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Collective choice rules in group decision making under fuzzy preferences and fuzzy majority: a unified OWA operator based approach

by

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Abstract: A general form of a collective choice rule in group decision making under fuzzy preferences and a fuzzy majority is proposed. It encompasses some well-known choice rules. Our point of departure is the fuzzy majority based linguistic aggregation rule (solution concept) proposed by Kacprzyk (1985a,b; 1986). This rule is viewed here from a more general perspective, and the fuzzy majority – meant as a fuzzy linguistic quantifier – is dealt with by using Yager's (1988) OWA operators. The particular collective choice rules derived via the general scheme proposed are shown to be applicable in the case of nonfuzzy preferences too.

Keywords: group decision making, fuzzy preferences, fuzzy majority, OWA operators.

1. Introduction

In this paper we consider the problem of group decision making which, for the purposes of our paper, can be briefly stated as follows. We have a set of M options, $S = \{s_1, \dots, s_M\}$, and a set of N individuals, $X = \{x_1, \dots, x_N\}$. Each individual x_k from X provides his or her preferences over S. Since these preferences may be not clear – cut, their representation by *individual fuzzy preference relations* is strongly advocated (see, e.g., the articles in Kacprzyk and Fedrizzi, 1986, 1988, 1989; Fodor and Roubens, 1994; Ovchinnikov, 1990; etc.).

A *fuzzy preference relation* of an individual x_k, R_k, is given by its membership function $\mu_{R_k} : S \times S \to [0,1]$ such that $\mu_{R_k} \in [0,1]$ denotes the strength of preference. If card S is small enough (as assumed here), R_k may be represented by a matrix $[r^k_{ij}]$, $r^k_{ij} = \mu_{R_k}(s_i, s_j)$; $i,j = 1, \dots, M$; $k = 1, \dots, N$. R_k is commonly assumed (also here) reciprocal, i.e. $r^k_{ij} + r^k_{ji} = 1$; moreover, $r^k_{ii} = 0$, for all i, j, k.

The fuzzy preference relations, similarly as their nonfuzzy counterparts, are evidently a point of departure for devising a multitude of solution concepts.

Basically, two lines of reasoning may be followed here (see Kacprzyk, 1985a,b; 1986):

– a direct approach

$$\{R_1, \ldots, R_N\} \rightarrow \text{ solution, and}$$

– an indirect approach

$$\{R_1, \ldots, R_N\} \rightarrow R \rightarrow \text{ solution,}$$

that is, in the first case we determine a *solution* just on the basis of individual fuzzy preference relations, and in the second case we form first a social fuzzy preference relation R which is then used to find a solution.

A solution (a nonfuzzy or, more generally and adequately in this setting, fuzzy set of options) is here not clearly understood. For instance, Kacprzyk (1985a,b; 1986) introduced the core for the direct approach and the *consensus winner* for the indirect approach, using a fuzzy majority represented by a linguistic quantifier. For a comprehensive account of a wider variety of solution concepts, see, e.g., Nurmi (1987, 1988).

It is easy to see that the direct and indirect approach mentioned above are equivalent to the determination of a (collective) choice function (see Aizerman and Aleskerov, 1995) acting from S to a family of all nonfuzzy or fuzzy subsets of S, including clearly the empty set (nonfuzzy or fuzzy).

In this paper we propose a general scheme of collective choice rule that covers a number of well-known rules. Our point of departure is the choice (aggregation) rule as proposed by Kacprzyk (1985a,b; 1986). We reconsider this rule on a more abstract level and use the OWA operators instead of the originally employed linguistic quantifiers in the sense of Zadeh (1983). We obtain a general form of a choice rule. We consider individual fuzzy preference relations as a point of departure. Thus, since nonfuzzy preference relations are a special case of fuzzy preference relations, then all collective choice rules derived from our general scheme are applicable either to nonfuzzy (classic) or fuzzy preferences.

Now, we will first introduce the notation used and recall the concept of a fuzzy preference relation. Second, we will briefly discuss the concept of a linguistic quantifier, and a fuzzy majority to be used in a specific, fuzzy (linguistic) collective choice rule. Finally, we will propose a general form of a collective choice rule and discuss some special cases.

2. Fuzzy preference relations

For generality, we assume that the preferences to be aggregated are represented as fuzzy preference relations. Clearly, nonfuzzy preference relations are their special case so that our discussion will be applicable to the fuzzy and nonfuzzy case. Fuzzy relations allow for a more natural expression of individual preferences that are often not clear-cut but to a degree. The following notation will be used. Let $S = \{s_1, \dots, s_M\}$ be a finite set of options and $X = \{x_1, \dots, x_N\}$ the set of individuals. A fuzzy preference relation R is a fuzzy subset of $S \times S$ characterized by the membership function:

$$\mu_R(s_i, s_j) = \begin{cases} 1 & \text{definite preference} \\ c \in (0.5, 1) & \text{preference to some extent} \\ 0.5 & \text{indifference} \\ d \in (0, 0.5) & \text{preference to some extent} \\ 0 & \text{definite preference.} \end{cases}$$

The degree of preference, $\mu_R(s_i, s_j)$, is here interpreted in a continuous manner, i.e. when the value of $\mu_R(s_i, s_j)$ function changes from the one slightly below 0.5 to the one slightly above 0.5, there is no abrupt change of its meaning – both values more or less correspond to the indifference. In the other words, the particular values of a membership function $\mu_R(s_i, s_j)$ express some uncertainty as to the actual preferences, highest in the case of 0.5 and lowest in the case of 1.0 and 0.0.

The particular values of a membership function $\mu_R(s_i, s_j)$ may be interpreted in a different way. For example, Nurmi (1981) assumes, that $\mu_R(s_i, s_j) > 0.5$ means a definite preference of s_i over s_j and the particular values from the $(0.5, 1]$ interval express the intensity of this preference. In what follows, we refer to this interpretation as Nurmi's interpretation.

Usually, the fuzzy preference relation is assumed to meet certain conditions, most often those of reciprocity and transitivity. For our main result, none of these properties are directly relevant. Nevertheless, for some parts of our presentation it is suitable to assume the reciprocity property, i.e., we will assume that the following condition holds

$$\mu_R(s_i, s_j) + \mu_R(s_j, s_i) = 1, \ \forall i \neq j.$$

Such relations are known as fuzzy tournaments, see Nurmi and Kacprzyk (1991). Assuming a reasonable (small) cardinality of S, it is convenient to represent a preference relation R_k of the individual k as a matrix (table):

$$[r_{ij}^k] = [\mu_{R_k}(s_i, s_j)], \ \forall i, j, k.$$

3. Fuzzy majority, linguistic quantifiers and the OWA operators

Fuzzy majority constitutes a natural generalization of the concept of majority in the case of a fuzzy setting within which a group decision making problem is considered. A fuzzy majority was introduced into group decision maing under fuzziness by Kacprzyk (1985a,b; 1986), and then considerably extended in the works of Fedrizzi, Herrera, Herrera-Viedma, Kacprzyk, Nurmi, Verdegay, Zadrożny etc. (see, e.g., a review by Kacprzyk and Nurmi, 1988, and papers cited in the bibliography).

Basically, Kacprzyk's (1985a,b; 1986) idea was to equate fuzzy majority with fuzzy linguistic quantifiers which often appear in a natural language discourse. Linguistic (fuzzy) quantifiers exemplified by expressions like "most", "almost all", etc. allow for a more flexible quantification than the classical general and existential quantifiers.

There exist a few approaches to the linguistic quantifiers modeling. Basically, we are looking for the truth of a proposition:

"Most objects posses a certain property"
that may be formally expressed as follows:

$$\underset{x \in X}{Q} P(x), \tag{1}$$

where Q denotes a fuzzy linguistic quantifier (in this case "most"), $X = \{x_1, \ldots, x_m\}$ is a set of objects, $P(\cdot)$ corresponds to the property. It is assumed that the property P is fuzzy and its interpretation may be informally equated with a fuzzy set and its membership function, i.e.:

$$\operatorname{truth}(P(x_i)) = \mu_P(x_i)$$

The first approach, proposed by Zadeh (1983, 1987), is called calculus of linguistically quantified propositions. Here, a linguistic quantifier is represented as a fuzzy set $Q \in F([0,1])$, where $F(A)$ denotes the family of all fuzzy sets defined on A. For our purposes and for some practical reasons, its membership function should be assumed piece-wise linear. Thus the fuzzy set corresponding to the fuzzy quantifier Q ("most") may be defined by, e.g., the following membership function:

$$\mu_Q(y) = \begin{cases} 1 & \text{for } y \geq 0.8 \\ 2y - 0.6 & \text{for } 0.3 < y < 0.8 \\ 0 & \text{for } y \leq 0.3. \end{cases} \tag{2}$$

The truth of the proposition (1) is determined from:

$$\operatorname{truth}(QP(x)) = \mu_Q\Big(\sum_{i=1}^{m} \mu_P(x_i)/m\Big) \tag{3}$$

where $m = \operatorname{card}(X)$.

Another approach to the modeling of fuzzy linguistic quantifiers is by using Yager's OWA (ordered weighted averaging) operators (1988, 1994) (see also Yager and Kacprzyk's, 1997 volume).

An OWA operator O of dimension n is defined as:

$$O : \Re^n \to \Re$$

$$W = [w_1, \ldots, w_n], \quad w_i \in [0,1], \quad \sum_{i=1}^{n} w_i = 1$$

$$O(a_1, \ldots a_n) = \sum_{j=1}^{n} w_j b_j, \quad b_j \text{ is } j\text{-th largest of the } a_i.$$

Thus, an OWA operator is fully defined by its vector of weights W. The correspondence between an OWA operator (its vector of weights) and a fuzzy linguistic quantifier in Zadeh's sense is given by the well-known formula:

$$w_i = \mu_Q\left(\frac{i}{n}\right) - \mu_Q\left(\frac{i-1}{n}\right). \tag{4}$$

Basically, using this formula we may define an OWA operator that behaves (in the sense of its aggregating behavior) similarly to a Zadeh's linguistic quantifier given by the membership function μ_Q.

The OWA operators provide us with a convenient, compact and simple, representation of the classical quantifiers, i.e. the general and existential, respectively:

$$\begin{aligned} \forall &\to W = [0, \ldots, 0, 1] \quad O_\forall \\ \exists &\to W = [1, 0, \ldots, 0] \quad O_\exists \end{aligned}$$

For our purposes, related to group decision making, the following vectors of weights define some other OWA operators that correspond to:

- *classic crisp majority* (at least a half) O_{maj}

  $W = [0, \ldots, 0, 1, 0, \ldots, 0] \quad w_{(n/2)+1}$ or $w_{(n+1)/2} = 1$

- *average* O_{avg}

  $W = [1/n, \ldots, 1/n]$

- *most* O_{most}
  the weight vector may be, e.g., calculated using (1) and (4).

4. Collective choice rules under fuzzy majority

As we may remember from Section 1, a collective choice rule describes how to determine a set of preferred options starting from the set of individual preference relations. Thus, it may be informally represented as follows:

$$\{R_1, \ldots, R_N\} \to 2^S$$

Notice that this expression reflects the direct approach to the determination of a solution. In fact, for our discussion later on its is not important if we assume the collective choice function to be derived directly as above, i.e. via $\{R_1, \ldots, R_N\} \rightarrow 2^S$ or via the indirect approach $\{R_1, \ldots, R_N\} \rightarrow R \rightarrow 2^N$. It is only important that the individual preferences should somehow be aggregated so as to produce a set of options satisfying preferences of all involved parties according to some rationality principles. Here, we do not care if there are some intermediate steps in the process of choice. For example, the rule may first require creation of a group (collective) preference relation and only then – using this relation – select a set of options. Moreover, some interesting and popular rules are meant just for producing group preference relations leaving the choice of a "best" option as irrelevant or obvious (e.g., social welfare functions – see Sen, 1970).

In cases where the group preference relations are required to be linear orderings we will assume that the option(s) that is (are) first in that ordering are selected.

One of the most popular rules of aggregation is the simple majority rule (known also as the Condorcet rule) – see Nurmi (1987). Basically, it is assumed to work for linear orderings and produce group linear ordering (what is not always possible, in general). Thus, this rule may be described by the following formulas:

$$R(s_i, s_j) \Leftrightarrow \mathrm{Card}\{k : R_k(s_i, s_j)\} \geq \mathrm{Card}\{k : R_k(s_j, s_i)\} \tag{5}$$

$$S_0 = \{s_i \in S : \underset{i \neq j}{\forall} R(s_i, s_j)\}, \tag{6}$$

where $\mathrm{Card}\{A\}$ denotes cardinality of the set A and S_0 is the set of collectively preferred options. As a counterpart for this rule in the fuzzy case Nurmi (1981) proposed the following rule:

$$R(s_i, s_j) \Leftrightarrow \mathrm{Card}\{k : \mu_{R_k}(s_i, s_j) > \alpha \geq 0.5\} \geq \text{threshold} \tag{7}$$

$$S_0 = \{s_i \in S : \neg \underset{j}{\exists} R(s_j, s_i)\}. \tag{8}$$

Therefore, Nurmi (1981) restated (5) adapting it to the case of a fuzzy relation R and employing a more flexible concept of majority defined by a threshold. Notice that in (8) still the strict quantifying is used (referred here to the concept of a non-domination).

Kacprzyk (1985a,b; 1986) interpreted the rule (5)–(6) by employing the concept of a fuzzy majority equated with a linguistic quantifier. He introduced the concept of a Q-core that may be informally stated in a slightly modified version, as the $Q1/Q2$-core (see Zadrożny, 1996) as:

$CC_{Q1,Q2}$: *Set of options, which are for most (Q2) of individuals "better" than most (Q1) of the rest of options from the set S.*

$$CC_{Q1,Q2} \in F(S), \quad \mu_{CC_{Q1,Q2}}(s_i) \rightarrow \underset{s_j}{Q1} \underset{x_k \in X}{Q2} R_k(s_i, s_j). \tag{9}$$

Then, using Zadeh's fuzzy linguistic quantifiers, we obtain:

$$h_i^j = \frac{1}{N}\sum_{k=1}^{N} r_{ij}^k \qquad h_i = \frac{1}{M-1}\sum_{\substack{j=1\\ j\neq i}}^{M} \mu_{Q2}(h_j^k) \tag{10}$$
$$\mu_{CC_{Q1,Q2}}(s_i) = \mu_{Q1}(h_i),$$

where h_i^j denotes the degree to which, in the opinion of all individuals, option s_i is better than option s_j; h_i denotes the degree to which, in the opinion of all individuals, option s_i is better than most $(Q1)$ other options; $\mu_{Q1}(h_j)$ denotes the degree (to be determined) to which, in the opinion of most $(Q2)$ individuals, option s_i is better than most $(Q1)$ other options.

Formula (9) serves as a prototype for our generic collective choice rule proposed in the next section.

5. A classification of collective choice rules

It turns out that the $Q1/Q2$-core rule given by (9) and (10) may me viewed as a generic form for many well-known aggregation rules that employ, more or less explicitly, only classical quantifiers. Thus, in order to cover them by our generic rule given by (9) and (10), we would rather use OWA operators instead of linguistic quantifiers in Zadeh's sense. Thus, using the notation from Section 3, first transform (9) into:

$$\underset{s_j}{Q1}\ \underset{x_k\in X}{Q2}\ R_k(s_i,s_j) \rightarrow O^j_{most}O^k_{most}R_k(s_i,s_j)$$

In our next discussion j and k will be indexing the set of options and individuals, respectively. Thus, $O^j_{most}(O^k_{most})$ denotes an OWA operator aggregating some values for all options (individuals) and governed by the weight vector indicated by the lower index, i.e. corresponding to the linguistic quantifier *most* with weights determined by (4).

Now, the generic collective choice rule (CCR) proposed in this paper may be expressed as follows:

$$\mu_{CCR}(s_i) = O_1O_2R_k(s_p,s_q).$$

This form has a number of "degrees of freedom". Namely, specific collective choice rules may be recovered by specifying:

1. what are the upper indexes of the OWA operators, i.e., if we first aggregate over the individuals and then over the options or in the opposite way,
2. what are weight vectors of both OWA operators,
3. whether the pair of option indexes (p,q) corresponds to (i,j) or to (j,i)

Therefore, we can basically distinguish four types of collective choice rules:

I. $\mu_{CCR}(s_i) = O_1^kO_2^jR_k(s_i,s_j)$
II. $\mu_{CCR}(s_i) = O_1^jO_2^kR_k(s_i,s_j)$

III. $\mu_{CCR}(s_i) = O_1^k O_2^j R_k(s_j, s_i)$
IV. $\mu_{CCR}(s_i) = O_1^j O_2^k R_k(s_j, s_i)$

In order to identify the classical rules covered by this generic scheme, which are meant to provide a nonfuzzy set of options as a solution, we have to propose a way to determine such a nonfuzzy set of preferred options having a fuzzy set represented by the membership function μ_{CCR}. This may be done in the following way:

- for type I and II rules choose s_i such that $\mu_{CCR}(s_i) = \max_j \mu_{CCR}(s_j)$
- for type III and IV rules choose s_i such that $\mu_{CCR}(s_i) = \min_j \mu_{CCR}(s_j)$.

Now we can mention some well-known rules covered by our generic form of the collective choice rule. In the sequel we will use some specific OWA operators as defined at the end of Section 3. Most of these rules assume the individual preferences in the form of linear orderings and we will comment upon them in these terms.

First, let us list some rules which may be classified as type I as well as type II:

1. $O_\forall O_\forall$ – a "consensus solution",
2. $O_{avg} O_{avg}$ – Borda's rule.

On the other hand, the following rule may be classified as type III or IV:

3. $O_\exists O_\exists$ – the minimax degree set (Nurmi) (1981).

Now, let us show some type I rules:

4. $O_{avg}^k O_\forall^j$ – the plurality voting,
5. $O_{maj}^k O_\forall^j$ – the qualified plurality voting,
6. $O_{avg}^k O_{maj}^j$ – the approval voting-like,
   provided that O_{maj}^j models the individuals' behavior;
   O_{most}^j leads to a cumulative variant,
7. $O_\forall^k O_{maj}^j$ – the "consensus+approval voting".

Some examples of type II rules are:

8. $O_\forall^j O_{maj}^k$ – the simple majority (Condorcet's rule),
9. $O_\forall^j O_\exists^k$ – the Pareto rule,
10. $O_{avg}^j O_{maj}^k$ – Copeland's rule.

An example of a type III rule is:

11. $O_{most}^k O_{avg}^j$ – Kacprzyk's (1985a,b; 1986) Q-minimax set.

And finally, some type IV rules are:

12. $O_\exists^j O_{avg}^k$ – the minimax set (see Nurmi, 1981)

13. $O^j_\forall O^k_{maj}$ – the Condorcet loser,
14. $O^j_\exists O^k_\forall$ – the Pareto inferior options.

Thus, the generic scheme proposed in this paper covers some classical rules, particularly well–known in the context of voting (see Nurmi, 1987). Some of those rules are not, strictly speaking, collective choice rules. For example, rules 13 and 14 produce sets of options that may be viewed as being collectively rejected rather than collectively selected.

6. Concluding remarks

In the paper we proposed a generic form of a collective choice rule. By using the OWA operators that can easily represent both fuzzy and nonfuzzy majorities that correspond to some specific sets of weights, the generic collective choice rule proposed can represent, as its specific cases, both choice rules derived in the context of group decision making under fuzziness (see Kacprzyk and Nurmi, 1988, for a comprehensive review) and also many classical collective choice rules (see Nurmi, 1987). The generic form of a collective choice rule makes also possible to identify new rules, as exemplified by rule 7.

The proposed scheme may give a better insight into the preference aggregation process aimed at deriving group decision making solution concepts. Although some classical rules are defined using crisp quantifiers and nonfuzzy individual preference relations, their counterpart emerging from our generic form may be directly applied to fuzzy preferences.

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