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D-stability for a class of discrete descriptor systems with multiple time delays

by

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Abstract: In this paper, the research on discrete descriptor systems is extended to include discrete multiple time-delay descriptor systems. The impulse-free and D-stability problem for a class of discrete descriptor systems with multiple time delays is investigated. A delay-dependent criterion is first derived to guarantee that the system is proper. A delay-dependent stability criterion in terms of spectral radius is then presented to ensure the D-stability of the system. Furthermore, a delay-dependent criterion is proposed to guarantee that the system is regular, impulse-free, and D-stable. Finally, a numerical example is provided to illustrate our main results.

Keywords: D-stability, descriptor system, time delay, proper, asymptotically stable.

Notation

A^{-1}:	inverse of matrix A
$\overline{d}$:	$\{0, 1, 2, \ldots, d\}$
$\underline{d}$:	$\{1, 2, \ldots, d\}$
$\|A\|$:	$[\|a_{ij}\|]$, with matrix $A = [a_{ij}]$
$A \leq B$:	$a_{ij} \leq b_{ij}$ for all i, j, with $A = [a_{ij}]$ and $B = [b_{ij}]$
$\rho(A)$:	spectral radius of matrix A
$\deg[f(z)]$:	degree of polynomial $f(z)$

1. Introduction

Many practical systems are descriptor systems. They appear in engineering systems, economics, network analysis, time-series analysis, mechanical systems, singularly perturbed systems, etc. (Dai, 1989). They endow the systems with several special features that are not found in classical systems. Therefore, it is inevitable and challenging to investigate the stability problem of descriptor systems. Descriptor systems have been studied by many researchers in recent years (see, for example, Tornambe, 1996, Tarbouriech and Castelan, 1995, Yu and Muller, 1994, Qiu and Davison, 1992, and the references therein).

On the other hand, it is well known that the time-delay phenomena always exist in various engineering systems, such as chemical processes, long transmission lines, pneumatic systems, hydraulic systems, and electric networks. Their existence frequently causes the instability of the system. Therefore, the stability problem of the time-delay systems has been a main concern of the researchers over the years (Lien, Hsieh and Sun, 1998, Phoojaruenchanachai, Uahchinkul and Prempraneerach, 1998, Sun and Hsieh, 1998).

To achieve the various aspects of system performance, the dynamic response of a linear time-invariant system can be modified by means of placing the poles in predetermined locations (Fang, Lee and Chang, 1994, Lee and Lee, 1987). Consequently, the technique of pole-assignment has been considered during the past years. On the other hand, due to the existence of uncertainties, the locations of poles cannot be placed at a specific location. Therefore, assigning the poles at a specific region instead of a specific location is more practical (Hsiao, Pan and Teng, 1999, Rachid, 1990, Vicino, 1989). For the above reasons, it is practical to consider the D-stability problem for a class of multiple time-delay descriptor systems. This is due not only to theoretical interests but also to the relevance of these phenomena in the field of control engineering applications.

Some recent results concerning this topic have been reported. For example, Fang et al. (1994) proposed a stability criterion for discrete-time descriptor systems. Chou et al. (1999) presented a less conservative stability criterion compared with those of Fang et al. (1994). Lee et al. (1992) proposed D-stability criteria based on the spectral norm inequality for discrete systems with a time delay. A sufficient condition based on the pulse-response sequence matrix of the nominal systems has been proposed to guarantee pole location within a specific disk for discrete systems (Chou, 1991). Su and Shyr (1994) extended the stability criterion in Chou (1991) to derive a D-stability criterion for discrete time-delay systems. However, the evaluation of pulse-response sequence matrix in Chou (1991) as well as Su and Shyr (1994) is very complicated for the discrete time-delay system. It is the purpose of this paper to investigate the D-stability problem of the discrete multiple time-delay descriptor systems by evaluating the spectral radius instead of the pulse-response sequence matrix. To the authors' knowledge, the D-stability problem of discrete descriptor systems with multiple time delays has not yet been well explored.

2. Problem formulation and preliminaries

Consider the following discrete descriptor system with multiple time delays:

$$Ex(k+1) = \sum_{i=0}^{d} A_i x(k-h_i) + Bu(k), \tag{1a}$$

$$y(k) = Cx(k), \tag{1b}$$

where $x(k) \in \mathrm{R}^n$, $u(k) \in \mathrm{R}^m$, $y(k) \in \mathrm{R}^r$ are the states, input, and output of the system, respectively; E, $A_i \in \mathrm{R}^{n\times n}$, $B \in \mathrm{R}^{n\times m}$, $C \in \mathrm{R}^{r\times n}$ are constant matrices; $h_0 = 0$, h_i, $i \in \underline{d}$, are non-negative integers and the matrix E may be a singular matrix, i.e., $\mathrm{rank}(E) \leq n$.

First, consider the nominal delay-free system

$$Ex(k+1) = A_0 x(k) \tag{2}$$

DEFINITION 1 (Lin, Yang and Lam, 2000, Fang, Lee and Chang, 1994, Verghese, Levy and Kailath, 1981) *The system $Ex(k+1) = A_0 x(k)$, or the pair (E, A_0), is said to be*

- *(a) regular if $\det(\alpha E - A_0) \neq 0$ for some complex number α;*
- *(b) impulse-free if $\deg[\det(zE - A_0)] = \mathrm{rank}(E)$, i.e., there is no impulsive motion or dynamical infinite mode;*
- *(c) asymptotically stable if $x(k) \to 0$ as $k \to \infty$ for any $x(0) \in \mathrm{R}^n$.*

REMARK 1 *It is noted that the system (2) is impulse-free if and only if $(zE - A_0)^{-1}$ is proper (Fang, Lee and Chang, 1994), i.e., the order of the numerator in $(zE - A_0)^{-1}$ is not greater than that of its denominator. It is asymptotically stable if and only if $\det|zE - A_0)| > 0$ for all $|z| \geq 1$ (Fang, Lee and Chang, 1994, Lewis, 1986).*

DEFINITION 2 *The system (1) is said to be $D(\alpha, r)$-stable if all roots of the characteristic equation*

$$\det\left(zE - A_0 - \sum_{i=1}^{d} A_i z^{-h_j}\right) = 0$$

are within the disk $D(\alpha, r)$ centered at $(\alpha, 0)$ with radius r, $|\alpha| + r < 1$. A disk $D(\alpha, r)$ centered at $(\alpha, 0)$ with radius r is shown in Figure 1.

DEFINITION 3 *The system (1) is proper if its transfer function matrix $M(z)$ is proper.*

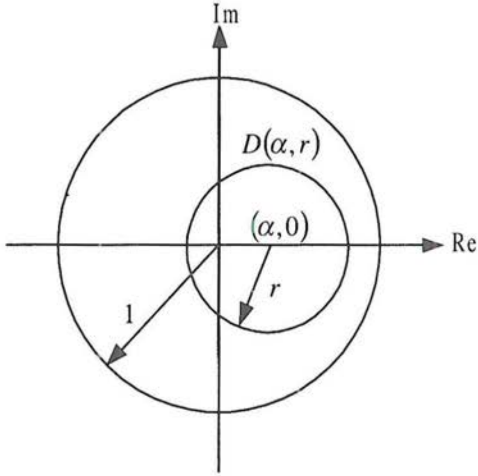

Figure 1. A disk $D(\alpha, r)$ centered at $(\alpha, 0)$ with radius r.

DEFINITION 4 *The discrete descriptor system (1) is said to be regular, impulse-free, and $D(\alpha, r)$-stable if the transfer matrix $M(z)$ of system (1) is proper and all roots of the characteristic equation of system (1) are within the disk $D(\alpha, r)$ centered at $(\alpha, 0)$ with radius r, $|\alpha| + r < 1$.*

Before proceeding, several lemmas are given in the following.

LEMMA 1 (Ortega, 1972) *For any $m \times m$ matrices A, B, and C, if $|B| < C$, then*

$$\rho(AB) \leq \rho(|A| \cdot |B|) \leq \rho(|A| \cdot C).$$

LEMMA 2 (Hsiao, Pan and Teng, 1999) *For any matrix $A \in R^{m \times m}$, if $\rho(A) < 1$, then*

$$|\det(I \pm A)| > 0.$$

LEMMA 3 (Chen, 1984) *Let $H(z)$ be a square rational matrix and be decomposed uniquely as*

$$H(z) = H_p(z) + H_{sp}(z),$$

where $H_p(z)$ is a polynomial matrix and $H_{sp}(z)$ is a strictly proper rational matrix, i.e., the order of the numerator in each element of the matrix is less than the order of its denominator. Then $H^{-1}(z)$ is proper if and only if $H_p^{-1}(z)$ exists and is proper.

LEMMA 4 (John, 1967) *If $f(z)$ is analytic in a bounded domain D and continuous in the closure of D, then $|f(z)|$ takes its maximum on the boundary of D.*

LEMMA 5 (Gorecki, Fuksa, Grabowski and Korytowski, 1989) *The zero-input response of the time-delay system*

$$x(k+1) = \sum_{i=0}^{d} F_i x(k-i)$$

is asymptotically stable if and only if

$$|\det [zI - F(z)]| > 0, \qquad |z| \geq 1,$$

where $F(z) = \sum_{i=0}^{d} F_i z^{-i}$.

3. Stability criterion

It is obvious that (1) can be rewritten as

$$Ex(k+1) = A_0 x(k) + \sum_{i=1}^{d} A_i x(k-h_i) + Bu(k), \tag{3a}$$

$$y(k) = Cx(k). \tag{3b}$$

The transfer matrix $M(z)$ of system (1) is

$$\begin{aligned} M(z) &= C\left(zE - A_0 - \sum_{i=1}^{d} A_i z^{-h_i}\right)^{-1} B \\ &= C\left[I - (zE - A_0)^{-1} \sum_{i=1}^{d} A_i z^{-h_i}\right]^{-1} (zE - A_0)^{-1} B \\ &= C\,[I - T(z) \cdot \Xi(z)]^{-1}\, T(z)\, B, \end{aligned} \tag{4a}$$

where

$$T(z) = (zE - A_0)^{-1} \tag{4b}$$

and

$$\Xi(z) = \sum_{i=1}^{d} A_i z^{-h_i}. \tag{4c}$$

If the matrix $T(z)$ is proper, then $T(z)$ can be expanded as

$$T(z) = \Psi + T_{sp}(z), \tag{5}$$

where Ψ is a constant matrix and $T_{sp}(z)$ is a strictly proper matrix.

In the following, a delay-dependent criterion is proposed to ensure that the discrete time-delay descriptor system is proper.

THEOREM 1 *Suppose that the matrix $T(z)$ of the nominal pair (E, A_0) in (4b) is proper. Then the discrete multiple time-delay descriptor system (1) is proper if*

$$\rho\left[\Psi \cdot \Xi(z)\right] < 1 \qquad \text{for all } |z| \geq 1, \tag{6}$$

where Ψ and $\Xi(z)$ are determined in (5) and (4c), respectively.

Proof. Since the matrix $T(z)$ is proper, we have

$$\begin{aligned}\left[I - T(z)\,\Xi(z)\right]^{-1} &= \left\{I - \left[\Psi + T_{sp}(z)\right] \cdot \Xi(z)\right\}^{-1} \\ &= \left[I - \Psi \cdot \Xi(z) - T_{sp}(z) \cdot \Xi(z)\right]^{-1}\end{aligned} \tag{7}$$

in view of (5). By Lemma 2, if (6) holds, then $\left|\det\left[I - \Psi \cdot \Xi(z)\right]\right| > 0$ for all $|z| \geq 1$. Hence $\left[I - \Psi \cdot \Xi(z)\right]^{-1}$ exists and is proper in view of Lemma 3. Similarly, by the fact that $T_{sp}(z)\,\Xi(z)$ is strictly proper, we have that $\left[I - \Psi \cdot \Xi(z) - T_{sp}(z) \cdot \Xi(z)\right]^{-1}$ is proper. Hence, the discrete multiple time-delay descriptor system (1) is proper in view of (4) and (7). This completes our proof. ∎

COROLLARY 1 *Suppose that the matrix $T(z)$ in (4b) is proper. Then the discrete descriptor system (1) is proper if*

$$\rho[\Psi \cdot \Xi(e^{-j\theta})] < 1, \qquad \theta \in [0,\ 2\pi], \tag{8}$$

where Ψ and $\Xi\left(e^{-j\theta}\right)$ are determined in (5) and (4c), respectively.

Proof. Let $z = \eta^{-1}$. Then (6) is equivalent to

$$\rho\left[\Psi \cdot \Xi\left(\eta^{-1}\right)\right] < 1, \qquad |\eta| \leq 1.$$

Since $\Xi(\eta^{-1})$ is analytic in the bounded region $|\eta| \leq 1$ and continuous in the closed bounded region $D \equiv \{\eta \mid |\eta| \leq 1\}$, $\rho[\Psi \cdot \Xi(\eta^{-1})]$ is analytic and continuous in D. Consequently, $\left|\rho[\Psi \cdot \Xi(\eta^{-1})]\right|$, or equivalently $\rho[\Psi \cdot \Xi(\eta^{-1})]$, takes its maximum on the boundary of $\eta \leq 1$ by Lemma 4. Hence, from Theorem 1, the discrete multiple time-delay descriptor system (1) is proper if (8) holds. This completes our proof. ∎

THEOREM 2 *Suppose the nominal pair (E, A_0) is $D(\alpha, r)$-stable with $|\alpha| + r < 1$ and $|\alpha| < r$. Then the discrete descriptor system (1) is $D(\alpha, r)$-stable if*

$$\rho\left[T\left(\alpha + re^{-j\theta}\right) \cdot \Xi\left(\alpha + re^{-j\theta}\right)\right] < 1, \qquad \theta \in [0,\ 2\pi]. \tag{9}$$

Proof. From Remark 1 and Definition 2, if the nominal pair (E, A_0) is $D(\alpha, r)$-stable, then we have

$$\left|\det\left(zE - A_0\right)\right| = \left|\det T^{-1}(z)\right| > 0, \qquad |z - \alpha| \geq r. \tag{10}$$

Moreover, according to (4), we obtain

$$\left|\det\left(zE - A_0 - \sum_{i=1}^{d} A_i z^{-h_i}\right)\right| = \left|\det\left[T^{-1}(z) - \Xi(z)\right]\right|$$
$$= \left|\det T^{-1}(z)\right| \cdot \left|\det\left[I - T(z)\cdot \Xi(z)\right]\right|. \tag{11}$$

Hence, if the nominal pair (E, A_0) is $D(\alpha, r)$-stable, then the discrete descriptor system (1) is $D(\alpha, r)$-stable if

$$\left|\det\left[I - T(z)\cdot\Xi(z)\right]\right| > 0, \qquad |z-\alpha| \geq r \tag{12}$$

in view of (4), (10), (11), and Definition 2. Let $\beta = (z-\alpha)/r$, i.e., $z = \alpha + r\beta$. Then (12) can be rewritten as

$$\left|\det\left[I - T(\alpha + r\beta)\cdot\Xi(\alpha + r\beta)\right]\right| > 0, \qquad |\beta| \geq 1. \tag{13}$$

As a result of (12), and Lemma 2, if

$$\rho\left[T(\alpha + r\beta)\cdot\Xi(\alpha + r\beta)\right] < 1, \qquad |\beta| \geq 1,$$

then (13) holds and hence the system (1) is $D(\alpha, r)$-stable. Let $\delta = \beta^{-1}$. Then (13) can be rewritten as

$$\left|\det\left[I - T\left(\alpha + r\delta^{-1}\right)\cdot\Xi\left(\alpha + r\delta^{-1}\right)\right]\right| > 0, \qquad |\delta| \leq 1. \tag{14}$$

It is clear that $T\left(\alpha + r\delta^{-1}\right)$ is analytic in a bounded region $|\delta| \leq 1$ in view of (10). Moreover, the multiple roots of $\Xi\left(\alpha + r\delta^{-1}\right)$ are at $\delta = -r/\alpha$, $r > |\alpha|$. Hence, $\Xi\left(\alpha + r\delta^{-1}\right)$ is also analytic in $|\delta| \leq 1$. Consequently, $\rho\left[T\left(\alpha + r\delta^{-1}\right)\cdot\Xi\left(\alpha + r\delta^{-1}\right)\right]$ is analytic in $|\delta| \leq 1$, continuous in the closure of $|\delta| \leq 1$. Hence $\left|\rho\left[T\left(\alpha + r\delta^{-1}\right)\cdot\Xi\left(\alpha + r\delta^{-1}\right)\right]\right|$, or equivalently $\rho\left[T\left(\alpha + r\delta^{-1}\right)\cdot\Xi\left(\alpha + r\delta^{-1}\right)\right]$, takes its maximum on the boundary of $|\delta| \leq 1$ by Lemma 4. Therefore, according to Lemma 2 and Lemma 4, if (9) holds, then (14) and hence (13) hold. Consequently, the discrete descriptor system (1) is $D(\alpha, r)$-stable. This completes our proof. ■

COROLLARY 2 *Suppose the nominal pair* (E, A_0) *is asymptotically stable. Then the discrete descriptor system (1) is asymptotically stable if* $\rho[T(e^{-j\theta})\cdot\Xi(e^{-j\theta})] < 1$, $\theta \in [0, 2\pi]$.

Proof. This result follows immediately from Theorem 2 by setting $\alpha = 0$ and $r = 1$. ■

REMARK 2 *Suppose the nominal pair* (E, A_0) *is regular, impulse-free, and* $D(\alpha, r)$*-stable with* $|\alpha| < r$. *Then the discrete descriptor system (1) is regular, impulse-free, and* $D(\alpha, r)$*-stable with* $|\alpha| < r$ *if*

$$\max\left\{\rho[T(\alpha + re^{-j\theta})\cdot\Xi(\alpha + re^{-j\theta})],\ \rho[\Psi\cdot\Xi(e^{-j\theta})]\right\} < 1, \qquad \theta \in [0, 2\pi],$$

in view of Remark 1, Corollary 1, and Theorem 2.

4. Numerical Example

Consider the discrete descriptor system (1) with

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_0 = \begin{bmatrix} 0.54 & 0.05 & 0 \\ 0 & 0.5 & 0 \\ -1 & 0 & 1 \end{bmatrix},$$
$$A_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0.2 & -0.1 & 0 \\ 1.5 & -1.2 & 0.1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \quad h_0 = 0, \quad h_1 = 1.$$

From (5), $T(z)$ can be decomposed as

$$T(z) = \Psi + T_{sp}(z), \tag{15}$$

where $\Psi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, and

$$T_{sp}(z) = \left(z^2 - 1.04z + 0.27\right)^{-1} \cdot \begin{bmatrix} z-0.5 & 0.05 & 0 \\ 0 & z-0.54 & 0 \\ z-0.5 & 0.05 & 0 \end{bmatrix}.$$

Moreover, according to (4c), we have

$$\Xi(z) = A_1 z^{-1} = \begin{bmatrix} 0.1z^{-1} & 0 & 0 \\ 0.2z^{-1} & -0.1z^{-1} & 0 \\ 1.5z^{-1} & -1.2z^{-1} & 0.1z^{-1} \end{bmatrix}. \tag{16}$$

Since

$$\max_{\theta \in [0,2\pi]} \rho\left[\Psi \cdot \Xi\left(e^{-j\theta}\right)\right] = 0.1 < 1, \tag{17}$$

the discrete descriptor system (1) is proper in view of Corollary 1. In what follows, the $D(\alpha, r)$- stability problem with $\alpha = 0.3$ and $r = 0.5$ is investigated according to Theorem 2. Since

$$\max_{\theta \in [0,2\pi]} \rho\left[T\left(\alpha + re^{-j\theta}\right) \cdot \Xi\left(\alpha + re^{-j\theta}\right)\right] = 0.7656 < 1, \tag{18}$$

the discrete descriptor system (1) is $D(0.3, 0.5)$-stable in view of Theorem 2. In order to verify our result, the poles of discrete descriptor system (1) are found as

$$\{-0.1,\ 0.7182,\ 0.2294 \pm j \cdot 0.2215,\ -0.1369\}.$$

Hence, the distances between the poles and $(0.3, 0)$ are given by

$$\{0.4,\ 0.4182,\ 0.2325,\ 0.2325,\ 0.4369\}.$$

It is obvious that all poles of system (1) are within the disk $D(0.3, 0.5)$. Thus, the inequality (18) guarantees the $D(0.3, 0.5)$-stability of discrete descriptor system (1). Furthermore, according to (17) and (18), the discrete descriptor system (1) is regular, impulse-free, and $D(0.3, 0.5)$-stable in view of Remark 2. The impulse response of discrete descriptor system (1) is shown in Figure 2.

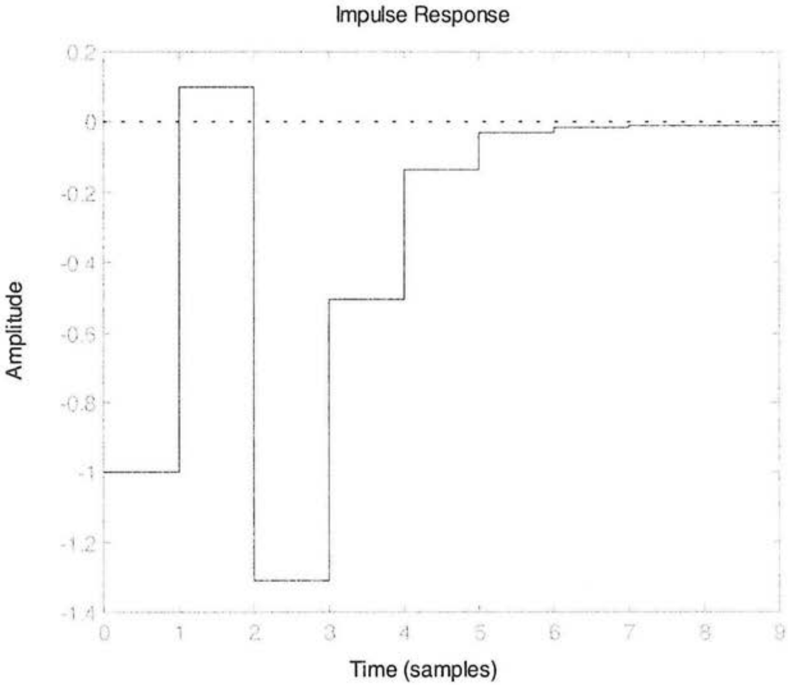

Figure 2. The impulse response of discrete descriptor system in example.

5. Conclusions

The impulse-free and D-stability problem of discrete descriptor systems with multiple time delays has been investigated in this paper. It is obvious that the systems considered in this paper are much more general than those examined in recent researches. We have proposed a delay-dependent criterion to guarantee that the discrete descriptor system is proper. A delay-dependent stability criterion has also been presented to ensure the D-stability of the discrete descriptor system. Moreover, a stability criterion deduced from the D-stability criterion has been proposed to guarantee the asymptotic stability of the discrete descriptor system. Finally, a delay-dependent criterion has been presented to guarantee that the system is regular, impulse-free, and D-stable.

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