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Searching for an equivalence between decision rules and concordance-discordance preference model in multicriteria choice problems

by

Salvatore Greco[1], Bartłomiej Prędki[2] and Roman Słowiński[2]

[1] Faculty of Economics, University of Catania,
Corso Italia, 55, 95129 Catania, Italy

[2] Institute of Computing Science, Poznań University of Technology,
Piotrowo 3a, 60-965 Poznań, Poland

Abstract: Solving multicriteria decision problems, like choice and ranking, requires the use of decision maker's DM's (Decision Maker's) preference model. In this paper, we investigate some issues of equivalence between the preference model in terms of "*if...*, *then...*" decision rules and a concordance-discordance preference model based on the use of the outranking relation. The decision rule model is attractive for at least two reasons: (i) it is intelligible and speaks the language of the DM, (ii) the preference information coming from the DM is a set of decision examples. The decision rules are induced from rough approximations of the preference relation specified in decision examples. Then, from the set of decision rules representing the DM's preferences, criteria weights and veto thresholds are inferred, setting up an equivalent preference model following from concordance-discordance tests proposed in ELECTRE methods. A simple example will illustrate the interest of such an equivalence.

Keywords: decision rules, multicriteria choice, ranking, concordance-discordance, rough sets, ELECTRE.

1. Introduction

Solving multicriteria decision problems, such as choice and ranking, requires the use of DM's (Decision Maker's) preference model. It is usually a (utility) function or a binary relation - its construction requires some preference information from the DM, like substitution ratios among criteria, importance weights, or indifference, preference and veto thresholds. Acquisition of this preference information from the DM is not easy and, moreover, the resulting preference terms of "*if...*, *then...*" decision rules induced from decision examples provided by the DM has two advantages over the classical models: (i) it is intelligible and speaks the language of the DM, (ii) the preference information comes from observation of DM's decisions. The preference information expressed through decision examples is easier and more natural than the preference information expressed in terms of specific model parameters mentioned above.

More precisely, decision examples concern a subset of actions, called *reference actions*, relatively well-known to the DM. He/she is thus able to tell for each pair of reference actions (a, b) whether a is at least as good as b or not. This corresponds to the presence (S) or absence (S^c) of the *outranking relation* for the pair (a, b). Taking into account that pairs of reference actions are also characterized by difference of evaluations on n criteria, each difference is translated to a *marginal preference intensity* on the corresponding criterion.

There is, however, a problem with *inconsistency* often present in the set of decision examples. These inconsistencies cannot be considered as simple error or noise – they follow from hesitation of the DM, unstable character of his/her preferences and incomplete determination of the family of criteria. They can convey important information that should be taken into account in the construction of the DM's preference model. Rather to correct or ignore these inconsistencies, we propose to take them into account in the preference model construction using the rough set concept, see Pawlak (1985), Słowiński (1992, 1993), Słowiński, Stefanowski, Greco, Matarazzo (2000). For this purpose, the original rough sets theory have been extended in two ways: (i) substituting the classical indiscernibility relation by a dominance relation, which permits taking into account the preference order in domains (scales) of criteria, and (ii), substituting the data table by a pairwise comparison table (PCT), where each row corresponds to a pair of actions described by marginal preference intensities on particular criteria, which permits approximation of a comprehensive preference relation in multicriteria choice and ranking problems. The extended rough set approach is called *dominance-based rough set approach* Greco, Matarazzo, Słowiński (1998, 1999, 2000, 2001).

Using the rough set approach to the analysis of the PCT, we obtain dominance-based rough approximations of presence (S) and absence (S^c) of the outranking relation. The dominance-based rough set approach answers several questions related to the approximation: (a) is the set of decision examples consistent? (b) what are the non-redundant subsets of criteria ensuring the same quality of approximation as the whole set of criteria? (c) what are the criteria which cannot be eliminated from the approximation without decreasing the quality of approximation? (d) what minimal "*if ...*, *then...*" decision rules can be induced from the approximations? The resulting decision rules constitute a preference model. As was formally proved by Greco, Matarazzo, Słowiński (2002), it is more general than the classical utility function or any nontransitive and nonadditive preference model.

Decision rules derived from rough approximations are then applied to a set of actions concerned by the choice or ranking problem. As a result, one obtains a four-valued outranking relation, see Tsoukias, Vincke (1995), on this set. In order to obtain a recommendation, it is advisable to use an exploitation procedure based on the net flow score of the actions, Greco, Matarazzo, Słowiński, Tsoukias (1998).

In this paper, we investigate some issues of equivalence between the decision rule model induced from rough approximations of S and S^c specified in decision examples, and the relational preference model following from concordance-discordance tests proposed in the ELECTRE methods.

The paper is organized as follows. In Section 2, we specify the conditions of the considered equivalence. Then, in Section 3, we present concordance-discordance approach of the ELECTRE methods, and in Section 4 we define the PCT representing the preference information given by the DM. In Section 5, we briefly sketch the dominance-based rough set approach to the analysis of PCT. Section 6 is devoted to generation of decision rules and Section 7 shows how to interpret the decision rules in terms of criteria weights and veto thresholds compatible with concordance-discordance tests. An illustrative example showing the interest of the stated equivalence is presented in Section 8. The last section includes conclusions.

2. Conditions of the investigated equivalence

The multicriteria decision problem concerns a choice of the best action from among a finite set A of actions evaluated by a family of criteria $G = \{g_1, \ldots, g_n\}$. We assume, without loss of generality, that all criteria are of the "gain" type, i.e. the greater the better.

The preference information acquired from the DM is a set of decision examples, that is, pairwise comparisons of some reference actions from the set $B \subseteq A$. The comparison states either a presence or an absence of the outranking relation (S and S^c, respectively) for a given pair of reference actions.

In order to investigate an equivalence of decision rules and concordance-discordance tests, used in ELECTRE methods Roy (1985, 1993) for construction of the outranking relation over a set of actions, we assume, moreover, that the DM provides for each criterion $g_i \in G$ an indifference threshold q_i, $i = 1, \ldots, n$ defined as linear function of criterion value. The indifference thresholds constitute intra-criteria preference parameters.

Given the pairwise comparisons of reference actions, the dominance-based rough set methodology will be used to set up lower and upper approximations of the relations S and S^c, and then decision rules will be induced from lower approximations of S and S^c, giving a set of all non-ambiguous rules supporting the hypothesis of the presence or the absence of outranking.

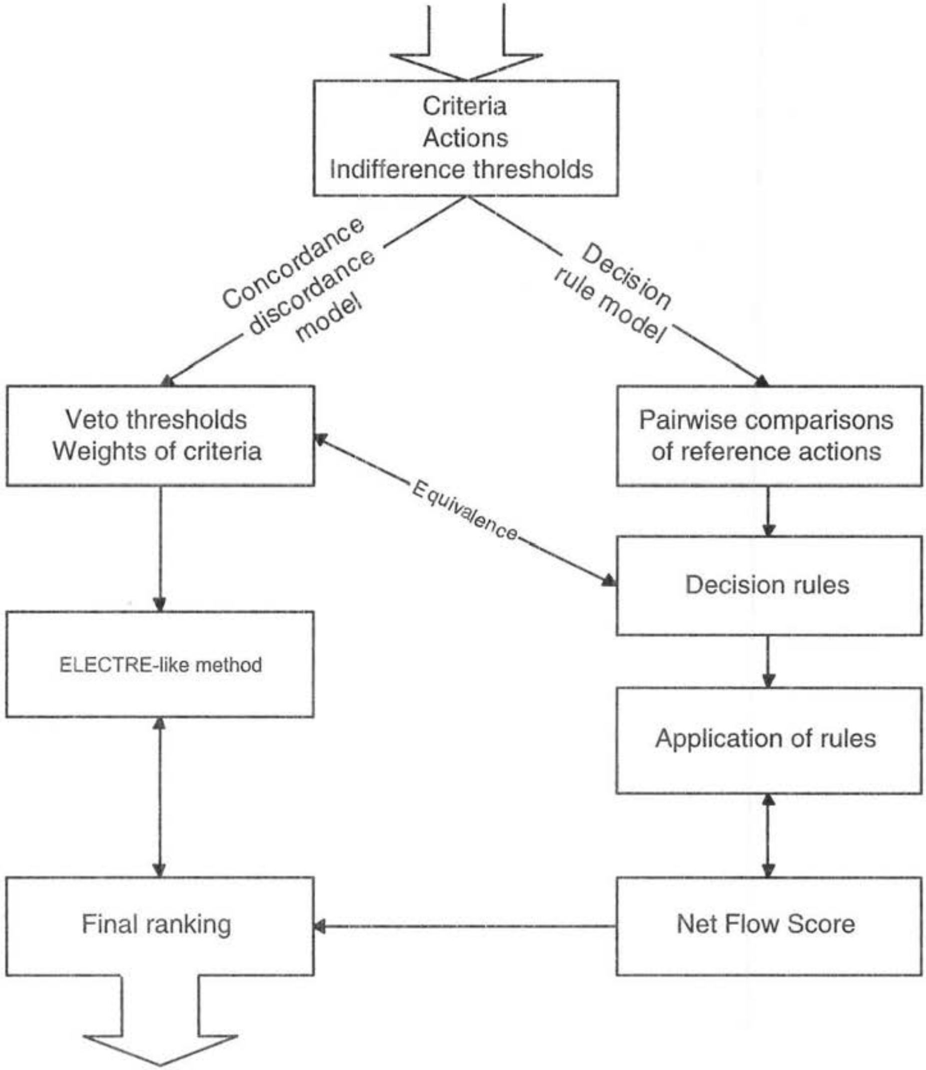

Figure 1. Process flow of multicriteria choice problem and equivalence between models.

To establish the equivalence between a subset of decision rules covering all pairwise comparisons of reference actions, we propose to translate these rules into a set of concordance and discordance conditions corresponding to the outranking relation S. From these conditions follows a set of inequalities defining a feasible space for the vector of criteria weights and veto thresholds. The weights and the veto thresholds constitute inter-criteria preference parameters.

If they are acceptable for the user, it implicitly means that the subset of considered decision rules represents the user's preferences in agreement with the concordance-discordance test.

The process leading from the problem definition to the solution in the form of final ranking is presented in Fig. 1. Usually the decision maker will choose only one path, the one which he/she is more confident with. We do not propose that each problem should be solved in both ways, we are just looking for the equivalence of the both models representing the preference information of the decision maker. Our aim is the equivalence between decision rules and the concordance-discordance model that works in both directions: it is possible to select rules according to preferential parameters of relational model and to find preferential parameters corresponding with the set of decision rules. The comparison of both final rankings may ultimately prove the researched equivalence.

3. Acquisition of preference information and construction of the pairwise comparison table (PCT)

For a subset $B \subseteq A$ of reference actions that the DM finds representative for the decision problem, assume he/she is willing to express the preferences by pairwise comparison of reference actions.

For each pair $(x, y) \in B \times B$, the DM is asked to select one of the three possibilities:

1. action x is as good as y, i.e. xSy
2. action x is worse than y, i.e. $xS^c y$
3. DM is not willing to compare those actions.

S is a binary relation called outranking, and S^c is another binary relation stating the absence of outranking. It is called non-outranking.

An $m \times (n+1)$ Pairwise Comparison Table S_{PCT} is then created on the basis of this information. Its first n columns correspond to criteria from the set G. The last, $n+1$ column of S_{PCT}, specifies binary relation S or S^c. The m rows are pairs from $B \times B$. For each pair in S_{PCT}, a difference on criterion values is put in the corresponding column. If the DM refused to compare two actions, such a pair does not appear in S_{PCT}.

Each difference of evaluations for a pair $(x, y) \in B \times B$ is translated in S_{PCT} into a degree of intensity of preference of x over y, denoted by $T_i(x, y)$, $i = 1, \ldots, n$, and defined as follows:

$$T_i(x,y) = \begin{cases} 0 & \text{if } g_i(x) \geq g_i(y) - q_i(x) \\ g_i(x) - g_i(y) & \text{otherwise} \end{cases}.$$

Note that $T_i(x, y) \leq 0$; $T_i(x, y) = 0$ meaning that x outranks y on criterion g_i, while $T_i(x, y) < 0$ meaning that x does not outrank y on criterion g_i.

Remark also that there may be some cases of incomparability in S_{PCT}. This is when the DM states both xSy and $xS^c y$ for $(x, y) \in B \times B$.

Thus, the pairs of reference actions can be presented in a *pairwise comparison table* (PCT), where rows correspond to pairs of reference actions and columns to preference intensities on n criteria and to the presence or absence of the comprehensive outranking relation.

4. Concordance-discordance test for construction of the outranking relation in ELECTRE methods

To construct the outranking relation S, the ELECTRE methods use two tests – concordance test and discordance test, Roy (1993).

In concordance test, for each pair $(x, y) \in B \times B$ a concordance coefficient $C(x, y)$ is calculated

$$C(x,y) = \frac{\sum_{i=1}^{n} w_i c_i(x,y)}{\sum_{i=1}^{n} w_i},$$

where $c_i(x, y)$ is the marginal concordance coefficient defined as follows:

$$c_i(x,y) = \begin{cases} 1 & g_i(x) \geq g_i(y) - q_i(x) \\ 0 & g_i(x) \leq g_i(y) - p_i(x), \; i = 1, \ldots, n, \\ \text{between 0 and 1} & \text{otherwise} \end{cases}$$

where q_i is indifference threshold and p_i is preference threshold for criterion g_i. Typically, these thresholds are bi-linear functions of criterion values of the worse of two actions being compared.

In our investigation of equivalence, we decided not to use the preference thresholds p_i for g_i, thus the definition of the marginal concordance coefficient boils down to:

$$c_i(x,y) = \begin{cases} 1 & g_i(x) \geq g_i(y) - q_i(x) \\ 0 & \text{otherwise} \end{cases}, \quad i = 1, \ldots, n.$$

The concordance coefficient represents a relative strength of the coalition of criteria being in favor of S. The concordance test is passed if $C(x, y) \geq \lambda$, where $0.5 \leq \lambda \leq 1$, is a cutting level defined by the user.

In discordance test, for each pair $(x, y) \in B \times B$ and for each criterion $g_i \in G$, the difference $g_i(y) - g_i(x)$ is compared with the veto threshold v_i, being another preferential parameter. The discordance test is passed if $g_i(y) - g_i(x) < v_i$ for each criterion $g_i \in G$.

If both concordance and discordance tests are passed, one concludes that the outranking relation xSy is true.

5. Dominance-based rough set analysis of the PCT

In order to express the outranking and non-outranking relations in terms of intensities of preference on particular criteria, the dominance-based rough set approach will be used on the S_{PCT}, see Greco, Matarazzo, Słowiński (1999).

The dominance relation D is defined as follows:
for each $(x, y), (w, z) \in B \times B$ and $P \subseteq G$, $(x, y)D_P(w, z)$ if for each criterion $g_i \in P$ action x is preferred over y at least with the same intensity as w is preferred over z, i.e.

$$T_i(x, y) \geq T_i(w, z) \quad \text{for each } g_i \in P.$$

Based on dominance relation D_P, for each pair $(x, y) \in B \times B$ two auxiliary sets can be defined: positive dominance (D_P^+) and negative dominance (D_P^-):

$$D_P^+(x, y) = \{(w, z) \in B \times B : (w, z)D_P(x, y)\},$$
$$D_P^-(x, y) = \{(w, z) \in B \times B : (x, y)D_P(w, z)\}.$$

Lower approximation of outranking relation S in S_{PCT} is defined as:

$$\underline{P}(S) = \{(x, y) \in B \times B : D_P^+(x, y) \subseteq S\}.$$

Analogously, lower approximation of non-outranking relation S^c in S_{PCT} is defined as:

$$\underline{P}(S^c) = \{(x, y) \in B \times B : D_P^-(x, y) \subseteq S^c\}.$$

It is also possible to use the Variable Consistency Model on S_{PCT}, Greco, Matarazzo, Słowiński, Stefanowski (2001), allowing that some of the pairs in positive or negative dominance sets belong to the opposite relation but at least f percent of pairs belong to the correct one:

$$\underline{P}(S) = \left\{(x, y) \in B \times B : \frac{|D_P^+(x, y) \cap S|}{|D_P^+(x, y)|} \geq f\right\},$$
$$\underline{P}(S^c) = \left\{(x, y) \in B \times B : \frac{|D_P^-(x, y) \cap S^c|}{|D_P^-(x, y)|} \geq f\right\}.$$

6. Generation of decision rules

We propose to induce decision rules from lower approximations of outranking relation S and and S^c defined in Section 5. Any rule generation approach may be used at this stage, although we suggest using an algorithm inducing all possible rules (e.g. DOMAPRIORI, Stefanowski, 2001) and selecting then a feasible subset of rules. Induced rules have then the following syntax:

$D_{\geq}$-rule: "if $T_{i1}(x, y) \geq t_{i1}$ and $\ldots T_{ip}(x, y) \geq t_{ip}$, then xSy", where $\{g_{i1}, \ldots, g_{ip}\} = P \subseteq G$.

$D_{\leq}$-rule: "if $T_{i1}(x,y) \leq t_{i1}$ and $\dots T_{ip}(x,y) \leq t_{ip}$, then $xS^c y$", where $\{g_{i1},\dots,g_{ip}\} = P \subseteq G$.

By adopting the Variable Consistency Model on S_{PCT} it is possible to obtain decision rules having the same syntax but with at least credibility f, $0 \leq f \leq 1$.

Our aim in inferring such rules is to use them to induce the preference model parameters (weights, cutting level and veto thresholds) for concordance-discordance tests, Greco, Matarazzo, Słowiński (2002).

To generate a set of rules shown above an algorithm inducing all possible rules is recommended, although then, the equivalence with the concordance-discordance test will be established for a subset of rules covering all the examples from the S_{PCT}. In choosing the subset of rules, we will privilege rules having the smallest intersection of sets of covered pairs in S_{PCT}.

7. Interpretation of the decision rules in terms of concordance-discordance tests

The decision rules obtained from rough set approach and belonging to the selected subset of rules are used as follows:

1. from each $D_{\geq}$-rule
   "if $T_{i1}(x,y) = 0$ and $\dots T_{ip}(x,y) = 0$, then xSy", where $\{g_{i1},\dots,g_{ip}\} = P \subseteq G$,
   we have from the concordance test:
   $$\sum_{g_i \in P} w_i \geq \lambda \sum_i w_i;$$
2. from each $D_{\geq}$-rule
   "if $T_{i1}(x,y) = 0$ and $\dots T_{ip}(x,y) = 0$ and $T_{j1}(x,y) \geq t_{j1}$ and $\dots T_{jr}(x,y) \geq t_{jr}$, then xSy",
   where $\{g_{i1},\dots,g_{ip}\} = P \subseteq G$, $\{g_{j1},\dots,g_{jr}\} = R \subseteq G$ with $P \cap R = \emptyset$ and $t_{jr} < 0$, we have:
   a. from the concordance test – $\sum_{g_i \in P\cup R} w_i \geq \lambda \sum_i w_i$,
   b. from the discordance test – for each criterion $g_j \in R$, $v_j > -t_j$;
3. from each $D_{\geq}$-rule
   "if $T_{j1}(x,y) \geq t_{j1}$ and $\dots T_{jr}(x,y) \geq t_{jr}$, then xSy",
   where $\{g_{j1},\dots,g_{jr}\} = R \subseteq G$,
   we have from the discordance test:
   for each criterion $g_i \in R$, $v_j > -t_j$;
4. from each $D_{\leq}$-rule
   "if $T_{i1}(x,y) \leq t_{i1}$ and $\dots T_{ip}(x,y) \leq t_{ip}$, then $xS^c y$",
   where $\{g_{i1},\dots,g_{ip}\} = P \subseteq G$,
   we have:
   a. if card$(P) = 1$, then from the discordance test $v_j \leq -t_j$; where

b. if card$(P) > 1$, then we have:

from the concordance test – $\sum_{g_i \in G-P} w_i < \lambda \sum_i w_i$,
from the discordance test – for each criterion $g_i \in P, v_i > -t_i$.

Remark that in point 2 above when calculating the strength of the subset of criteria in favor of S, we take into account criteria from both P and R. This is because the criteria from R, on which the veto did not occur, may also contribute to the concordance.

We look for the set of weights w_i, veto thresholds v_i, $i = 1, \ldots, n$, and the cutting level λ satisfying all the constraints specified in points 1–4. If there is no feasible set of preferential parameters, then another subset of rules should be considered. If the Variable Consistency Model were used, then the consistency level f could also be lowered.

If the feasible set of preferential parameters is not empty and the DM finds an acceptable set of weights w_i, veto thresholds v_i, $i = 1, \ldots, n$, and cutting level λ in this feasible set, then the corresponding subset of decision rules can be considered as equivalent preference model. Both can be used further, in ELECTRE method and in the decision rule approach, to work out the final recommendation on the complete set of actions A.

8. An illustrative example

We have chosen to test our approach on a real life example – construction of water supply system for a rural area, Roy, Słowiński, Treichel (1992). One of the tasks in this problem is setting a priority order in which water users are connected to the new water supply infrastructure.

Originally, this example contained 21 actions (locations) described by 7 criteria:

g_1: water deficiency,
g_2: farm production potential,
g_3: function and activity of user,
g_4: structure of settlement
g_5: water demands,
g_6: share of WSS in overall investment,
g_7: possibility of connection to another WSS.

Preferential parameters used in Roy, Słowiński, Treichel (1992) with respect to these criteria are presented in Table 1.

We used all of the 21 actions from the original problem, presented in Table 2.

In order to find the equivalence between the concordance-discordance approach and the decision rule approach we have solved the problem using ELECTRE III with the preferential information shown in Table 1.

Table 1. Criteria and preferential information of water supply problem, Roy, Słowiński, Treichel (1992)

Criterion	Weight	Indifference threshold	Preference threshold	Veto threshold	Direction of preference
g_1	8	2	3.5	7	gain
g_2	5	$0.05 * g_2$	$0.1 * g_2$	$0.5 * g_2$	gain
g_3	6	0	4	8	gain
g_4	6	$0.1 * g_4 + 0.07$	$0.2 * g_4 + 0.1$	$0.25 * g_4 + 0.4$	gain
g_5	6	$0.05 * g_5 + 20$	$0.1 * g_5 + 50$	$0.2 * g_5 + 100$	gain
g_6	2	2	4	10	gain
g_7	1	0	2.5	10	gain

Table 2. Actions and their performances in space of criteria

User	g_1	g_2	g_3	g_4	g_5	g_6	g_7
U1	0	2	12	0.82	270	0	5
U2	0	1.6	0	0.2	50	0	5
U3	1	1.5	0	0.17	21	5	5
U4	1	2.1	12	0.76	240	7	2
U5	0	2.2	8	0.3	171	2	2
U6	5	1.45	0	1.09	54	2	2
U7	0	1.9	4	0.26	45	5	2
U8	1	1.9	4	0.42	332	5	0
U9	3	1.45	8	0.17	28	2	0
U10	3	1.65	0	0.2	15	7	0
U11	7	1.65	0	0.21	27	5	0
U12	3	2	6	0.21	66	5	0
U13	3	1.75	4	0.17	78	5	0
U14	3	1.8	6	0.15	51	5	0
U15	5	1.4	0	0.33	24	7	0
U16	3	1.75	6	0.26	108	5	0
U17	1	2	8	0.19	87	7	0
U18	0	1.2	0	0.56	57	7	0
U19	1	2.1	12	0.31	117	7	0
U20	3	1.9	6	0.46	55	7	0
U21	3	1.7	0	0.19	18	7	0

The obtained median preorder is shown in Table 3.

Table 3. Median preorder of actions

Rank	Action	Rank	Action
1	**U4**	**12**	**U15**
2	U8	13	U13
3	**U1**	14	U14
4	U11	**15**	**U18**
5	**U20**	16	U21
6	U6	17	U7
7	U19	**18**	**U10**
8	**U5**	19	U2
9	U16		U9
10	U12	**20**	**U3**
11	U17		

In order to generate the PCT table, we have decided to select 8 (reference) actions, shown in bold in Table 3. When comparing the reference actions by pairs, we used the order in which they appear in the median preorder and the preferential information shown in Table 4.

Table 4. Preferential information used to generate decision rules

Criterion	Indifference threshold
g_1	2
g_2	$0.05 * g_2$
g_3	0
g_4	$0.1 * g_4 + 0.07$
g_5	$0.05 * g_5 + 20$
g_6	2
g_7	0

After constructing the approximations of S and S^c by dominance relation, we generated decision rules from lower approximations using the all-rule type algorithm. For further analysis we selected the strongest rules covering all pairs from PCT and having the smallest intersection of sets of covered pairs. These rules are shown in Table 5.

In the second column of Table 5, the number of the point from Section 7 relevant to the interpretation of the corresponding rule is given. First we will

Table 5. Decision rules

Rule no.	Relevant point	Rule body
1	4a	if $(T_1 \leq -7) \rightarrow S^c$
2	4a	if $(T_6 \leq -5) \rightarrow S^c$
3	4a	if $(T_2 \leq -0.55) \rightarrow S^c$
4	4b	if $(T_3 \leq -4)\&(T_4 \leq -0.13)\&(T_5 \leq -30) \rightarrow S^c$
5	4b	if $(T_1 \leq -3)\&(T_5 \leq -28) \rightarrow S^c$
6	4b	if $(T_1 \leq -3)\&(T_3 \leq -2) \rightarrow S^c$
7	4b	if $(T_1 \leq 0)\&(T_2 \leq -0.099)\&(T_4 \leq -0.26) \rightarrow S$
8	4b	if $(T_1 \leq -3)\&(T_2 \leq -0,099) \rightarrow S^c$
9	1	if $(T_1 = 0)\&(T_2 = 0)\&(T_4 = 0) \rightarrow S$

try to search for the veto thresholds for all criteria. Conditions on the veto thresholds, following from the interpretation of particular rules are given below:

$v_1 \geq 7$ (from rule 1),
$v_6 \geq 5$ (from rule 2),
$v_2 \geq 0.55$ (from rule 3),
$v_3 > 4$ (from rule 4),
$v_4 > 0.13$ (from rule 4),
$v_5 > 30$ (from rule 4),
$v_1 > 3$ (from rule 5),
$v_5 > 28$ (from rule 5),
$v_1 > 3$ (from rule 6),
$v_3 > 2$ (from rule 6),
$v_1 > 0$ (from rule 7),
$v_2 > 0.099$ (from rule 7),
$v_4 > 0.26$ (from rule 7),
$v_1 > 3$ (from rule 8),
$v_2 > 0.099$ (from rule 8).

The feasible values of the veto thresholds are shown in Table 6. For comparison with the ELECTRE model, the mean values of veto thresholds used to get the median preorder, are also shown in this table.

Table 6. Veto thresholds on particular criteria

Criterion	g_1	g_2	g_3	g_4	g_5	g_6	g_7
Veto thresholds from rule interpretation	≥ 7	≥ 0.55	> 4	> 0.26	> 30	≥ 5	–
Mean veto thresholds from ELECTRE III	7	0.825	8	0.555	134.7	10	10

Analysis of the rules from the viewpoint of weights gives the following set of inequalities:

$$w_2 + w_3 + w_4 + w_6 + w_7 < \lambda \Sigma w_i \qquad \text{(from rule 5)},$$
$$w_2 + w_4 + w_5 + w_6 + w_7 < \lambda \Sigma w_i \qquad \text{(from rule 6)},$$
$$w_3 + w_5 + w_6 + w_7 < \lambda \Sigma w_i \qquad \text{(from rule 7)},$$
$$w_3 + w_4 + w_5 + w_6 + w_7 < \lambda \Sigma w_i \qquad \text{(from rule 8)},$$
$$w_1 + w_2 + w_4 \geq \lambda \Sigma w_i \qquad \text{(from rule 9)}.$$

If we assume the value of cutting level λ equal to 0.6, one of the possible solutions of the above set of inequalities is:

$$w_1 = 8.9,$$
$$w_2 = 5.9,$$
$$w_3 = 5.2,$$
$$w_4 = 6.05,$$
$$w_4 = 5.2,$$
$$w_4 = 1.2,$$
$$w_4 = 1.$$

We have applied the considered subset of decision rules to the whole set A of 21 actions (users). To exploit the result of this application we have used the Net Flow Score exploitation procedure described in Greco, Matarazzo, Słowiński, Tsoukias (1998).

The use of this exploitation procedure gives the final preorder as in Table.

Table 7. Ranking of actions obtained using decision rules and Net Flow Score exploitation procedure

Rank	Action	Rank	Action
1	U4	11	U13
2	U1	12	U11
3	U19	13	U7
4	U8	14	U18
	U20	15	U21
5	U5	16	U10
6	U17	17	U9
7	U6	18	U15
8	U12	19	U2
9	U16	20	U3
10	U14		

The results obtained using the ELECTRE III method and the decision rules approach are very similar. Also the preferential parameters inferred from the rules (veto thresholds and weights) are in accordance with the original parameters used to generate the median preorder.

9. Conclusions

We tried to show that there exists an equivalence between the decision rule model and the concordance-discordance model used in ELECTRE methods. Theoretical considerations have been illustrated by an example in which we inferred the weights and veto thresholds from decision rules obtained using the dominance-based rough set approach and proved that they correspond to the preferential parameters of ELECTRE III.

The property of investigated equivalence may also be useful to generate a set of decision rules compatible with the concordance-discordance test used in ELECTRE for specific preferential parameters. This equivalence could be an additional argument for adopting a considered set of rules as the preference model.

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