-output linearizing controller on the basis of the discretized original PDE system. The finite-differences method was used for discretization. The output profiles obtained by using this traditional approach reveal oscillations, offset, and longer transient response. Since the controller developed by the traditional technique does not take into account the spatially varying nature of the chemical process it leads to poor performance.

Chapter 3 deals with control of spatially one-dimensional quasi-linear first-order hyperbolic PDEs with time varying uncertain variables and unmodelled dynamics. The manipulated variables and the controlled variables are assumed to be distributed in space. First, necessary and sufficient conditions for the complete elimination of the effect of uncertainty on the output in the closed-loop system, as well as the explicit controller synthesis formulas are derived.

ness of the state and achieves asymptotic output tracking with arbitrary degree of asymptotic attenuation of the effect of uncertain variables on the output of the closed-loop system. The controller is designed constructively using Lyapunov's direct method and requires the existence of the known bounding functions that capture the magnitude of the uncertain terms and the satisfaction of the matching condition. Next, the problem of the robustness of the synthesized controller with respect to the stable unmodelled dynamics is considered. This problem is studied within the context of control of two-time-scale hyperbolic PDE systems modelled in a singulary perturbed form. A robustness result of the bounded stability property of a reduced-order slow model PDE with respect to stable and fast singular perturbations is proved. This result is then used to establish that the controllers synthesized on the basis of the reduced slow-order model and achieving uncertainty decoupling or uncertainty attenuation continue to enforce these control objectives in the full-order closed-loop system, provided that the unmodelled dynamics are stable and sufficiently fast. The developed control method is tested on a nonisothermal fixed-bed reactor. In this reactor the reactant wave propagates through the bed with significantly larger speed than the heat wave and the heat of reaction is unknown and time varying.

Chapter 4 deals with the methodology for the synthesis of nonlinear output feedback controllers for spatially one-dimensional quasi-linear parabolic PDE systems. The eigenspectrum of spatial operator of these systems can be partitioned into a finite-dimensional slow component and an infinite-dimensional stable fast component. This motivates the solution of the controller synthesis problem for parabolic PDE on the basis of finite-dimensional systems accurately discribing their dynamic behavior. The considered parabolic PDE systems describe transport-reaction processes with significant diffusive and dispersion mechanisms. Singular perturbation methods are used to show that the discrepancy between the solutions of original PDE system and the solutions of approximating it ODE system of dimension equal to the number of slow modes is proportional to the degree of separation of the slow and the fast modes of the spatial operator. The approximating ODE system is obtained through the Galerkin method. Next, the method for construction of the Approximating Inertial Manifolds (AIM) for PDE systems is proposed. Inertial Manifold (IM) is defined as a graph of a Lipschitz function, mapping slow components, manipulated inputs and the perturbation parameter into fast components of the original PDE system. IM attracts every trajectory exponentially. Since the derivation of an explicit analytic form of the IM is a very difficult task, AIM is defined as an expansion, up to desired order, of the IM and the manipulated inputs in a power series with respect to the perturbation parameter. The AIMs are used to construct the ODE systems of dimensions equal to the number of slow modes, approximating the original PDE systems. It is proved that for almost all times the solutions of the original PDE are approximated by the solutions of the approximating ODEs with a desired accuracy. These ODE systems are used as a

these controllers assure the stability of the closed-loop system. Moreover, the output of the closed-loop system follows up to a desired accuracy a prespecified response for almost all times. The proposed methodology is employed to control the temperature profiles of a catalytic rod as well as of a nonisothermal tabular reactor with recycle around unstable steady states.

Chapter 5 is devoted to synthesis of nonlinear robust state and output feedback controllers for one-dimensional in space quasi-linear parabolic PDEs with time varying uncertain variables. The controllers constructed assure the boundedness of the state and output tracking with arbitrary degree of asymptotic attenuation of the effect of the uncertain variables on the output of the closed-loop system. First, using Galerkin method a system of ODEs of dimension equal to the number of slow modes approximating the original parabolic PDE system with uncertainty is introduced. This ODE system, as well as the Lyapunov method, are employed to synthesize the robust state feedback controller. It is shown that the degree of asymptotic attenuation of the effect of uncertain variables on the output enforced by these controllers is proportional to the degree of separation of the fast and slow modes of the spatial differential operator. If the degree of uncertainity attenuation is not sufficient, a sequential procedure based on AIM is proposed to synthesize the robust controllers that achieve the arbitrary degree of asymptotic uncertainity attenuation in the closed-loop parabolic system. Then, under the assumption that the number of measurements is equal to the number of slow modes a procedure for obtaining estimates for the states of the auxiliary ODE system from the measurements is proposed. The use of these state estimates in the robust state feedback controller leads to a robust output feedback controller. The constructed feedback controller has the desired stability properties in the closed-loop system, assuming that the separation between the slow and the fast eigenvalues is sufficiently large. The method developed is employed to design the robust output feedback controller for a catalytic rod with uncertainty. The calculated profiles of temperature are provided.

Chapter 6 is devoted to synthesis of nonlinear robust time-dependent controllers for quasi-linear parabolic PDE systems with one-dimensional time-dependent spatial domains. The dynamics of these PDE systems can be partitioned into the slow and fast parts. The diffussion-reaction processes of crystal growth, metal casting, gas-solid reaction, and coatings are modelled by these moving boundary problems. In these processes nonlinear behaviour arises typically from complex reaction mechanisms and their Arrehnius dependence on the temperature, while the motion of the boundaries is a result of phase change, chemical reaction or heat and mass transfer. In this chapter the considered class of parabolic PDEs is formulated as an evolution equation in an appriopiate Hilbert space. Using Galerkin method, combined with the notion of AIM, this evolution equation is approximated by the ODE system having solutions close, up to the desired accuracy, to the solutions of PDE system for almost all times. Then, via geometric control method, a nonlinear output feedback con-

a nonlinear robust time varying controller for quasi-linear parabolic PDE with time dependent spatial domain and uncertain variables is synthesized. The proposed controller assures stability, output tracking and uncertainty attenuation in the closed-loop system. The robust controllers are tested on a catalytic rod with time dependent spatial domain and uncertainty.

Chapter 7 contains the details of the implementation and the results of applications of the nonlinear control methods for parabolic PDE systems with fixed and moving spatial domains, developed in Chapters 4–6, to the rapid thermal chemical vapor deposition process as well as to the Czochralski crystal growth process. In both cases the proposed nonlinear controllers lead to the better output of the closed-loop PDE systems (i.e., uniform film depsition on the wafer, almost linear axial temperature profile in the crystal) than the linear controllers designed on the basic of the finite-dimensional approximation of the original PDE system.

Appendices A–E contain the proofs of Theorems and Lemmas from Chapters 2–6, respectively. Appendix F recalls the Karhunen–Loewe expansion method (or principal component analysis method) used to compute an optimal basis for a modal decomposition of a PDE system from an appropiately constructed set of data for this system obtained by detailed simulations. The list of references consists of 151 items and contains main positions dealing with control theory.

The book is well written and clearly presents both the fundamentals of the feedback control theory of PDE systems and the development of new techniques of design of the nonlinear and robust controllers for PDE systems. Using the results from the dynamics of PDE systems and from the nonlinear control theory the stability properties of the closed-loop systems with the designed controllers are proved. The proposed new methods of design of nonlinear controllers are applied to control of numerous transport-reaction chemical processes of industrial interest. The implementation issues of the proposed design methods and the obtained numerical results are explicitly discussed. The designed nonlinear controllers for PDE systems show much better performance than the linearized controllers designed traditionally. The book may be the source of inspiring ideas both for applied mathematicians and for process control engineers interested in theoretical and practical aspects of nonlinear control of PDE systems.

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