Minimization of electric and magnetic losses in the speed control of induction motors

by

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Abstract: The article presents a minimization method of electric energy losses in rotor and stator windings and in the magnetic core of the motor during the frequency speed control of induction motors which limits the amplitude of the stator current. To solve this problem, the hybrid algorithm of parametric optimization, the mathematical model of the induction motor and the Pontryagin maximum principle are used. In the mathematical model of the motor both electromagnetic transients and the motor magnetic material saturation are taken into consideration. The electric energy losses in the magnetic core and in the windings of the motor are also calculated. An application example of the method developed for the idle starting of an induction motor is given.

Keywords: optimal control, induction motors, genetic algorithms.

1. Introduction

The electric energy losses during the frequency speed control of induction motors can be divided into the losses in the stator and rotor windings (called electric losses or losses in the copper) and losses in the magnetic core of the motor caused by the eddy currents and the hysteresis of the magnetic material of the motor (called magnetic losses or losses in the iron). Electric losses depend on the amplitudes of the stator and rotor currents, while magnetic losses depend on the frequencies of these currents.

It is assumed in many papers (for example Schreiner, Gildebrand, 1973, Kawecki, Niewierowicz, 1996, 1999) that magnetic losses can be neglected since they are small in comparison to the losses in the copper. This is true, for example, for the ordinary starting of the induction motor. However, when

variation of the supply current (or of the supply voltage) in order to limit the amplitude of the stator current and/or to optimize the control from the point of view of an optimization index as, for example, the minimization of the control time, the minimization of the losses in the copper, the minimization of the losses in the iron etc., the contribution of magnetic losses in the total losses (in copper and in iron) cannot always be neglected. In these cases the magnetic losses can reach high values, far higher even than the electric losses (Kawecki, 1998). Therefore it is important to take into consideration both electric and magnetic losses, when minimizing electric energy losses during the speed control of induction motors.

The limitation of the amplitude of the stator current, must not only be considered important from the point of view of the cont out for the frequency converter that supplies the motor, but the fact that it avoids the possible saturation of the magnetic material of the induction motor has also to be contemplated. This second factor is important only when a mathematical model of the motor, which does not consider this saturation, is used in the synthesis of the optimal control. In this case, to avoid saturation, we must conform with the fact that the limitation of the stator current amplitude will be of relatively small value, which in turn will increase the control time, and the electric and magnetic losses will consequently increase too (Kawecki, Niewierowicz, 1999). For this reason, in this article a mathematical motor model is used, which takes the saturation of the magnetic material into consideration.

2. Mathematical model of the motor

The mathematical model of the induction motor is based on the following assumptions:

1) The supply source is a sinusoidal symmetric current frequency converter.
2) The induction motor is symmetric with the smooth rotor (of squirrel cage) and with the stator and rotor windings connected in star form, without the neuter driver.
3) The resistances and the inductances are constant (the influence of the current displacement effect and the thermal changes of the resistances can be neglected).

Under these assumptions, the mathematical model of the double-phase equivalent induction motor on the $d - q$ axes, rotating synchronously with the rotor, may be described by the following equations (Kovac, Rac, 1963, Krause, 1997, Sandler, Sarbatov, 1973):[1]

$$\frac{d\psi'_{2d}}{dt} = \frac{R'_2 X_o}{X_o + X'_2} i_1 \cos\xi - \frac{R'_2 \omega_n}{X_o + X'_2} \psi'_{2d} - \frac{R'_2 \omega_n}{X_o + X'_2} f(\psi_{od}) \tag{1}$$

$$\frac{d\psi'_{2q}}{dt} = \frac{R'_2 X_o}{X_o + X'_2} i_1 \sin\xi - \frac{R'_2 \omega_n}{X_o + X'_2} \psi'_{2q} - \frac{R'_2 \omega_n}{X_o + X'_2} f(\psi_{oq}) \tag{2}$$

$$\frac{d\omega_r}{dt} = \frac{3p^2 \omega_n X_o}{2JF(X_o + X'_2)} \left\{ \frac{Fi_1}{\omega_n} [\psi'_{2d} \sin\xi - \psi'_{2q} \cos\xi] \right.$$

$$\left. + X'_2 [\psi'_{2d} f(\psi_{od}) - \psi'_{2q} f(\psi_{oq})] \right\} - \frac{p}{J} M_o \tag{3}$$

$$f(\psi_{od}) = \begin{cases} 0 & \text{for } \psi_o = 0 \\ \dfrac{\psi_{od}}{\psi_o} f(\psi_o) & \text{for } \psi_o \neq 0 \end{cases} \tag{4}$$

$$f(\psi_{oq}) = \begin{cases} 0 & \text{for } \psi_o = 0 \\ \dfrac{\psi_{oq}}{\psi_o} f(\psi_o) & \text{for } \psi_o \neq 0 \end{cases} \tag{5}$$

$$\psi_o = \sqrt{\psi_{od}^2 + \psi_{oq}^2} \tag{6}$$

$$f(\psi_o) = \begin{cases} 0 & \text{for } i_o \leq i_{o\,max} \\ \dfrac{X_o}{\omega_n} (i_o - i_{o\,max}) & \text{for } i_o > i_{o\,max} \end{cases} \tag{7}$$

$$i_{o\,max} = \frac{u_m}{\sqrt{R_1^2 + (X_o + X_1)^2}} \tag{8}$$

$$i'_{2d} = \frac{1}{X_o + X'_2} [\omega_n \psi'_{2d} - X_o i_1 \cos\xi + \omega_n f(\psi_{od})] \tag{9}$$

$$i'_{2q} = \frac{1}{X_o + X'_2} [\omega_n \psi'_{2q} - X_o i_1 \sin\xi + \omega_n f(\psi_{oq})] \tag{10}$$

$$i'_2 = \sqrt{i'^2_{2d} + i'^2_{2q}} \tag{11}$$

$$i_{1d} = i_1 \cos\xi(t) \tag{12}$$

$$i_{1q} = i_1 \sin\xi(t) \tag{13}$$

$$i_o = \sqrt{(i_{1d} + i'_{2d})^2 + (i_{1q} + i'_{2q})^2} \tag{14}$$

$$u_{1d} = \frac{X_1}{\omega_n} \frac{di_1}{dt} \cos\xi + \frac{X_1}{\omega_n} i_1 \frac{d(\cos\xi)}{dt} + \frac{d\psi_{od}}{dt} + R_1 i_1 \cos\xi$$

$$- \omega_r \frac{X_1}{\omega_n} i_1 \sin\xi - \omega_r \psi_{oq} \tag{15}$$

$$u_{1q} = \frac{X_1}{\omega_n} \frac{di_1}{dt} \sin\xi + \frac{X_1}{\omega_n} i_1 \frac{d(\sin\xi)}{dt} + \frac{d\psi_{oq}}{dt} + R_1 i_1 \sin\xi$$

$$- \omega_r \frac{X_1}{\omega_n} i_1 \cos\xi - \omega_r \psi_{od} \tag{16}$$

$$u = \sqrt{u_{1d}^2 + u_{1q}^2} \tag{17}$$

$$\omega(t) = \omega_r(t) + \frac{d\xi}{dt} = \omega_r(t) + \beta'(t) = \omega_r(t) + \beta(t)\omega_n \tag{18}$$

$$\beta'(t) = \frac{d\xi}{dt} = \cos\xi\frac{d(\sin\xi)}{dt} - \sin\xi\frac{d(\cos\xi)}{dt} \tag{19}$$

$$\beta(t) = \frac{\beta'(t)}{\omega_n}. \tag{20}$$

3. Electric energy losses

The electric losses in the three-phase motor during a control time t_r, are expressed by the following relationship (Kawecki, Niewierowicz, 2002):

$$
\begin{aligned}
Q_e &= \frac{3}{2}\int_0^{t_r} R_1 i_1^2 dt \\
&+ \frac{R_2'}{(X_o + X_2')^2}\int_0^{t_r} \left\{ \begin{array}{l} [\omega_n(\psi_{2d}' + f(\psi_{od})) - X_o i_1 \cos\xi]^2 + \\ +[\omega_n(\psi_{2q}' + f(\psi_{oq})) - X_o i_1 \sin\xi)]^2 \end{array} \right\} dt
\end{aligned}
\tag{21}
$$

The magnetic losses in the core of the motor Q_m during the control time t_r, can be described by the following relationship (Turowski, 1993, Florillo, Novikov, 1990, Berotti et al., 1991, Amaro, Kaczmarek, 1995):

$$
\begin{aligned}
Q_m &= B_m^2 \int_0^{t_r} [A_z M_m \omega^2 + (B_z M_m - 2M_r A_z \omega_r)\omega \\
&+ M_r(A_z\omega_r - B_z)]\omega_r dt.
\end{aligned}
\tag{22}
$$

4. Optimization index

This paper's objective is to find the speed control of the induction motor, which minimizes the total electric energy losses (electric and magnetic losses) limiting the amplitude of the stator current to the desired value.

The optimization index Q must be a functional that takes into consideration both types of losses, and, as having two components, it then can be considered as a vector optimization index:

$$Q = \left[\begin{array}{c} Q_e \\ Q_m \end{array} \right] \tag{23}$$

In this paper, the transformation method of the vector optimization index to the scalar optimization index, is accepted. This consists in presenting the vector criterion (23) in the form of the linear combination of its components (Salukvadze, 1975):

$$Q = \lambda_1 Q_e + \lambda_2 Q_m \tag{24}$$

$$\lambda_1 > 0, \lambda_2 > 0 \;\; \} \tag{25}$$

Taking into consideration (21) and (22), the optimization index (24) will result in the following relationship:

$$Q = \int_0^{t_r} \left[\lambda_1 \frac{3}{2} \left\{ R_1 i_1^2 + \frac{R_2'}{(X_o + X_2')^2} \left\{ \begin{bmatrix} \omega_n(\psi_{2d}' + f(\psi_{od})) + \\ -X_o i_1 \cos \xi \end{bmatrix}^2 + \\ + \begin{bmatrix} \omega_n(\psi_{2q}' + f(\psi_{oq})) + \\ -X_o i_1 \sin \xi \end{bmatrix}^2 \right\} \right\} \right.$$
$$\left. + \lambda_2 B_m^2 \begin{bmatrix} A_z M_m \omega^2 + \\ +(B_z M_m - 2M_r A_z \omega_r)\omega + \\ +M_r(A_z \omega_r - B_z)\omega_r \end{bmatrix} \right] dt \qquad (26)$$

5. Optimal control

The optimization problem consists in finding a mathematical description of the induction motor speed control in the open loop system that minimizes the electric and magnetic losses, limiting the amplitude of the stator current. Supposing that the frequency converter, which supplies the motor, is a current source and that it is also a proportional system, the solution of the optimization problem consists in the search of the control:

$$i_1 = i_1(t) \qquad (27)$$
$$\xi = \xi(t) \qquad (28)$$

that minimizes the optimization index (26) and limits the stator current amplitude:

$$i_1 \leq i_1^o \qquad (29)$$

To solve this problem the Pontryagin maximum principle is used (Pontryagin, Boltianski, 1962, Athans, Falb, 1966).

Taking the mathematical model of the motor (1)–(3) and the optimization index (26) into consideration, the hamiltonian will take the following form:

$$H = -\lambda_1 \frac{3}{2} \left\{ i_1^2 R_1 + \frac{R_2'}{(X_o + X_2')^2} \begin{bmatrix} (\omega_n \psi_{2d}' - X_o i_1 \cos \xi + \omega_n f(\psi_{od}))^2 \\ +(\omega_n \psi_{2q}' - X_o i_1 \sin \xi + \omega_n f(\psi_{oq}))^2 \end{bmatrix} \right\}$$
$$- \lambda_2 B_m^2 \begin{bmatrix} A_z M_m \left(\omega_r + \frac{d\xi}{dt} \right)^2 + (B_z M_m - 2M_r A_z \omega_r) \left(\omega_r + \frac{d\xi}{dt} \right) \\ +M_r (A_z \omega_r - B_z) \omega_r \end{bmatrix}$$
$$+ V_1 \left[\frac{R_2' X_o}{X_o + X_2'} i_1 \cos \xi - \frac{R_2' \omega_n}{X_o + X_2'} \psi_{2d}' - \frac{R_2' \omega_n}{X_o + X_2'} f(\psi_{od}) \right]$$
$$+ V_2 \left[\frac{R_2' X_o}{X_o + X_2'} i_1 \sin \xi - \frac{R_2' \omega_n}{X_o + X_2'} \psi_{2q}' - \frac{R_2' \omega_n}{X_o + X_2'} f(\psi_{oq}) \right]$$
$$+ V_3 \left\{ \frac{3p^2 \omega_n X_o}{\omega_n} \left[\frac{i_1 F}{\omega_n} (\psi_{2d}' \sin \xi - \psi_{2q}' \cos \xi) + \right] \frac{M_o p}{\cdots} \right\} \qquad (30)$$

where V_1, V_2, V_3 are the conjugated variables that satisfy the following conjugated equations:

$$\frac{dV_1}{dt} = \frac{3\lambda_1 R_2' \omega_n}{(X_o + X_2')^2}[\omega_n \psi_{2d}' - X_o i_1 \cos\xi + \omega_n f(\psi_{od})]$$

$$+ \frac{R_2' \omega_n}{X_o + X_2'}V_1 + \frac{3p^2 X_o V_3}{2J(X_o + X_2')}\left[\frac{\omega_n X_2'}{F}f(\psi_{oq}) - i_1 \sin\xi\right] \tag{31}$$

$$\frac{dV_2}{dt} = \frac{3\lambda_1 R_2' \omega_n}{(X_o + X_2')^2}[\omega_n \psi_{2q}' - X_o i_1 \sin\xi + \omega_n f(\psi_{oq})]$$

$$+ \frac{R_2' \omega_n}{X_o + X_2'}V_2 + \frac{3p^2 X_o V_3}{2J(X_o + X_2')}\left[-\frac{\omega_n X_2'}{F}f(\psi_{od}) + i_1 \cos\xi\right] \tag{32}$$

$$\frac{dV_3}{dt} = \lambda_2 B_m^2 \left[\begin{array}{l} 2A_z\,(M_m - M_r)\,\omega_r + \\ +2A_z\,(M_m - M_r)\left(\cos\xi\frac{d\sin\xi}{dt} - \sin\xi\frac{d\cos\xi}{dt}\right) + \\ +B_z\,(M_m - M_r) \end{array}\right]$$

$$+ \frac{p}{J}\frac{\partial M_o}{\partial \omega_r}V_3 \tag{33}$$

The optimal control, if it exists, should maximize the hamiltonian (30) and therefore satisfy the following equations:

$$\frac{\partial H}{\partial i_1} = 0 \tag{34}$$

$$\frac{\partial H}{\partial \xi} = 0. \tag{35}$$

Solving the equation (34) for the hamiltonian (30) yields:

$$i_{1opts} = \frac{X_o R_2' \omega_n}{A}\{[\psi_{2d}' + f(\psi_{od})]\cos\xi + [\psi_{2q}' + f(\psi_{oq})]\sin\xi\}$$

$$+ \frac{R_2' X_o(X_o + X_2')}{3\lambda_1 A}[V_1 \cos\xi + V_2 \sin\xi]$$

$$+ \frac{p^2 X_o(X_o + X_2')}{2J\lambda_1 A}V_3[\psi_{2d}' \sin\xi - \psi_{2q}' \cos\xi]. \tag{36}$$

Taking into consideration the limitation (29), the amplitude of the stator current is described by the following equation (Kawecki, Niewierowicz, 1999):

$$i_{1opt} = \begin{cases} i_{1opts} & \text{for } i_{1opt} \leq i_1^o \\ i_1^o & \text{for } i_{1opt} > i_1^o. \end{cases} \tag{37}$$

For the Hamiltonian (30) the equation (35) will be as follows:

$$\sin\xi\left\{-\frac{3\lambda_1 R_2' \omega_n}{X_o + X_2'}\left[\psi_{2d}' + f(\psi_{od}) - V_1 R_2' + V_3 \frac{3p^2}{2J}\psi_{2q}'\right]\right\}$$

$$- \left\{\frac{3\lambda_1 R_2' \omega_n}{\ }\left[\ \right]\right\} = 0 \tag{38}$$

Considering the equation (38) as the scalar product of two vectors, one can write:

$$\sin \xi = \frac{g}{j} \qquad \cos \xi = \frac{h}{j}$$

$$g = \frac{3\lambda_1 R_2' \omega_n}{X_o + X_2'} [\psi_{2q}' + f(\psi_{oq})] + R_2' V_2 + \frac{3}{2} \frac{p^2}{J} V_3 \psi_{2d}'$$

$$h = \frac{3\lambda_1 R_2' \omega_n}{X_o + X_2'} [\psi_{2d}' + f(\psi_{od})] + R_2' V_1 - \frac{3}{2} \frac{p^2}{J} V_3 \psi_{2q}' \tag{39}$$

$$j = \sqrt{h^2 + g^2}.$$

The relationships (36), (37) and (39) describe the optimal control in the implicit shape (depending on the state variables and of the conjugated variables). To find the description of the optimal control in the explicit shape (27), (28), it is necessary to solve the system of the canonical equations compounded by the state equations (1), (2), (3) and the conjugated equations (31), (32), (33), which implies the knowledge of the motor load type and of the initial conditions of the state variables $\psi_{2d}'(0)$, $\psi_{2q}'(0)$, $\omega_r(0)$ and of the conjugated variables $V_1(0)$, $V_2(0)$, $V_3(0)$. Only the initial conditions of the state variables are known. The final value of the state variable $\omega_r(t_r)$ is known too. By means of the transversability conditions (Athans, Falb, 1966, Pontryagin, Boltianski, 1962, Boltianski, 1971), it is possible to prove that the final values of the conjugated variables $V_1(t_r)$, $V_2(t_r)$ are null.

To solve the canonical equations, it is necessary to solve the two-point boundary value problem, which consists in finding the initial values of the conjugated variables knowing the initial values of the state variables $\psi_{2d}'(0)$, $\psi_{2q}'(0)$, $\omega_r(0)$, the final values of the conjugated variables $V_1(t_r)$, $V_2(t_r)$, and the final value of the state variable $\omega_r(t_r)$. To solve the two-point boundary value problem, the use of a computer and an algorithm of parametric optimization are necessary.

6.   Two-point boundary value problem

The two-point boundary value problem belongs to a type of problems called the parametric optimization problems, in which the optimal values of some parameters are looked for, so that they guarantee the global extremum of an optimization index.

Taking as the parametric optimization index Q_p in the problem of the search for the initial values of the conjugated variables, one can choose a measure of distance between two points in space. One of these points is constant and determined by the final values $V_1(t_r) = 0$, $V_2(t_r) = 0$, $\omega_r(t_r) = \omega_{rtr}$ and the other is determined by the final values of the variables V_1, V_2, ω_r obtained

example, one can accept as the criterion of parametric optimization, the measure of distance between two points mentioned before, in the following form:

$$Q_p = |\omega(t_r) - \omega_{rtr}| + |V_1(t_r) - 0| + |V_2(t_r) - 0| \tag{40}$$

However, in the case of optimization of a control system, the initial conditions of the conjugated variables must have the values, with which by applying the optimal control, the extreme value of the accepted optimization index is obtained. Then, taking as criterion of the parametric optimization, one can also use the control system optimization index (26).

To solve the two-point boundary value problem, different algorithms of parametric optimization can be used. The classic algorithms work correctly, when the parametric optimization index has one single or several equal global extremes. Unfortunately, in most of the practical cases it is not previously known if the parametric optimization index fulfills the supposition mentioned. Using a classic algorithm for these cases, it is necessary to apply this algorithm many times, beginning the calculations from a different initial point, every time.
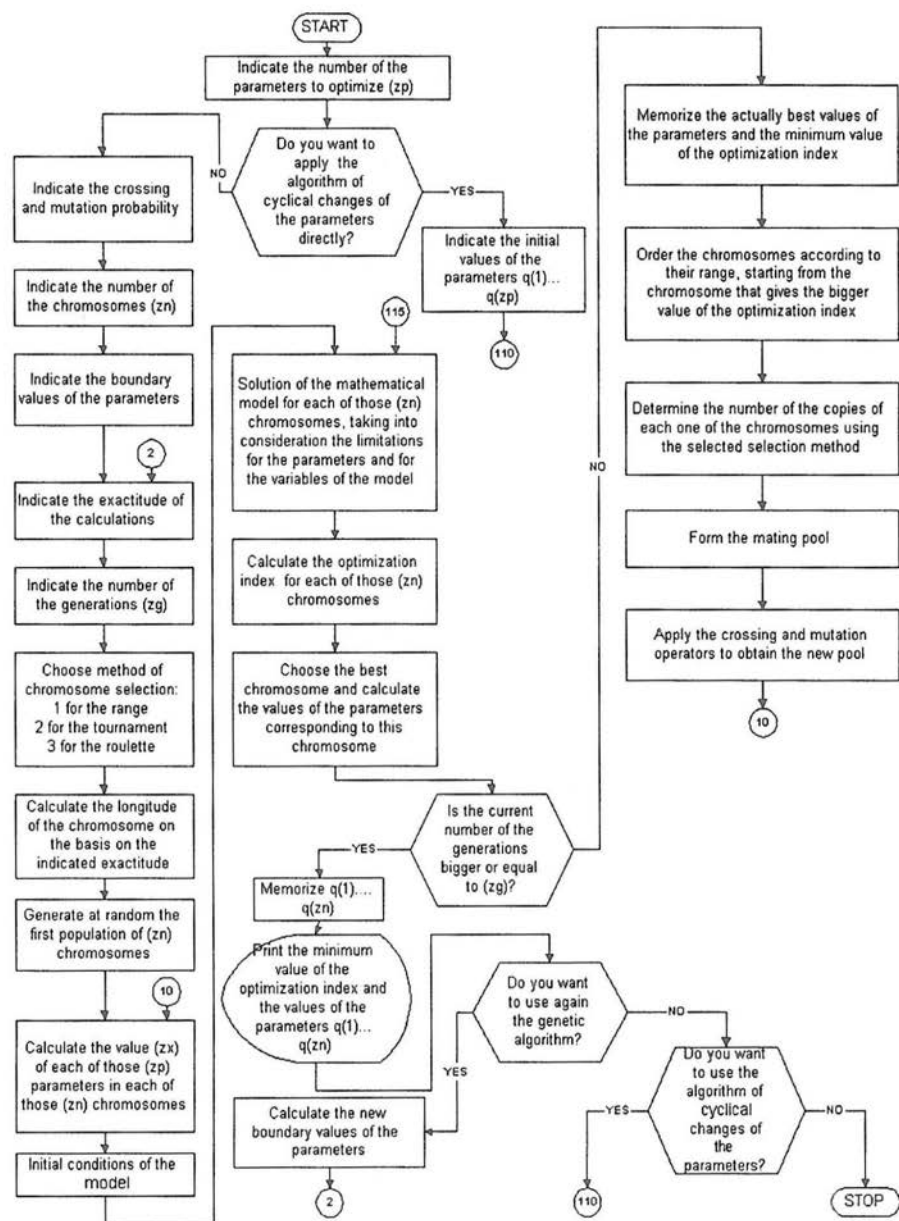
The genetic algorithms allow for finding with a certain precision, usually not very high, the global extreme of the optimization index, even when this index has the local extremes.

From what was previously exposed, the idea could be derived that both mentioned types of the parametric optimization algorithms should be applied, to find the global extreme of an optimization index, with possibly high precision. First, the genetic algorithm should be applied, to come near to the global extreme and afterwards a classic algorithm may be applied, taking the values of the parameters found by the genetic algorithm, as the start point. Carrying out this idea, the algorithm called the hybrid algorithm of parametric optimization, has been elaborated (Kawecki, 2000), consisting of two particular algorithms: the genetic algorithm with three chromosome selection methods (for the range, for the tournament and for the roulette) and the classic algorithm based on the method of Gauss-Seidel, also called the algorithm of the cyclical changes of the parameters.

In Figs. 1 and 2 the flow diagram of the hybrid algorithm is presented (all notations used in figures are given in the Appendix, at the end of the paper).

The two algorithms which compose the hybrid algorithm can be applied separately or jointly. In the case of the joint option, it is always necessary to apply the genetic algorithm first and then the algorithm of the cyclical changes of the parameters.

START

Indicate the number of the parameters to optimize (zp)

Do you want to apply the algorithm of cyclical changes of the parameters directly? — NO / YES

Indicate the initial values of the parameters q(1)... q(zp)

(110)

Indicate the crossing and mutation probability

Indicate the number of the chromosomes (zn)

Indicate the boundary values of the parameters

(2)

Indicate the exactitude of the calculations

Indicate the number of the generations (zg)

Choose method of chromosome selection:
1 for the range
2 for the tournament
3 for the roulette

Calculate the longitude of the chromosome on the basis on the indicated exactitude

Generate at random the first population of (zn) chromosomes

(10)

Calculate the value (zx) of each of those (zp) parameters in each of those (zn) chromosomes

Initial conditions of the model

(115)

Solution of the mathematical model for each of those (zn) chromosomes, taking into consideration the limitations for the parameters and for the variables of the model

Calculate the optimization index for each of those (zn) chromosomes

Choose the best chromosome and calculate the values of the parameters corresponding to this chromosome

Memorize q(1).... q(zn)

Print the minimum value of the optimization index and the values of the parameters q(1)... q(zn)

Calculate the new boundary values of the parameters

(2)

Is the current number of the generations bigger or equal to (zg)? — YES / NO

Do you want to use again the genetic algorithm? — NO / YES

Do you want to use the algorithm of cyclical changes of the parameters? — YES / NO

(110)

STOP

Memorize the actually best values of the parameters and the minimum value of the optimization index

Order the chromosomes according to their range, starting from the chromosome that gives the bigger value of the optimization index

Determine the number of the copies of each one of the chromosomes using the selected selection method

Form the mating pool

Apply the crossing and mutation operators to obtain the new pool
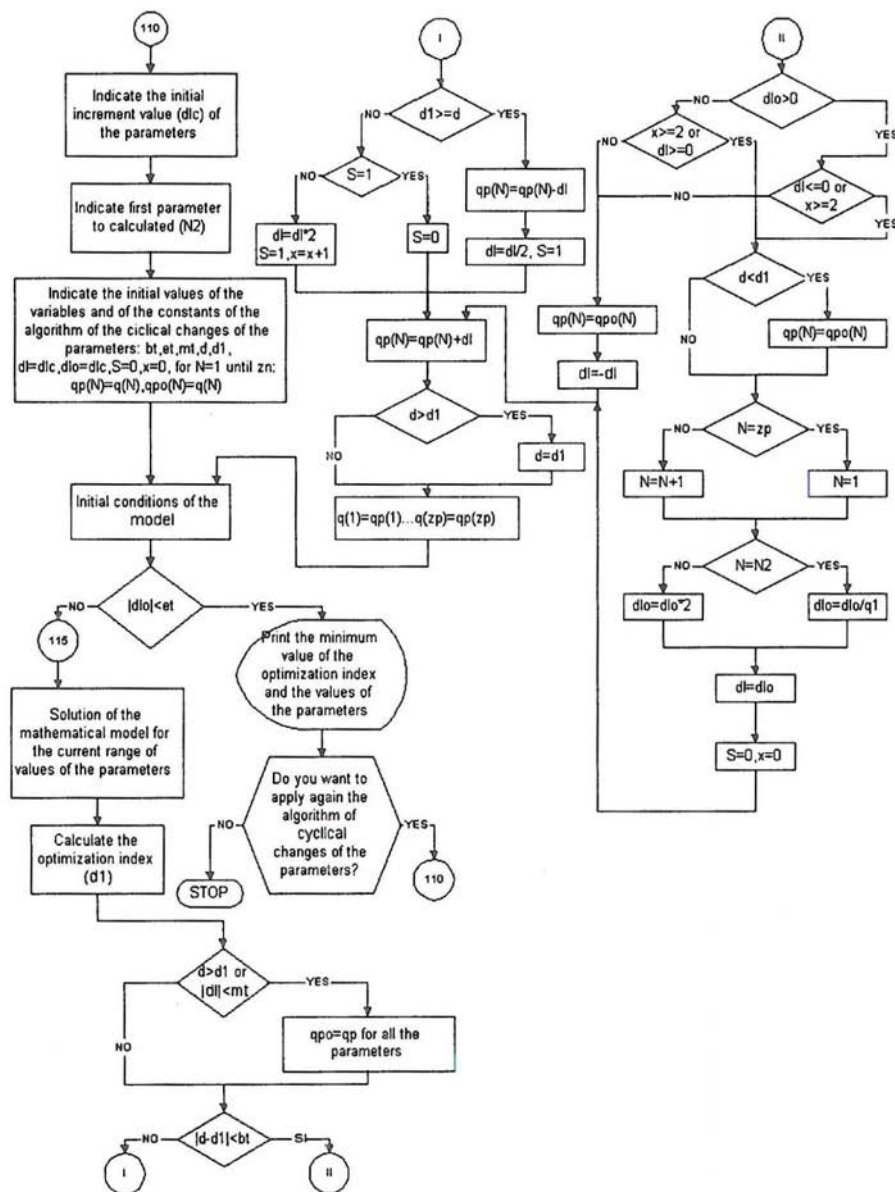
(10)

Fig. 2. Hybrid algorithm of parametric optimization (part 2)

7. Numerical example

In order to illustrate the developed design method of the optimal induction

example for the idle starting ($M_o = 0$) of a 500 h.p. three-phase induction motor is given, whose parameters are:

$$u_m = 1877.94[V], \quad p = 2, \quad \omega_n = 377[rd/s]$$
$$R_1 = 0.262[\Omega], \quad R'_2 = 0.187[\Omega], \quad X_1 = X'_2 = 1.206[\Omega]$$
$$X_o = 54.02[\Omega], \quad M_m = 2649.7[kg], \quad M_r = 657.34[kg]$$
$$i_{o\,max} = 34[A].$$

The magnetic cores of the stator and the rotor are built alike of the same anisotropic cold-rolled sheet, containing 4% of silicon. The parameters of this sheet are:

$$B_m = 1.5[T], \quad \rho_m = 7.65 \times 10^3[kg/m^3], \quad l = 0.001[m]$$
$$\gamma_c = 2.096 \times 10^6[S/m], \quad \varepsilon = 1.5[m^4/Hkg], \quad d = 0.0003[m]$$

The calculations are carried out for the coefficients λ_1 and λ_2 with the same values: $\lambda_1 = \lambda_2 = 0.5$ and for the limit values of the stator current amplitude: $i_1^o = 30[A]$ and $i_1^o = 150[A]$.

For $i_1^o = 30A$ the saturation of the magnetic material of the motor does not occur ($i_o < i_{o\,max}$) during the control, even though the mathematical model of the motor, used in the design of the optimal control, does not take into consideration the saturation of the magnetic material of the motor (Kawecki, Niewierowicz, 1999). For $i_1^o = 150[A]$ the optimal control designed on the basis of the motor model that does not consider saturation, would surely cause the saturation of the magnetic material of the motor, at least in some time intervals.

In order to solve the two-point boundary value problem, the hybrid algorithm of parametric optimization has been applied, using both as the parametric optimization index, the expression (41) and the functional (26).

When using the genetic algorithm, the following values were accepted:
— the crossing probability $= 0.95$
— the mutation probability $= 0.03$
— the number of chromosomes $= 200$
— the number of generations $= 50$.

The integration step used in the genetic algorithm as well as in the cyclical parameters changes algorithm was equal to $0.001s$ ($dt = 0.001s$).

By applying the hybrid algorithm for the parametric optimization index (40), many similar global minima for different ranges of the initial values of the conjugated variables were obtained. However, only a part of these ranges minimizes the total losses. This indicates that there exist many controls that minimize the parametric optimization index (40), but only some of these controls minimize the index (26). This is caused by the fact, that the Pontryagin maximum principle, for the non-linear systems, provides the necessary conditions but not the sufficient ones. When the parametric optimization index for

(26), several different ranges of the initial values of the conjugated variables $V_1(0)$, $V_2(0)$, $V_3(0)$ that give the same minimum value of the index (26) are also obtained, though the solution of the canonical equations for each of these ranges gives the same control. Therefore, to solve the two-point boundary value problem, it is more convenient to accept as the parametric optimization index, the index (26).

The obtained results are the following ones:

— For $i_1^o = 30A$, the following initial values of the conjugated variables were obtained:

$$V_1(0) = -4175.233 \quad V_2(0) = 1857.633 \quad V_3(0) = 726.539.$$

— For $i_1^o = 150A$, the following initial values of the conjugated variables were obtained:

$$V_1(0) = 6842.948 \quad V_2(0) = 4252.685 \quad V_3(0) = 610.875.$$

Using the initial conditions of the conjugated variables found, the behavior of the optimal control system has been simulated.

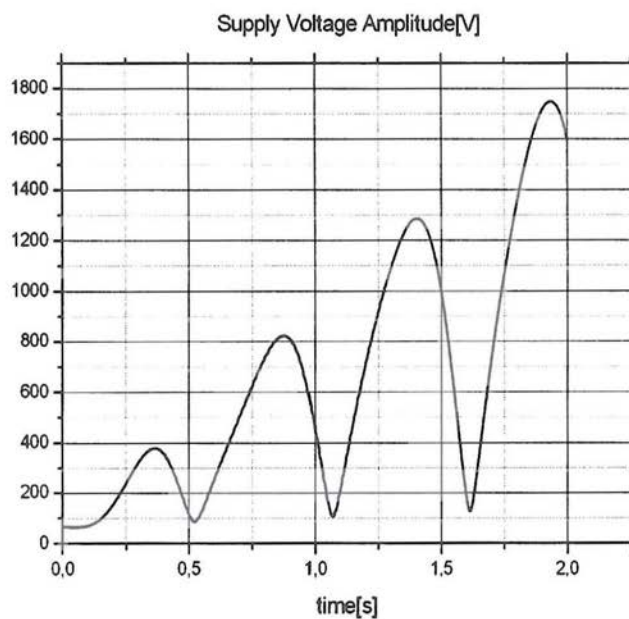The results of the simulation for $i_1^0 = 150\ A$ are shown in Figs. 3–11.

Stator Current Amplitude[A]
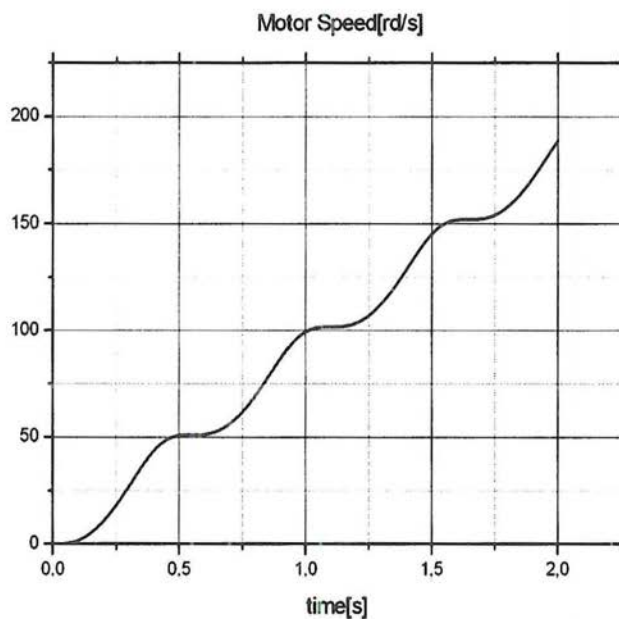
Stator Current Angular Frequency[rd/s]
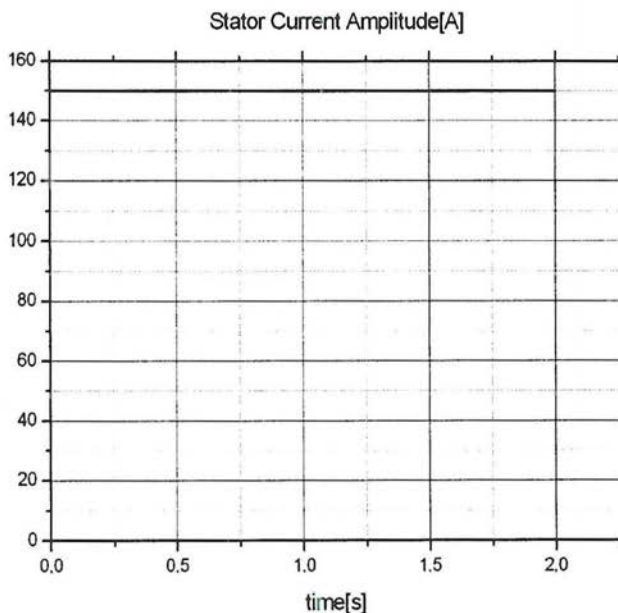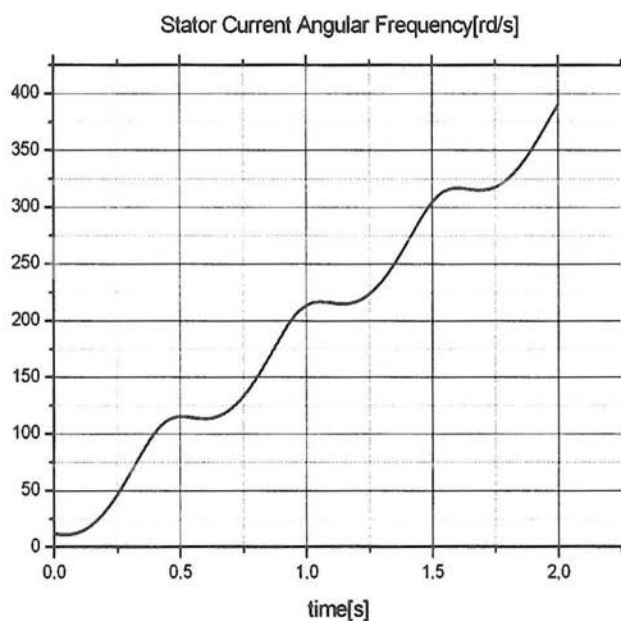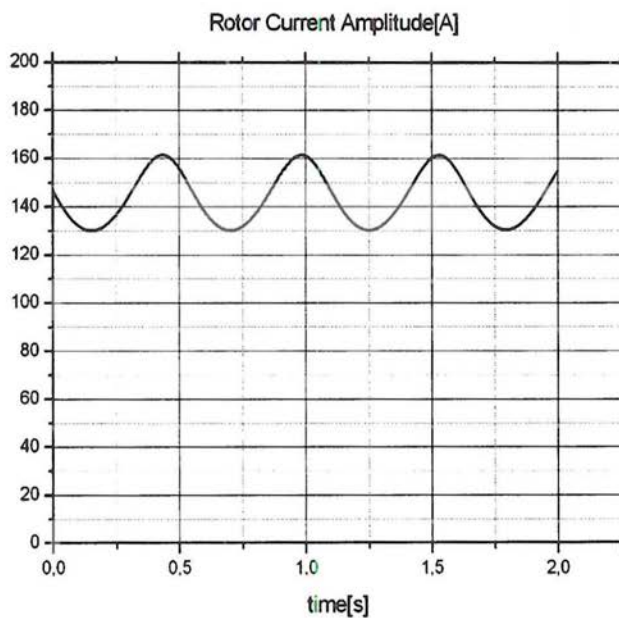

Fig. 4. Stator current frequency

Supply Voltage Amplitude[V]

Fig. 6. Motor speed
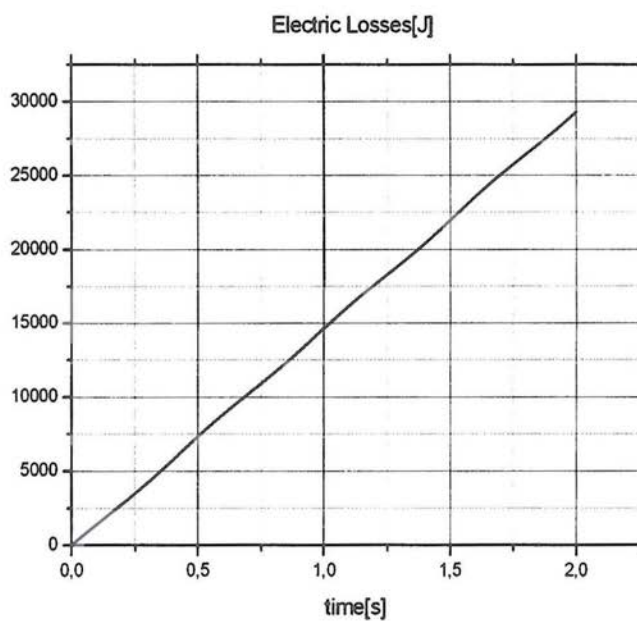
Magnetization Current Amplitude[A]
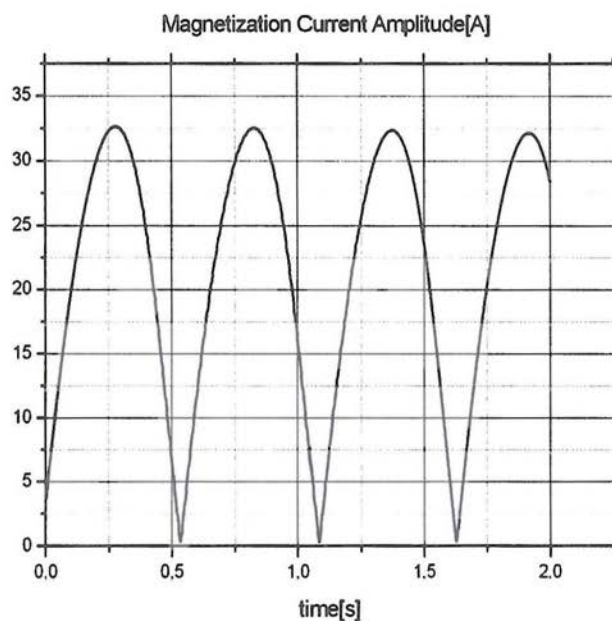
Fig. 8. Magnetization current amplitude
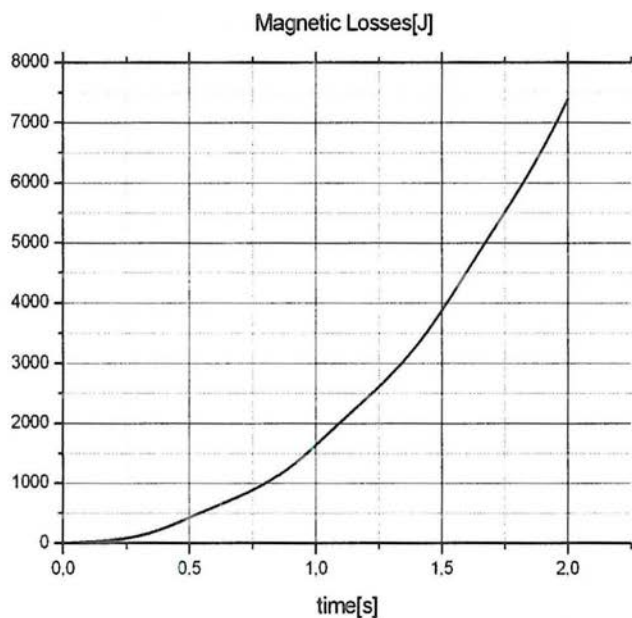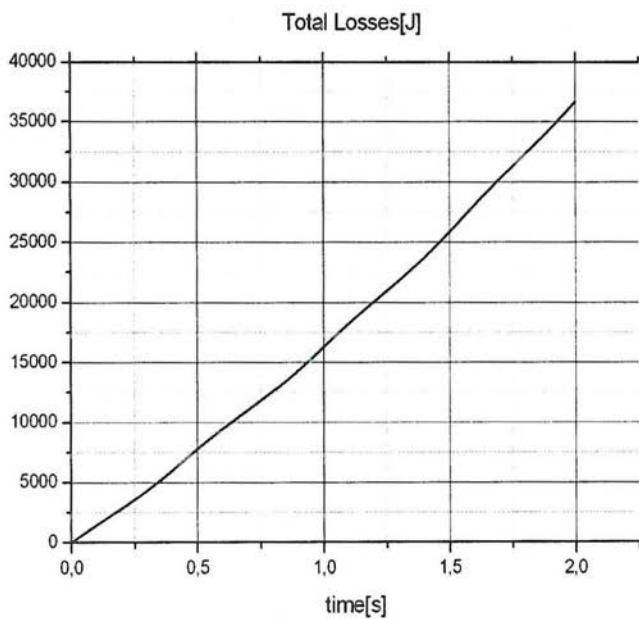
Electric Losses[J]

Fig. 10. Magnetic losses

The results of the calculations obtained during the simulations are shown in Table 1.

Table 1.

$i_1^0[A]$	$t_r[s]$	$Q_e[J]$	$Q_m[J]$	$0.5Q_e + 0.5Q_m$	$i_{om}[A]$
30	11.62	5530.0	37506.8	43036.8	21.31
150	2.0	29260.7	7394.7	36655.4	32.70

8. Conclusions

From the results obtained in this paper one can conclude that it is possible to obtain the general mathematical description of the optimal induction motor speed control that minimizes the total losses, but only in the implicit form (the relationships (36), (37) and (39)).

To determine the explicit optimal control (27), (28) it is necessary to solve the two-point boundary value problem, using a computer and a parametric optimization algorithm. This implies that the obtained explicit control is not good for general use but only for the given induction motor.

Based on the calculation results obtained for the investigated motor one can conclude that:

- To solve the two-point boundary value problem it is more convenient to use as parametric optimization index, the same optimization index (26) that expresses the total losses, since this allows for saving of the calculation time and even when the classic optimization algorithm alone is used, there will be a quite high probability that the minimum found is the global minimum.
- The optimal control found, guarantees not only the minimization of the electric and magnetic losses, limiting the stator current amplitude, but also the limitation of the magnetization current amplitude to the value that does not allow the magnetic material of the motor to be saturated, irrespective of the limit value of the stator current amplitude (see Fig. 8 and Table 1). This result was obtained because the motor's magnetic material saturation in the mathematical motor model was taken into consideration.
- The electric losses during the optimal starting increase almost linearly with time (see Fig. 9). This is due to the fact that the stator current amplitude is constant during the starting (Fig. 3) and the rotor current amplitude is almost constant, oscillating around the constant value (Fig. 7).
- The magnetic losses during the optimal starting increase almost parabolically with time (Fig. 10), since the frequency of the stator current and the angular speed of the rotor increase in an almost linear manner, oscillating around the direct line (Fig. 4).

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Appendix: List of symbols

$A = R_1(X_0 + X_2')^2 + R_2'X_0^2$

$A_z = \dfrac{\gamma_c d(d + 1.628l)}{24\rho_m}$

B_m — maximum magnetic induction measured in tesla [T]

$B_z = \frac{\varepsilon}{200\pi}$

d — thickness of the sheet measured in meters [m]

$F = (X_1 + X_o)(X_2' + X_o) - X_o^2 = X_1 X_o + X_1 X_2' + X_2' X_o$

H — Hamiltonian

i_1 — amplitude of the stator current

i_{1d}, i_{1q} — components of the vector of the stator current

i_2' — amplitude of the rotor current related to the stator circuit

i_{2d}', i_{2q}' — components of the vector of the rotor current related to the stator circuit

i_o — amplitude of the magnetization current

i_{om} — maximum value of the magnetization current amplitude, obtained during the motor starting

$i_{o\,max}$ — maximum value of the magnetization current amplitude when the magnetic material of the motor does not enter in saturation yet, according to its approximate curve of magnetization

i_{1opts} — optimal value of the stator current amplitude without the limitation of the stator current

i_1^0 — maximum admissible value of the stator current amplitude

J — inertial torque of the rotor

l — distance between the walls of the magnetic domains that form parallel bands of the thickness

$M_m = M_s + M_r$ — total mass of the magnetic core of the motor measured in kilograms [kg]

M_r — mass of the rotor measured in kilograms [kg]

M_s — mass of the stator measured in kilograms [kg]

M_o — load torque

p — number of pairs of poles

Q — optimization index

Q_e — electric losses

Q_m — magnetic losses

Q_p — parametric optimization index

R_1, R_2' — resistances of the stator winding and of the rotor winding related to

X_1, X_2' — dissipation reactances of one phase of the stator winding and of one phase of the rotor winding related to the stator circuit, in two-phase equivalent motor, calculated for the nominal frequency of the stator current, respectively

X_o — magnetizing reactance of a two-phase motor for nominal frequency of the stator current

t — time

u — amplitude of supply voltage

u_{1d}, u_{1q} — components of the vector of the supply voltage on the d-q axes

u_m — nominal supply voltage amplitude

V_1, V_2, V_3 — conjugate variables

$\beta'(t)$ — absolute slip

$\beta(t)$ — relative slip

ε — coefficient whose value depends on the sheet type (for example, for the transformer's sheet which has 4% of silicon, ε takes the value between 1.2 and 2 $[\mathrm{m}^4/\mathrm{Hkg}]$)

γ_c — specific conductivity of the sheet measured in siemens per meter $[\mathrm{S/m}]$

λ_1, λ_2 — constant coefficients in the optimization index

ω — angular frequency of the stator current

ω_n — nominal angular frequency of the stator current

ω_r — angular speed of the motor with one pair of poles

ω_{rtr} — desired value of the motor speed

ρ_m — specific density of the sheet measured in kilograms per cubic meter $[\mathrm{kg/m3}]$

ξ — angle between the stator current vector and "d" axis

ψ_{1d}, ψ_{1q} — components of the vectors of the magnetic flux linkage with the windings of the stator

ψ_{2d}', ψ_{2q}' — components of the vector of the magnetic flux linkage with the windings of the rotor related to the stator circuit

ψ_{od}, ψ_{oq} — components of the magnetization flux vector

ψ_o — amplitude of the magnetization flux

Following symbols are used in Figs. 1 and 2:

bt, et, mt — small numbers determining the calculation exactitude in the algorithm of the cyclical changes of parameters

d1 — parametric optimization index value in the present iterative step of the algorithm of the cyclical changes of parameters

d — parametric optimization index value in the preceding iterative step of the algorithm of the cyclical changes of parameters

dl — parameter increment value in the present iterative step of the algorithm of the cyclical changes of parameters

dlo — parameter increment value starting a cycle of the parameter changes in

dlc — parameter increment value beginning the execution of the algorithm of the cyclical changes of parameters

N — currently changed parameter number in the algorithm of the cyclical changes of parameters

N2 — the number of the first parameter destinated to change at the start of the calculation in the algorithm of the cyclical changes of parameters

q1 — integer number greater than 2^{zp}, divisor used in the algorithm of the cyclical changes of parameters for the decrease of parameter increment, finishing a cycle of the parameter changes

q(N) — initial value of the N – number parameter in the mathematical model used in the algorithm of the cyclical changes of parameters

qp(N) — initial value of the N – number parameter starting the iterative cycle of the parameters changes in the algorithm of the cyclical changes of parameters

qpo(N) — optimal value of the N—number parameter in the algorithm of the cyclical changes of parameters

S, x — flags of the algorithm of the cyclical changes of parameters

zn — number of the chromosomes

zp — number of the parameters for optimization

zg — number of the generations.