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Book review:

Advances in Dynamic Games and Applications

by

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"Advances in dynamic games and applications" is a collection of papers concerning dynamic games. The papers are divided into four thematic groups: *Dynamic Games: Theory*, *Stochastic Games*, *Solution Methods of Dynamic Games* and *Dynamic Games: Applications*. The first two parts are theoretical, the third part is devoted to algorithms and numerical solution approaches for dynamic games and the third part concerns several applications.

Part I contains six papers devoted to theory of dynamic games. These papers are:

T. Radzik and A. J. Goldman, *On Problems with Information in Some Games: Modelling the Strategies in Some Dynamic Games*. The paper deals with zero-sum two-player games of resource allocation played in continuous time. In such games there are problems with formulation of the normal form. The normal forms of games are formulated in the paper depending on whether players are "noisy" (player's actions are observable by the opponent) or "silent".

A. Haurie, *A Historical Perspective on Cooperative Differential Games*. This is a survey paper on cooperative differential games.

P. Bernhard and N. Hovakimyan, *Certainty Equivalence Principle and Minimax Team Problems*. In this paper a two player zero-sum dynamic game called team problem with uncertainty is considered. The authors assume that players do not exchange information. Instead of expected utility theory, uncertainty is dealt with minimax fashion – the authors look for guaranteed payoffs of the players. The results are applied to the linear-quadratic game.

G. Umbhauer, *Evolutionary Processes in Signalling Games: The Impact of Different Learning Schemes*. The paper deals with two-player game with asymmetric information and signalling. An evolutionary game approach to such games is examined. Depending on the learning scheme, different long-term behaviours can be obtained: either Hellwig or Riley equilibrium.

L. Mallozzi and J. Morgan, *Mixed Strategies for Hierarchical Zero-Sum Games*. In this paper, a Stackelberg zero-sum game with one leader and two followers is examined. There is no assumption of convexity of cost functions of the followers, which implies that the reactions sets are not singletons. The

fore plays to minimize the worst. For such game a concept of approximate mixed solution is introduced, sufficient conditions for existence are derived and convergence of the corresponding values is presented.

D. A. Carlson, *The Existence and Uniqueness of Equilibria in Convex Games with Strategies in Hilbert Spaces.* The paper examines the existence and uniqueness of Nash equilibria in convex, finitely-many-player games with strategies of each player in a separable Hilbert space. The existence is proven using an extension of the Kakutani fixed point theorem, while the uniqueness is obtained by the extension of Rosen's concept of strict diagonal convexity. The results are applied to open-loop dynamic games.

Part II contains five papers concerning stochastic games. These papers are:

N. Vieille, *The Existence of Equilibrium Payoffs in Two-Player Stochastic Games.* The paper contains a discussion of the main steps of the Vieille proof of the existence of equilibria in stochastic games.

P. Secchi and W. D. Sudderth, *Persistently Good Strategies for Nonleavable Stochastic Games with Finite State Space.* The authors prove the existence of persistently ε-optimal strategies for players engaged in nonleavable stochastic games with finite state space.

M-T. Nguyen, E. Altman, and V. Gaitsgory, *On Stochastic Hybrid Zero-Sum Games with Nonlinear Slow Dynamics.* This paper describes a zero-sum dynamic game with jumps at discrete moments, in which players control the transition probabilities of the controlled Markov process governing the jumps. The set of states and set of actions of the players are assumed to be finite. The values of such games, as the length of the intervals between jumps tends to 0, tend to the viscosity solution of a certain Hamilton-Jacobi-type equation.

H-U. Küenle, *On Multichain Markov Games.* The paper considers a two-player, zero-sum Markov game with Borel state and action spaces, unbounded stage costs and the average cost criterion. Under some assumption on transition probabilities, the author presents some results on the existence of ε-optimal strategies.

E. Altman, E. A. Feinberg, J. Filar, and V. Gaitsgory, *Perturbed Zero-Sum Games with Applications to Stochastic and Repeated Games.* This paper deals with perturbed matrix games and their applications to stochastic games with finitely many states and repeated games. The main result concerning perturbed games is that for some class of perturbations the sets of their solutions converge to some subsets of solutions of the corresponding lexicographic games. The applications concern the following types of games: infinite-horizon stochastic games with perturbed transition probabilities and payoffs with the transition probabilities controlled by only one player, finite-horizon stochastic games with perturbed transition probabilities and payoffs with the transition probabilities controlled by both players, and repeated games zero-sum with the criterion of weighted discounted accumulated payoff.

Part III consists of four papers devoted to algorithms for solving the dynamic

S. S. Kumkov and V. S. Patsko, *Construction of Singular Surfaces.* The paper presents an algorithm for construction and classification of singular surfaces in differential zero-sum games with fixed terminal time and linear dynamics. The objective function of player 1, who minimizes it, depends on two coordinates of the state at the terminal time and it is convex. The main procedure of the algorithm is based on construction of the levelsets of the value function using backward induction. The numerical results for two examples are provided as illustration.

M. Falcone, P. Lanucara, and M. Marinucci, *Parallel Algorithms for the Isaacs Equation.* The paper presents an algorithm of construction an approximation scheme for the Hamilton-Jacobi-Isaacs equation for a zero-sum differential game in $\mathbb{R}^n$. The algorithm uses a domain decomposition technique. The results converge to the viscosity solution of the equation. The efficiency is illustrated by numerical tests.

A. Haurie and F. Moresino, *Computation of S-adapted Equilibria in Piecewise Deterministic Games via Stochastic Programming Methods.* The authors propose a numerical technique to approximate some equilibria in a stochastic game of oligopoly. Jumps occur at discrete times (they constitute a disturbance Markov chain) and the information structure is S-adapted – players take into account initial state, time and the history of the Markov chain. The authors approximate some equilibria by a sequence of variational inequality problems.

P. Bernhard, S. Crepey, and A. Rapaport, *Comparison of two Numerical Approaches for the Barrier and Value of a Simple Pursuit-Evasion Game.* In the paper a well known pursuit-evasion problem – one-dimensional second order servomechanism problem – is presented in a new approach, called game in distance, in which oriented distance criterion is used instead of the capture time criterion. The theoretical and numerical results for the traditional and new approach are compared. The theory of viscosity solutions is used to solve the Bellman-Hamilton-Jacobi-Isaacs equation.

Part IV consists of three papers on various applications of dynamic games. These papers are:

M. H. Breitner, U. Rettig, and O. von Stryk, *On Optimal Missile Guidance Upgrades with Dynamic Stackelberg Game Linearizations.* The authors apply the dynamic Stackelberg game approach to the problem of interactions between an intercepting missile (the leader) and maneuverable ballistic missile (the follower) in the final, critical homing stage. The trajectories are computed using a direct collocation method. The results are illustrated by many numerical examples, which show that the upgraded guidance improves the interception capability.

V. S. Patsko and V. L. Turova, *Homicidal Chauffeur Game: Computation of Level Sets of the Value Function.* The authors consider two dynamic zero-sum homicidal chauffeur games: one, being the classical case with capture time criterion, while the other has the same dynamics but is a surveillance-evasion game with pedestrian criterion: minimizing escape time from the detection set being a two dimensional cone. The paper describes computation of levelsets of

A. Wiszniewska-Matyszkiel, *"The Tragedy of the Commons" Modelled by Large Games.* The paper is devoted to dynamic games with continuum of players modelling exploitation of a common ecosystem. An interesting decomposition theorem is proven, allowing to reduce the problem of finding a dynamic equilibrium to finding equilibria in some static games corresponding to the dynamic game, which is counterintuitive if compared to dynamic games with finitely many players. The theoretical results are illustrated by two examples of exploitation of a rainforest. In the case of unstable rainforest in every equilibrium players destroy the system at finite time.

The book is the sixth volume of the Annals of the International Society of Dynamic Games – the monograph series devoted to dynamic games and their applications. Some of the papers are based on a selection from presentations made at the 1998 ISDG symposium.

This book is addressed to readers working in game theory or optimization theory as well as their applications. Nowadays, game theory spans such disciplines as mathematics, economics, electrical and electronic engineering, operations research, computer science, theoretical ecology, environmental science, political science, etc. Therefore, there is quite a large group of potential readers.

Dynamic games, whose first informal beginning dates back to Steinhaus (1925) and whose bloom started in the 1960-ies describe all situations of coupled choice by many decision makers over time in an environment changing in time in response to their decisions. Dynamic games now attract researchers from many disciplines: they are mathematically interesting, they have applications in economics, social and political science, and, last but not least, they have applications in engineering – military applications, which were the main basis of the birth of dynamic games, and also in other areas, among others telecommunication and computer science.

The papers presented in the book, except for the Haurie's survey paper on cooperative dynamic games and the Vieille's paper on stochastic games, present novel research results. The problems concerned represent well the wide scope of the discipline.

The readers are required to be familiar with mathematical apparatus used in dynamic games, but do not have to be mathematicians.

Perhaps adding a short introduction to each part with description of the problems considered and basic terminology would make this volume easier to read also for researchers who are not game-theorists, but look for new interesting problems and solutions.

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