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Stability of nominally stable uncertain singularly perturbed systems with multiple time delays

by

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Abstract: In this paper, the robust asymptotic stability problem for a class of nominally stable uncertain singularly perturbed systems with multiple non-commensurate time delays is considered. A delay-dependent criterion is first proposed in this paper to guarantee robust asymptotic stability of the system under consideration. Based on this result, the range of allowable bounds of the perturbation matrices preserving the closed-loop stability can easily be found. Moreover, a simple criterion is also proposed to guarantee asymptotic stability of the nominal system. Furthermore, a simple estimation of the stability bound ε^* is proposed such that the nominal system is asymptotically stable for any $\varepsilon \in (0, \varepsilon^*)$. It can be seen that the stability bound proposed in this paper is less conservative than that presented in recent research. Finally, a numerical example is provided to illustrate our main results.

Keywords: singularly perturbed system, asymptotic stability, delay-dependent criterion.

1. Introduction

Most of the dynamic systems contain some uncertainties that may arise, to name a few, from modeling errors and/or linearization approximation (Phoojaruenchanachai et al., 1998). Moreover, the time-delay factors always exist in various engineering systems, such as long transmission lines, electric networks, chemical processes, pneumatic systems, or hydraulic systems. Its existence frequently causes the undesirable system responses. Therefore, the robust stability problem of time-delay systems has been a main concern of the researchers over the years (Phoojaruenchanachai et al., 1998, Lien et al., 1998).

Many physical systems contain some small parameters such as capacitances, small time constants, masses, etc. These small parameters tend to increase the order of the dynamic systems and thus complicate the system analysis. Fortunately, the singular perturbation method provides us with a powerful tool for the order reduction and separation of time scales; see, for example, Hsiao et al. (1999), Kokotovic et al. (1986), Shi et al. (1998), Xu et al. (1997), and the references therein. Shao and Sawan (1993) proposed a robust stability criterion for a class of linear time-invariant singularly perturbed systems, which have parametric uncertainties bounded by the H_∞-norm, but in which the time delay has not yet been considered and the upper bounds of the perturbation matrices depend on specific system matrices. Shao and Rowland (1995) proposed some stability criteria for a class of singularly perturbed systems with single time delay in the slow states. Moreover, the time delays in the fast states and the uncertain perturbation of the system matrices have not been considered. Pan et al. (1996) proposed a frequency-domain stability criterion for linear time-invariant singularly perturbed systems with multiple time delays in which the uncertainty was not investigated. However, the factors of uncertainties and time delays do exist in most of the dynamic systems. Consequently, it is crucial to take them into consideration. This is due not only to theoretical interest but also to the relevance of this topic for the control engineering applications.

To the authors' knowledge, the robust stability problem of nominally stable uncertain singularly perturbed systems with multiple non-commensurate time delays subject to unstructured perturbations has not yet been well explored. Consequently, it is the purpose of this paper to investigate the robust stability problem of uncertain multiple time-delay singularly perturbed systems. Moreover, the range of allowable bounds of the perturbation matrices preserving the closed-loop stability will be proposed. Furthermore, a simple estimation of the stability bound ε^* will be proposed such that the nominal system is asymptotically stable for any $\varepsilon \in (0, \varepsilon^*)$. It can be shown that the stability bound ε^* is less conservative than that of Shao and Rowland (1995).

2. Problem formulation and preliminaries

First, we define some notation that will be used throughout this paper

$\|A\| :=$ spectral norm of matrix A; $\|A\| := [\lambda_{\max}(A^*A)]^{1/2}$,

$H(s) \in S(C_+) := H(s)$ is analytic in C_+, where C_+ is the right-half s−plane,

$\|H(s)\|_\infty := H_\infty$-norm of $H(s)$; $\|H(s)\|_\infty = \sup_{w \in \Re} \|H(jw)\|$.

In this paper, we will consider the following uncertain singularly perturbed system with multiple time delays:

$$\dot{x}(t) = \sum_{i=0}^{n}(A_{1i} + \Delta A_{1i})x(t-h_i) + \sum_{i=0}^{n}(A_{2i} + \Delta A_{2i})z(t-h_i), \tag{1a}$$

$$\varepsilon\dot{z}(t) = \sum_{i=0}^{n}(A_{3i} + \Delta A_{3i})x(t-h_i) + \sum_{i=0}^{n}(A_{4i} + \Delta A_{4i})z(t-h_i), \tag{1b}$$

$$x(t) = \xi_d(t), \quad z(t) = \varphi_d(t), \quad t \in [-\tau, 0],$$

where A_{1i}, A_{2i}, A_{3i} and A_{4i}, are constant matrices with appropriate dimensions, $h_0 = 0$, h_i are non-negative numbers, τ is the maximum of h_i , and ΔA_{ji} are the perturbation matrices, $j \in \underline{4}$. The positive scalar ε is the singular perturbation parameter, which often occurs naturally due to the presence of small parameters in various physical systems.

LEMMA 1 (SHAO AND ROWLAND 1995) *If $H(s)$ is an $n \times n$ complex matrix, $H(s) \in S(C_+)$, and $\|H(s)\|_\infty \le \beta$, where β is a constant and $0 \le \beta < 1$, then $[I - H(s)]^{-1} \in S(C_+)$ with $\|[I - H(s)]^{-1}\|_\infty \le (1-\beta)^{-1}$.*

LEMMA 2 *Suppose that*

$$Q = \begin{bmatrix} d_1 & d_2 \\ d_3 & d_4 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix},$$

where P_i, $i \in \underline{4}$, are constant matrices with appropriate dimensions and $\|P_i\| \le d_i$, $d_i \ge 0$. Then we have $\|P\| \le \|Q\|$.

Proof. Let $\tilde{x} = [x_1^T \; x_2^T]^T$. By the definition of the induced norm, we have

$$\begin{aligned}
\|P\| &= \sup_{\|\tilde{x}\|=1} \left\| \begin{bmatrix} P_1x_1 + P_2x_2 \\ P_3x_1 + P_4x_2 \end{bmatrix} \right\| \\
&= \sup_{\|\tilde{x}\|=1} \{\|P_1x_1 + P_2x_2\|^2 + \|P_3x_1 + P_4x_2\|^2\}^{1/2} \\
&\le \sup_{\|\tilde{x}\|=1} \{\|P_1\|^2\|x_1\|^2 + 2\|P_1\| \cdot \|P_2\| \cdot \|x_1\| \cdot \|x_2\| + \|P_2\|^2\|x_2\|^2 \\
&\quad + \|P_3\|^2\|x_1\|^2 + 2\|P_3\| \cdot \|P_4\| \cdot \|x_1\| \cdot \|x_2\| + \|P_4\|^2\|x_2\|^2\}^{1/2} \\
&\le \sup_{\|\tilde{x}\|=1} \{d_1^2\|x_1\|^2 + 2d_1d_2 \cdot \|x_1\| \cdot \|x_2\| + d_2^2\|x_2\|^2 + d_3^2\|x_1\|^2 + 2d_3d_4 \cdot \|x_1\| \cdot \|x_2\| \\
&\quad + d_4^2\|x_2\|^2\} \\
&= \sup_{\|\tilde{x}\|=1} [(d_1\|x_1\| + d_2\|x_2\|)^2 + (d_3\|x_1\| + d_4\|x_2\|)^2]^{1/2} \\
&= \sup_{\|\tilde{x}\|=1} \left\| \begin{bmatrix} d_1 & d_2 \\ d_3 & d_4 \end{bmatrix} \cdot \begin{bmatrix} \|x_1\| \\ \|x_2\| \end{bmatrix} \right\| = \|Q\|.
\end{aligned}$$

■

3. Stability of the uncertain system

Consider the following nominal system of the uncertain singularly perturbed system (1) described as

$$\dot{x}(t) = \sum_{i=0}^{n} A_{1i}x(t-h_i) + \sum_{i=0}^{n} A_{2i}z(t-h_i), \tag{2a}$$

$$\varepsilon\dot{z}(t) = \sum_{i=0}^{n} A_{3i}x(t-h_i) + \sum_{i=0}^{n} A_{4i}z(t-h_i), \tag{2b}$$

$$x(t) = \xi_n(t), \quad z(t) = \varphi_n(t), \quad t \in [-\tau, 0].$$

Let $\phi_n(t) = [x(t)^T \; z(t)^T]^T$. Then, the nominal system (2) can be rewritten as

$$\dot{\phi}_n(t) = \sum_{i=0}^{n} A_i\phi_n(t-h_i), \tag{3a}$$

where

$$A_i = \begin{bmatrix} A_{1i} & A_{2i} \\ A_{3i}/\mathcal{E} & A_{4i}/\mathcal{E} \end{bmatrix}. \tag{3b}$$

Taking the Laplace transform of (3), we have

$$\Phi_n(s) = \Big(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\Big)^{-1}\phi_n(0).$$

Obviously, the nominal system (2) is asymptotically stable if and only if

$$\Big(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\Big)^{-1} \in S(C_+). \tag{4}$$

Now let $\phi_d(t) = [x(t)^T \; z(t)^T]^T$, then the uncertain system (1) can be rewritten as

$$\dot{\phi}_d(t) = \sum_{i=0}^{n} A_i\phi_d(t-h_i) + \sum_{i=0}^{n} \Delta A_i\phi_d(t-h_i), \tag{5a}$$

where

$$\Delta A_i = \begin{bmatrix} \Delta A_{1i} & \Delta A_{2i} \\ \Delta A_{3i}/\mathcal{E} & \Delta A_{4i}/\mathcal{E} \end{bmatrix}, \quad i \in \bar{n}. \tag{5b}$$

THEOREM 1 *Assume that the nominal system (2) is asymptotically stable for $\varepsilon \in (0, \varepsilon^*)$. Then, for a given singular perturbation parameter $\varepsilon = \varepsilon_0 \in (0, \varepsilon^*)$, the uncertain system (1) is asymptotically stable if*

$$\sum_{i=0}^{n} \|D_i\|_\infty < \Big\|\Big(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\Big)^{-1}\Big\|_\infty^{-1}, \tag{6a}$$

where

$$D_i = \begin{bmatrix} d_{1i} & d_{2i} \\ d_{3i}/\mathcal{E}_0 & d_{4i}/\mathcal{E}_0 \end{bmatrix} \tag{6b}$$

with $\|\Delta A_{ji}\|_\infty \le d_{ji}$ *and* d_{ji}, $j \in \underline{4}$, *non-negative constants.*

Proof. Taking the Laplace transform of (5a) yields

$$\Phi_d(s) = \left[I - \left(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\right)^{-1} \sum_{i=0}^{n} \Delta A_i e^{-h_i s}\right]^{-1} \left(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\right)^{-1} \phi_d(0).$$

According to (5b), (6b), Lemma 2, and the fact that $\|\Delta A_i e^{-h_i s}\|_\infty = \|\Delta A_i\|_\infty$ (Shao and Rowland 1995), we obtain

$$\left\| \left(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\right)^{-1} \sum_{i=0}^{n} \Delta A_i e^{-h_i s} \right\|_\infty \le \left\| \left(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\right)^{-1} \right\|_\infty \left(\sum_{i=0}^{n} \|D_i\|_\infty\right).$$

Due to (6a) and Lemma 1, we have

$$\left[I - \left(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\right)^{-1} \sum_{i=0}^{n} \Delta A_i e^{-h_i s}\right]^{-1} \in S(C_+).$$

Moreover, since the nominal system (2) is asymptotically stable, we obtain

$$\left(sI - \sum_{i=0}^{n} A_i e^{-h_i s}\right)^{-1} \in S(C_+)$$

in view of (4). Consequently, we have $\Phi_d(s) \in S(C_+)$, i.e., the uncertain singularly perturbed system (1) is asymptotically stable. This completes our proof. ∎

4. Stability of the nominal system

In Theorem 1, it is required that the nominal system be asymptotically stable. In this section, we propose a simple criterion to guarantee the stability of the nominal system (2). In this section, A_{40} is assumed to be Hurwitz.

The slow and fast subsystems of the nominal system (2) are first derived as follows. By setting $\varepsilon = 0$, the slow subsystem of (2) is obtained as

$$\dot{x}_s(t) = \sum_{i=0}^{n} A_{1i} x_s(t-h_i) + \sum_{i=0}^{n} A_{2i} z_s(t-h_i), \tag{7a}$$

$$0 = \sum_{i=0}^{n} A_{3i} x_s(t-h_i) + \sum_{i=0}^{n} A_{4i} z_s(t-h_i), \tag{7b}$$

where $x_s(t)$ and $z_s(t)$ are the slow components of $x(t)$ and $z(t)$, respectively. Taking the Laplace transform of (7) and letting

$$A_{jn}(s) = \sum_{i=0}^{n} A_{ji} e^{-h_i s}, \qquad j \in \underline{4}, \tag{8}$$

we obtain

$$Z_s(s) = -A_{4n}^{-1}(s) A_{3n}(s) X_s(s), \quad X_s(s) = M_s(s) x_s(0), \tag{9a}$$

where

$$M_s(s) = \{sI - [A_{1n}(s) - A_{2n}(s) A_{4n}^{-1}(s) A_{3n}(s)]\}^{-1}. \tag{9b}$$

Let $z_f(t) = z(t) - z_s(t)$ and $x(t) = x_s(t)$. Note that the slow varying state $z_s(t)$ is almost constant with respect to the fast state $z_f(t)$. Thus, we have $\dot{z}_f(t) + \dot{z}_s(t) \approx \dot{z}_f(t)$. Consequently, according to (7b), (2b) can be approximated by

$$\varepsilon\, \dot{z}_f(t) = \sum_{i=0}^{n} A_{4i} z_f(t-h_i). \tag{10}$$

Hence the dynamics of the fast subsystem (10) is independent of the slow varying states $x_s(t)$ and $z_s(t)$. Taking the Laplace transform of (10) yields

$$Z_f(s) = M_f(s,\varepsilon) \varepsilon z_f(0), \quad M_f(s,\varepsilon) = [\varepsilon s I - A_{4n}(s)]^{-1}. \tag{11}$$

LEMMA 3 *Assume*

$$\|(sI - A_{40})^{-1}\|_\infty \cdot \Big\| \sum_{i=1}^{n} A_{4i} e^{-h_i s} \Big\|_\infty = \lambda < 1. \tag{12}$$

Then we have

(a) $A_{4n}^{-1}(s) \in S(C_+)$.

(b) *The fast subsystem (10) is asymptotically stable for all* $\varepsilon > 0$.

(c) $[sI - A_{4n}(s)]^{-1} \in S(C_+)$. (13)

(d) *The slow subsystem (7) is asymptotically stable if*

$$\|\Xi(s)\|_\infty < 1, \tag{14a}$$

where

$$\Xi(s) = [sI - A_{4n}(s)]^{-1}[A_{1n}(s) - A_{2n}(s)A_{4n}^{-1}(s)A_{3n}(s) - A_{4n}(s)]. \quad (14b)$$

Proof. (a) Making use of (8), we obtain

$$A_{4n}^{-1}(s) = \Big(I + A_{40}^{-1}\sum_{i=1}^{n} A_{4i}e^{-h_i s}\Big)^{-1} \cdot A_{40}^{-1}.$$

Owing to (12), we have

$$\Big\|A_{40}^{-1}\sum_{i=1}^{n} A_{4i}e^{-h_i s}\Big\|_\infty \le \|(sI - A_{40})^{-1}\|_\infty \cdot \Big\|\sum_{i=1}^{n} A_{4i}e^{-h_i s}\Big\|_\infty < 1.$$

By virtue of Lemma 1, we have $(I + A_{40}^{-1}\sum_{i=1}^{n} A_{4i}e^{-h_i s})^{-1} \in S(C_+)$. Hence we get $A_{4n}^{-1}(s) \in S(C_+)$.

(b) Based on (11), we have

$$\begin{aligned} M_f(s,\varepsilon) &= [\varepsilon sI - A_{4n}(s)]^{-1} \\ &= \Big[I - (\varepsilon sI - A_{40})^{-1}\Big(\sum_{i=1}^{n} A_{4i}e^{-h_i s}\Big)\Big]^{-1}(\varepsilon sI - A_{40})^{-1}. \end{aligned}$$

Since A_{40} is Hurwitz and $\|(\varepsilon sI - A_{40})^{-1}\|_\infty = \|(sI - A_{40})^{-1}\|_\infty$, from Lemma 1 and (12), we obtain

$$(\varepsilon sI - A_{40})^{-1} \in S(C_+), \Big[I - (\varepsilon sI - A_{40})^{-1}\Big(\sum_{i=1}^{n} A_{4i}e^{-h_i s}\Big)\Big]^{-1} \in S(C_+),$$

with

$$\Big\|I - (\varepsilon sI - A_{40})^{-1}\sum_{i=1}^{n} A_{4i}e^{-h_i s}\Big\|_\infty \le (1-\lambda)^{-1}. \quad (15)$$

Hence we obtain $M_f(s,\varepsilon) \in S(C_+)$ for all $\varepsilon > 0$.

(c) The result follows immediately from part (b) by setting $\varepsilon = 1$.

(d) From (9b), we have

$$M_s(s) = [I - \Xi(s)]^{-1} \cdot [sI - A_{4n}(s)]^{-1}.$$

According to Lemma 1, if the inequality (14a) holds, then $[I - \Xi(s)]^{-1} \in S(C_+)$. Moreover, $[sI - A_{4n}(s)]^{-1} \in S(C_+)$ in view of (13). Consequently, we have $M_s(s) \in S(C_+)$. By the result of part (a), we have $A_{4n}^{-1}(s) \in S(C_+)$. This shows that the slow subsystem (7) is asymptotically stable in view of (7), (8), and (9). ■

LEMMA 4 (Pan et al., 1996) *Suppose that the slow subsystem (7) is asymptotically stable and (12) holds. Then the nominal system (2) is asymptotically stable if*

$$\|H_1(s,\varepsilon)\|_\infty < 1, \tag{16a}$$

where

$$H_1(s,\varepsilon) = M_s(s)A_{2n}(s)A_{4n}^{-1}(s)\{\varepsilon s[\varepsilon sI - A_{4n}(s)]^{-1}\}A_{3n}(s). \tag{16b}$$

THEOREM 2 *Suppose that the slow subsystem (7) is asymptotically stable and (12) holds. Then the nominal system (2) is asymptotically stable for any* $\varepsilon \in (0,\varepsilon^*)$, *where*

$$\varepsilon^* = \frac{1-\lambda}{\|sM_s(s)A_{2n}(s)A_{4n}^{-1}(s)\|_\infty \|(sI - A_{40})^{-1}\|_\infty \|A_{3n}(s)\|_\infty}. \tag{17}$$

Proof. By Lemma 3, we have $M_s(s)$, $A_{4n}^{-1}(s)$, $[\varepsilon sI - A_{4n}(s)]^{-1} \in S(C_+)$. Using (16b), we obtain

$$\begin{aligned} &\|H_1(s,\varepsilon)\|_\infty \\ &\le \varepsilon \|sM_s(s)A_{2n}(s)A_{4n}^{-1}(s)\|_\infty \|[\varepsilon sI - A_{4n}(s)]^{-1}\|_\infty \|A_{3n}(s)\|_\infty. \end{aligned} \tag{18}$$

As a consequence of (8), (15), and the fact that $\|(\varepsilon sI - A_{40})^{-1}\|_\infty = \|(sI - A_{40})^{-1}\|_\infty$, we get

$$\|[\varepsilon sI - A_{4n}(s)]^{-1}\|_\infty \le \frac{1}{1-\lambda}\|(sI - A_{40})^{-1}\|_\infty. \tag{19}$$

Consequently, as a result of (17), (18), and (19), if $\varepsilon < \varepsilon^*$, then we have

$$\begin{aligned} &\|H_1(s,\varepsilon)\|_\infty \\ &\le \frac{\varepsilon}{1-\lambda}\|sM_s(s)A_{2n}(s)A_{4n}^{-1}(s)\|_\infty \|(sI - A_{40})^{-1}\|_\infty \|A_{3n}(s)\|_\infty < 1. \end{aligned}$$

Hence the nominal system (2) is asymptotically stable according to Lemma 4.■

REMARK 1 *Consider the special system with a single time delay (Shao and Rowland 1995) described in our notation as*

$$\begin{aligned} \dot{x}(t) &= A_{10}x(t) + A_{11}x(t-h_1) + A_{20}z(t), \\ \varepsilon\dot{z}(t) &= A_{30}x(t) + A_{31}x(t-h_1) + A_{40}z(t). \end{aligned}$$

By Theorem 1.2 in Shao and Rowland (1995), the stability bound ε_1^* *is given by*

$$\begin{aligned} \varepsilon_1^* = [&\|(sI - A_{40})^{-1}\|_\infty \cdot (\|A_{30}\|_\infty + \|A_{31}\|_\infty) \cdot \|sM_s(s)\|_\infty \\ &\cdot \|A_{20}A_{40}^{-1}\|_\infty]^{-1}, \end{aligned}$$

where

$$M_s(s) = [sI - (A_{10} - A_{20}A_{40}^{-1}A_{30}) - (A_{11} - A_{20}A_{40}^{-1}A_{31}) \cdot e^{-h_1 s}]^{-1}.$$

Since λ in (12) is zero, it is clear that (12) is trivially satisfied. By Theorem 2 of this paper, the stability bound ε_2^* is given by

$$\varepsilon_2^* = [\|sM_s(s)A_{20}A_{40}^{-1}\|_\infty \|(sI - A_{40})^{-1}\|_\infty \|(A_{30} + A_{31}e^{-h_1 s})\|_\infty]^{-1}.$$

Since

$$\begin{aligned} &\|sM_s(s)A_{20}A_{40}^{-1}\|_\infty \|(sI - A_{40})^{-1}\|_\infty \|(A_{30} + A_{31}e^{-h_1 s})\|_\infty \\ &\le \|sM_s(s)\|_\infty \|A_{20}A_{40}^{-1}\|_\infty \|(sI - A_{40})^{-1}\|_\infty (\|A_{30}\|_\infty + \|A_{31}e^{-h_1 s}\|_\infty) \\ &= \|sM_s(s)\|_\infty \|A_{20}A_{40}^{-1}\|_\infty \|(sI - A_{40})^{-1}\|_\infty (\|A_{30}\|_\infty + \|A_{31}\|_\infty), \end{aligned}$$

We have $\varepsilon_1^* \le \varepsilon_2^*$. Hence, the result of our Theorem 2 is less conservative than Theorem 1.2 in Shao and Rowland (1995).

REMARK 2 *The utility of our results is stated as follows. First, we may use Theorem 2 to obtain an ε^*-bound for the stability of the nominal system (2). Then we may use Theorem 1 to check the stability of the uncertain system (1).*

5. Numerical example

Consider the following uncertain time-delay singularly perturbed system:

$$\dot{x}(t) = \sum_{i=0}^{2}(A_{1i} + \Delta A_{1i})x(t - h_i) + \sum_{i=0}^{2}(A_{2i} + \Delta A_{2i})z(t - h_i), \tag{20a}$$

$$\varepsilon\dot{z}(t) = \sum_{i=0}^{2}(A_{3i} + \Delta A_{3i})x(t - h_i) + \sum_{i=0}^{2}(A_{4i} + \Delta A_{4i})z(t - h_i), \tag{20b}$$

where

$$A_{10} = \begin{bmatrix} -1.6 & 0.3 \\ 0.6 & 0.3 \end{bmatrix}, \; A_{11} = \begin{bmatrix} -2.6 & 0.8 \\ -0.8 & -1.2 \end{bmatrix}, \; A_{12} = \begin{bmatrix} -1.3 & -1.5 \\ 1.2 & -0.5 \end{bmatrix},$$

$$A_{20} = \begin{bmatrix} -2.5 & -1.5 \\ 0.6 & 0.7 \end{bmatrix}, \; A_{21} = \begin{bmatrix} -0.7 & 0.8 \\ 0.3 & -0.5 \end{bmatrix}, \; A_{22} = \begin{bmatrix} 0.5 & -0.7 \\ 0.2 & 1.2 \end{bmatrix},$$

$$A_{30} = \begin{bmatrix} -1.8 & -1.4 \\ 0.3 & -0.3 \end{bmatrix}, \; A_{31} = \begin{bmatrix} -1.2 & 2.5 \\ 0.3 & -1.1 \end{bmatrix}, \; A_{32} = \begin{bmatrix} -1.3 & 2.2 \\ 0.5 & -0.4 \end{bmatrix},$$

$$A_{40} = \begin{bmatrix} -4.6 & -1.5 \\ 1.2 & -2.5 \end{bmatrix}, \; A_{41} = \begin{bmatrix} -0.6 & 0.5 \\ -0.3 & 0.2 \end{bmatrix}, \; A_{42} = \begin{bmatrix} -0.4 & -0.5 \\ 0.2 & -0.8 \end{bmatrix},$$

$$(x_1(0), x_1(\tau)) = (x_2(0), x_2(\tau)) = (z_1(0), z_1(\tau)) = (z_2(0), z_2(\tau)) = (2, 0),$$

$$\tau \in [-0.15, 0), \quad h_0 = 0, \quad h_1 = 0.1, \quad h_2 = 0.15.$$

Since

$$\lambda = \|(sI - A_{40})^{-1}\|_\infty \cdot \left\| \sum_{i=1}^{2} A_{4i} e^{-h_i s} \right\|_\infty = 0.5480, \qquad \|\Xi(s)\|_\infty = 0.6726$$

the inequalities (12) and (14a) are satisfied. By Theorem 2, the stability bound of the nominal system is given by $\varepsilon^* = 0.2418$. Moreover, the perturbation bound in (6a) is given by

$$\left\| \left(sI - \sum_{i=0}^{2} A_i e^{-h_i s} \right)^{-1} \right\|_\infty^{-1} = -0.6370. \tag{21}$$

Suppose the singular perturbation parameter of uncertain system (20) is given by $\varepsilon_0 = 0.2316$. According to Theorem 1, the uncertain system (20) is asymptotically stable if upper bounds of the perturbation matrices d_{ji}, $j \in \underline{4}$, satisfy the inequality (6a). For instance, suppose the perturbation matrices are given by

$$\Delta A_{10} = \begin{bmatrix} -0.02 & 0.01 \\ -0.03 & 0.01 \end{bmatrix}, \quad \Delta A_{11} = \begin{bmatrix} -0.06 & 0.02 \\ -0.02 & 0.02 \end{bmatrix},$$
$$\Delta A_{12} = \begin{bmatrix} -0.03 & -0.02 \\ 0.02 & -0.02 \end{bmatrix}, \quad \Delta A_{20} = \begin{bmatrix} -0.03 & -0.01 \\ -0.01 & -0.02 \end{bmatrix},$$
$$\Delta A_{21} = \begin{bmatrix} -0.02 & 0.01 \\ 0.02 & -0.02 \end{bmatrix}, \quad \Delta A_{22} = \begin{bmatrix} 0.02 & -0.01 \\ 0.02 & -0.03 \end{bmatrix},$$
$$\Delta A_{30} = \begin{bmatrix} -0.01 & -0.03 \\ -0.01 & -0.01 \end{bmatrix}, \quad \Delta A_{31} = \begin{bmatrix} -0.02 & 0.01 \\ 0.01 & -0.02 \end{bmatrix},$$
$$\Delta A_{32} = \begin{bmatrix} -0.02 & 0.03 \\ -0.01 & -0.02 \end{bmatrix}, \quad \Delta A_{40} = \begin{bmatrix} -0.03 & -0.02 \\ 0.02 & -0.03 \end{bmatrix},$$
$$\Delta A_{41} = \begin{bmatrix} -0.01 & 0.01 \\ -0.01 & 0.02 \end{bmatrix}, \quad \Delta A_{42} = \begin{bmatrix} -0.01 & -0.03 \\ 0.02 & -0.02 \end{bmatrix}.$$

Then we have

$$\|\Delta A_{10}\|_\infty = 0.0396, \quad \|\Delta A_{20}\|_\infty = 0.0362, \quad \|\Delta A_{30}\|_\infty = 0.0341,$$
$$\|\Delta A_{11}\|_\infty = 0.0683, \quad \|\Delta A_{21}\|_\infty = 0.0356, \quad \|\Delta A_{31}\|_\infty = 0.0300,$$
$$\|\Delta A_{12}\|_\infty = 0.0370, \quad \|\Delta A_{22}\|_\infty = 0.0413, \quad \|\Delta A_{32}\|_\infty = 0.0383,$$
$$\|\Delta A_{40}\|_\infty = 0.0361, \quad \|\Delta A_{41}\|_\infty = 0.0230, \quad \|\Delta A_{42}\|_\infty = 0.0362,$$

Choose $d_{ji} = \|\Delta A_{ji}\|_\infty$, $j \in \underline{4}$, $i \in \overline{2}$. From (6b), we have

$$\sum_{i=0}^{2} \|D_i\|_\infty = 0.6352. \tag{22}$$

As a result of (21) and (22), the inequality (6a) holds and hence the uncertain system (20) is asymptotically stable. For $\varepsilon_0 = 0.2316$, the time responses of system (20) are depicted in Fig. 1.

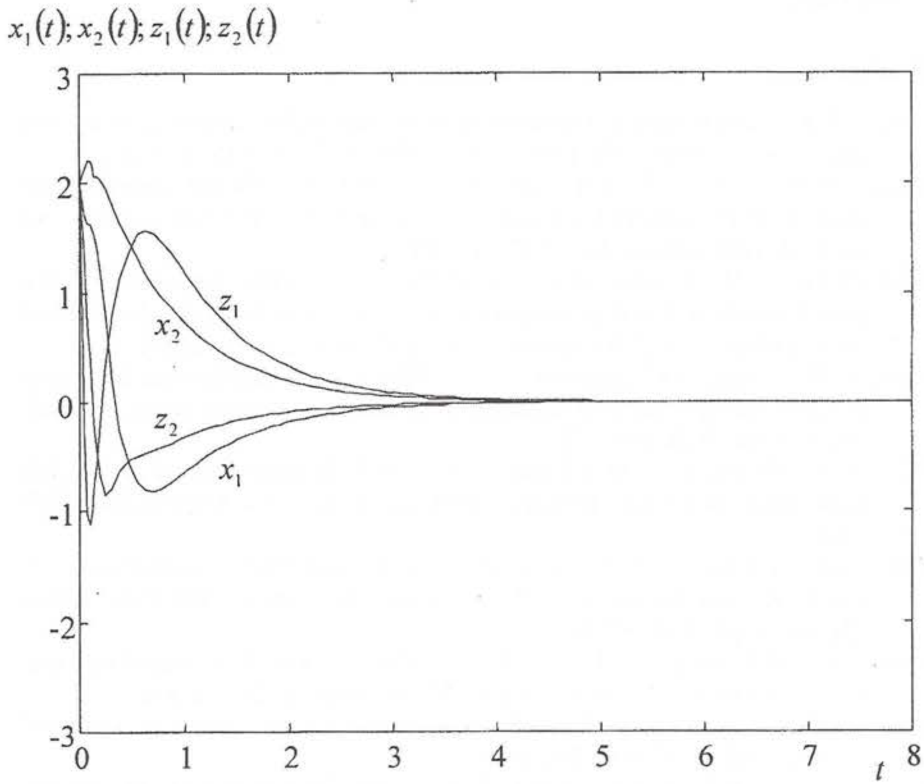

Figure 1. Time responses of system (20) for $\varepsilon_0 = 0.2316$

6. Conclusions

A delay-dependent criterion has been proposed in this paper to guarantee the robust stability of a class of nominally stable uncertain singularly perturbed systems with multiple non-commensurate time delays. Based on this result, the range of allowable bounds of the perturbation matrices preserving the closed-loop stability can easily be found. Furthermore, a simple estimation of the stability bound ε^* has also been proposed such that the nominal system is asymptotically stable for any $\varepsilon \in (0, \varepsilon^*)$. We have shown that the stability bound ε^* is less conservative than that of Shao and Rowland (1995). A numerical example has been provided to illustrate our main results. It is not clear whether our results can be immediately extended to singularly perturbed systems with distributed delays or time-varying delays. For these systems, some other methodologies might be needed. These constitute interesting future research topics.

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