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Book review:

Semi-Markov Processes and Reliability

by

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The theory of semi-Markov processes is a well-developed part of stochastic analysis. The main reason of this development is a possibility of applying processes of this type, especially in engineering problems. Semi-Markov processes often find applications in reliability. They are also used in economic sciences for catastrophic risk transfer considerations, in insurance and many other fields.

Semi-Markov processes were introduced by P. Lévy and W.L. Smith in the 1950s. Many mathematicians have dealt with this subject ever since. As key persons, who have developed the theory of the considered processes, one should mention R. Pyke, E.H. Moore, R. Schaufele, E. Çinlar and others. Important results were also obtained by N. Limnios and G. Oprişan. Some of them are presented in the book.

Semi-Markov processes are commonly considered as generalizations of Markov processes and therefore one may say that the main advantage of their development is to "allow non-exponential distributions for transitions between states". The second advantage is the fact that semi-Markov processes contain several types of stochastic processes used before. Both features of the considered processes are emphasized by the authors.

The book covers theoretical problems concerning semi-Markov processes as well as their applications in reliability. It contains Preface, six chapters and three appendices. Chapter 1 is dedicated to the general theory of stochastic processes. Particular attention is paid to Markov chains and processes, Markov jump-time processes, and Markov Gaussian processes, renewal processes and renewal theory. Chapter 1 is completed by the definition and basic properties of the regenerative processes.

The theory of Markov renewal processes, presented in Chapter 2, is closely related to semi-Markov processes and can be treated as a generalization of renewal theory. To show the above relation, the definition and some basic properties of semi-Markov kernel Q are introduced, and Markov renewal process (Jn,Sn) associated to Q and (J-X)-process are considered. An interesting (also from probabilistic point of view) convergence theorem for Markov renewal processes is proved. Moreover, a state space merging method proposed by V.S. Korolyuk

In Chapter 3 semi-Markov processes are defined and their basic properties are shown. A necessary and sufficient condition for the existence of a solution of the Markov renewal equation is given. Sufficient conditions for the existence of its unique solution are provided and the representation of this solution is shown. For a semi-Markov process $(Z(t), t \in R_+)$ with values in a measurable space (E,E) and for a Borel function $g : E \to R_+$, properties of real functionals of the integral form

$$W(t) = \int_0^1 g(Z(u))du, \quad t \in R_+,$$

are discussed. Functionals of this type are used in reliability. They are often interpreted as the accumulative rewards until time t. Furthermore, in Chapter 3 backward and forward recurrence times are defined and examined. The chapter is completed by some limit theorems. The most interesting are Functional Central Limit Theorem proved by S. Grigorescu and G. Oprian, as well as two theorems describing asymptotic behavior of : Strong Law of Large Numbers and Functional Central Limit Theorem, both proved by the authors.

Chapter 4 provides a detailed exposition of the theory of semi-Markov and Markov renewal processes with discrete state spaces. At the beginning, the authors present discrete counterparts of basic definitions introduced in previous chapters. The next discussed subjects are classification of states of a Markov renewal process, Markov renewal equation and limit theorems. Additionally, some properties of finite state space semi-Markov processes are examined. The results concerning statistical estimation obtained by E.H. Moore, R. Pyke, B. Ouhbi and N. Limnios terminate the chapter.

Chapter 5 is dedicated to readers who are particularly interested in applications of semi-Markov processes in reliability. At the beginning of this chapter a brief exposition of reliability theory is given. In the essential part of Chapter 5 considerations are focused on reliability modeling in the finite state space case and on methods of obtaining transition probabilities. Reliability and performability modeling of the system, whose evolution is described by a semi-Markov process with an arbitrary measurable state, are discussed.

Chapter 6 provides several examples of reliability modeling. For the first application, concerning a three-state system, reliability indicators are calculated. The semi-Markov kernels are shown for a system with mixed constant repair time and a system with multiphase repair. Additionally, for the system with multiphase repair the form of transition function matrix and the availability are presented. The next example is an n-component series system. For this system the asymptotic availability is obtained. A maintenance model, developed in the case of Markov process, was generalized by the authors to the semi-Markov process case. The next two discussed applications are an n-component parallel system an a two-component system with cold standby. Moreover, advanced Markov renewal shock models are examined. Stochastic Petri nets, representing of discussed examples. Chapter 6 also contains description of three algorithms for Monte-Carlo simulations realizing semi-Markov processes and examples of their applications to three-state systems and maintained systems.

Three appendices are dedicated to readers not familiar with probability and they make the book self-contained. In Appendix A measures and probability are discussed and a special attention is paid to conditional distributions. Laplace-Stielties transform is the subject of Appendix B. In Appendix C weak convergence of probability measures is defined and some important theorems, including Prohorov's Theorem, are presented.

It was probably assumed that the book would not be complete course on semi-Markov processes and reliability. Therefore, it does not contain all aspects of theory and all known types of applications; it is just an approach of the authors to the presented subject. However, it seems that the choice of presented theorems and indicated examples is relevant to the assumptions they made. According to the opinion of the authors, this book will be useful for students, teachers, and researchers and for all those who are interested in semi-Markov processes.

The fact that the authors look at the semi-Markov processes via applications is also expressed in the Preface, where they claim that the theory of stochastic processes "cannot be properly understood just as pure mathematics, separated from the body of experience and examples that have brought it to life". I think that pure mathematicians should express a slightly different opinion. Stochastic processes are also interesting just as a theory and they need to be properly understood by the researches proving new theorems and facts. Certainly, inspiration is often derived from the real life. However, unused theoretical notions and facts sometimes find their applications in the future.

A meaningful advantage of this book is putting in order the knowledge concerning the presented subject. N. Limnios, G. Oprian took up the mathematical side of the task. They present detailed proofs of a large majority of theorems. Precise definitions make understanding of notions easier. Nevertheless, it seems that some notions and theoretical facts presented in several chapters would require more comments. Their absence may be a difficulty for a reader not acquainted with the subject. One can also find several misprints. However the general impression from the reading of this book remains very positive.

I think that the book is valuable for researches applying semi-Markov processes, especially in reliability, and also for those readers who would like to learn the fundamentals of semi-Markov processes.

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