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Hybrid modelling and performance evaluation of switched discrete-event systems[1]

by

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Abstract: Motivated by the integrated complexity of real-time intelligent control and optimization of industrial/manufacturing processes, this paper discusses hybrid modelling and asymptotic periodic behavior of a class of switched discrete event systems, and shows how to evaluate the asymptotic performance/efficiency of such systems. We prove that, under some mild conditions, the switched discrete event system will achieve asymptotic periodic dynamics, and its performance/efficiency can be evaluated by calculating the eigenvalue of certain matrix in max-plus algebra. Illustrative examples are provided.

Keywords: complex systems, switched systems, discrete event systems, max-plus algebra, hybrid modelling, periodic behavior, performance evaluation.

1. Introduction

Modern technologies have created some open complex gigantic systems characterized by large-scale, high-dimensions, hierarchy, parallelism, networking, multi-patterns, uncertainties, nonlinearities, hybrid dynamics, time-delays, interconnections and interactions. Typical examples include contemporary integrated manufacturing systems (CIMS), air traffic systems, computer communication systems, etc. It is widely believed that informationization is an important driving force in modernization, whereas automation is a bridge between information technology and modern industry/society (Cheng Wu, 2000). Motivated

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by the integrated complexity of real-time intelligent control and optimization of industrial/manufacturing processes (Cheng Wu, 2000; Cohen et al., 1984, 1985; Ramadge and Wonham, 1987; Cunninghame-Green, 1979), this paper studies the asymptotic behavior of a class of human-machine interactive reconfigurable manufacturing processes, and establishes a simple efficient method to evaluate the asymptotic steady-state performance/efficiency of such complex systems.

Based on max-plus algebra, a class of discrete event processes can be described by linear recursive equations (Cohen et al., 1984, 1985; Ramadge and Wonham, 1987). Such a system exhibits asymptotic periodic behavior, and its steady-state performance/efficiency (i.e., the mean time of production cycles) can be evaluated by calculating the eigenvalue of the system matrix in max-plus algebra. On the other hand, control techniques based on switching among different subsystems have been explored extensively in recent years, where they have been shown to achieve better robustness, flexibility, and dynamic performance (Liberzon and Morse, 1999; Wang and Xie, 2002a, b, c, d, e, 2004).

This paper proposes a new model for a class of switched discrete event systems. Such a model consists of a finite set of discrete event subsystems, and a switching rule that orchestrates the switching among them. We show that the switched system can be transformed into an ordinary discrete event system without switching and under certain conditions the switched system exhibits asymptotic periodic behavior, i.e., after a finite-time transient process, the system achieves steady-state periodic dynamics, and its performance/efficiency can be evaluated by calculating the eigenvalue of certain matrix in max-plus algebra.

2. Preliminaries

In this section, we first define some basic operations in max-plus algebra, Cunninghame-Green (1979). Denote

$$R_\epsilon = R \cup \{-\infty\}$$
$$\epsilon = -\infty$$

and for any $x, y \in R_\epsilon$, define

$$x \oplus y = \max\{x, y\}$$
$$x \otimes y = x + y$$

A matrix $A = (a_{ij}) \in R_\epsilon^{n\times n}$ is said to be irreducible, Horn and Johnson (1990), if $\forall i, j$, $\exists (i_1 = i, i_2, \ldots, i_{k-1}, i_k = j)$, s.t. $a_{ii_2} + a_{i_2 i_3} + \ldots + a_{i_{k-1} j} > -\infty$.
For any matrices $A, B \in R_\epsilon^{n\times n}$, define, Cunninghame-Green (1979),

$$A \oplus B = (a_{ij} \oplus b_{ij})$$
$$A \otimes B = (\bigoplus_{k=1}^{n} (a_{ik} \otimes b_{kj})) =: AB$$

Given any matrix $A \in R_e^{n\times n}$, the corresponding directed graph (digraph) is a graph with n nodes, and there is a directed arc from node j to node i with weight a_{ij} if and only if $a_{ij} \neq -\infty$.

In a digraph, a circuit is a directed path that starts and ends at the same node. In a circuit, the sum of the weights of all its arcs divided by the number of arcs is called the mean weight. The circuit with the maximal mean weight in a digraph is called the critical circuit.

A zero vector is a vector with all its entries equal to $-\infty$.

For an irreducible matrix $A \in R_e^{n\times n}$, if there exist a real number λ and a nonzero vector $h \in R_e^{n\times 1}$ such that $Ah = \lambda h$, then λ and h are called the eigenvalue and eigenvector of A, respectively.

LEMMA 2.1 *(Cohen et al., 1984, 1985) For an irreducible matrix $A \in R_e^{n\times n}$, there is a unique eigenvalue λ, and it equals the mean weight of the critical circuit of its corresponding digraph.*

LEMMA 2.2 *(Cohen et al., 1984, 1985) For an irreducible matrix $A \in R_e^{n\times n}$, there exist positive integers k_0 and d such that*

$$A^{k+d} = \lambda^d A^k, \quad k \geq k_0$$

where d is called the period order of A.

LEMMA 2.3 *(Cohen et al., 1984, 1985) For an irreducible matrix $A \in R_e^{n\times n}$, suppose its eigenvalue is λ, and its period order is d. Then there exists a positive integer k_0 such that the solution of*

$$X(k+1) = AX(k)$$

satisfies

$$X(k+d) = \lambda^d X(k), \quad k \geq k_0.$$

This shows that the system will exhibit periodic behavior asymptotically. The mean period is exactly equal to the eigenvalue of A. Hence, the eigenvalue of A is an important performance index of the system.

3. Switched Systems

For notational simplicity, we first discuss switching between two subsystems (Liberzon and Morse, 1999). That is, the switched system is governed by

$$X(k+1) = A_i X(k) \tag{1}$$

where $A_i \in R_e^{n\times n}$, and the switching law is

$$i = \begin{cases} 1 & k \text{ even} \end{cases}$$

Namely

$$\begin{aligned} X(1) &= A_1 X(0) \\ X(2) &= A_2 X(1) \\ X(3) &= A_1 X(2) \\ X(4) &= A_2 X(3) \\ &\cdots \end{aligned}$$

That is

$$\begin{aligned} X(2) &= A_2 X(1) = A_2 A_1 X(0 \\ X(4) &= A_2 X(3) = A_2 A_1 X(2) \\ &\cdots \end{aligned}$$

Let

$$Y(k) = X(2k) \tag{2}$$

Then

$$\begin{aligned} Y(1) &= A_2 A_1 Y(0) \\ Y(2) &= A_2 A_1 Y(1) \\ &\cdots \\ Y(k+1) &= A_2 A_1 Y(k). \end{aligned} \tag{3}$$

In this way, we transform a switched system into a non-switched system. Thus, the following problem naturally arises: Suppose A_1 and A_2 are irreducible matrices, is their product $A_2 A_1$ still irreducible?

The answer is NO in general case. Consider the two irreducible matrices

$$A_1 = \begin{bmatrix} \epsilon & 1 & \epsilon \\ \epsilon & \epsilon & 1 \\ 1 & \epsilon & \epsilon \end{bmatrix}, \qquad A_2 = \begin{bmatrix} \epsilon & \epsilon & 1 \\ 1 & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon \end{bmatrix}.$$

Then, their product is

$$A_2 A_1 = \begin{bmatrix} 2 & \epsilon & \epsilon \\ \epsilon & 2 & \epsilon \\ \epsilon & \epsilon & 2 \end{bmatrix}$$

Clearly, $A_2 A_1$ is reducible. However, if every main diagonal entry of A_1 (or A_2) is not the null element ϵ, then the answer to the question above is YES.

THEOREM 3.1 *Suppose $A, B \in R_e^{n \times n}$ are irreducible matrices, with all the main*

Proof. Without loss of generality, suppose all the main diagonal entries of A are not equal to ϵ. Then, for any $1 \leq s, t \leq n$, $(AB)_{st} \neq \epsilon$ whenever $(B)_{st} \neq \epsilon$. Moreover, since B is irreducible, by definition, AB is irreducible, too. ■

THEOREM 3.2 *Suppose $A_1, A_2 \in R_\epsilon^{n\times n}$ are irreducible matrices, with all the main diagonal entries of A_1 (or A_2) not equal to ϵ. Then, there exist positive number λ, positive integers d and k_0, such that the switched system (1) satisfies*

$$X(k+d) = \lambda^d X(k), \quad k \geq k_0.$$

Proof. By the transformation (2), the switched system (1) can be transformed into a non-switched system (3). That is

$$Y(k+1) = A_2 A_1 Y(k)$$

By Theorem 3.1, A_2A_1 is irreducible. Hence, by Lemma 3.3 and by the transformation (2), we get the result. ■

EXAMPLE 3.1 *Consider two irreducible matrices*

$$A = \begin{bmatrix} 2 & \epsilon & 3 \\ 6 & 2 & \epsilon \\ \epsilon & 4 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} \epsilon & 3 & \epsilon \\ \epsilon & \epsilon & 2 \\ 4 & \epsilon & \epsilon \end{bmatrix}.$$

It is easy to see that $\lambda(A) = \frac{13}{3}$, $\lambda(B) = 3$. Moreover

$$AB = \begin{bmatrix} 7 & 5 & \epsilon \\ \epsilon & 9 & 4 \\ 7 & \epsilon & 6 \end{bmatrix}$$

is also irreducible, and $\lambda(AB) = 9$. Note that $\lambda(AB) > \lambda(A) + \lambda(B)$. But this inequality is not always true in general case.

EXAMPLE 3.2 *Consider two irreducible matrices*

$$A = \begin{bmatrix} 10 & 1 & \epsilon \\ \epsilon & 1 & 1 \\ 1 & \epsilon & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 & \epsilon \\ \epsilon & 1 & 1 \\ 1 & \epsilon & 10 \end{bmatrix}.$$

It is easy to see that $\lambda(A) = 10$, $\lambda(B) = 10$. Moreover

$$AB = \begin{bmatrix} 11 & 11 & 2 \\ 2 & 2 & 11 \\ 2 & 2 & 11 \end{bmatrix}$$

is also irreducible, and $\lambda(AB) = 11$. Hence $\lambda(AB) < \lambda(A) + \lambda(B)$

4. Some extensions

More complicated switching laws can be accommodated for performance evaluation. Suppose $A_i \in R_e^{n\times n}$, $i = 1, 2, \dots, m$ are irreducible matrices, with all their main diagonal entries not equal to ϵ. This switched system is governed by

$$X(k+1) = A_i X(k) \tag{4}$$

with switching law

$$i = \begin{cases} 1 & k = 0, 1, 2, \dots, k_1 mod(K) \\ 2 & k = k_1 + 1, k_1 + 2, \dots, k_2 mod(K) \\ 3 & k = k_2 + 1, k_2 + 2, \dots, k_3 mod(K) \\ \vdots & \vdots \\ m & k = k_{m-1} + 1, k_{m-1} + 2, \dots, k_m mod(K) \end{cases}$$

where $K = k_m + 1$.

In this case, the transformed system is

$$Y(k+1) = A_m^{k_m - k_{m-1}} \dots A_3^{k_3 - k_2} A_2^{k_2 - k_1} A_1^{k_1 + 1} Y(k)$$

and

$$Y(k) = X(Kk).$$

Similar asymptotic periodic properties can be established as follows.

THEOREM 4.1 *Suppose $A_i \in R_e^{n\times n}$, $i = 1, 2, \dots, m$ are irreducible matrices, with all their main diagonal entries not equal to ϵ. Then, for any positive integers l_1, $i = 1, 2, \dots, m$, $A_m^{l_m} \dots A_3^{l_3} A_2^{l_2} A_1^{l_1}$ is irreducible, too.*

THEOREM 4.2 *Suppose $A_i \in R_e^{n\times n}$, $i = 1, 2, \dots, m$ are irreducible matrices, with all their main diagonal entries not equal to ϵ. Then, there exist positive number λ, positive integers d and k_0, such that the switched system (4) satisfies*

$$X(k+d) = \lambda^d X(k), \qquad k \geq k_0.$$

Note that even if the matrix A is irreducible, its power A^l can be reducible for some integer l. For example, let

$$A = \begin{bmatrix} \epsilon & 1 & \epsilon \\ \epsilon & \epsilon & 1 \\ 1 & \epsilon & \epsilon \end{bmatrix}$$

Then

$$A^3 = \begin{bmatrix} 3 & \epsilon & \epsilon \\ \epsilon & 3 & \epsilon \\ \epsilon & \epsilon & 3 \end{bmatrix}$$

which is reducible. This is why we assume that all the main diagonal entries

5. Illustrative example

Inspired by the mathematical models of a class of reconfigurable manufacturing processes (Cheng Wu, 2000; Cohen et al., 1984, 1985; Ramadge and Wonham, 1987; Cunninghame-Green, 1979), we will illustrate our results in two different switching cases.

Consider a system switched alternatively between two subsystems, more specifically,

$$X(k+1) = \begin{cases} A_1X(k) & k \text{ even} \\ A_2X(k) & k \text{ odd} \end{cases}$$

with

$$A_1 = \begin{bmatrix} 1 & 1 & \epsilon \\ \epsilon & 2 & 1 \\ 2 & \epsilon & 3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 4 & \epsilon & 2 \\ 1 & 3 & \epsilon \\ \epsilon & 2 & 1 \end{bmatrix}.$$

It is easy to get

$$A_2A_1 = \begin{bmatrix} 5 & 5 & 5 \\ 2 & 5 & 4 \\ 3 & 4 & 4 \end{bmatrix}.$$

It can be verified that they are all irreducible (Cohen et al., 1984, 1985; Cunninghame-Green, 1979), and $\lambda(A_1) = 3$, $\lambda(A_2) = 4$, $\lambda(A_2A_1) = 5$.

The evolution of the switched system can be easily calculated:

$$X(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad X(1) = A_1X(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad X(2) = A_2A_1X(0) = \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix},$$

$$X(3) = A_1A_2A_1X(0) = \begin{bmatrix} 6 \\ 7 \\ 7 \end{bmatrix}, \quad X(4) = A_2A_1A_2A_1X(0) = \begin{bmatrix} 10 \\ 10 \\ 9 \end{bmatrix},$$

$$X(5) = A_1A_2A_1A_2A_1X(0) = \begin{bmatrix} 11 \\ 12 \\ 12 \end{bmatrix}, \; X(6) = A_2A_1A_2A_1A_2A_1X(0) = \begin{bmatrix} 15 \\ 15 \\ 14 \end{bmatrix},$$

$$X(7) = A_1A_2A_1A_2A_1A_2A_1X(0) = \begin{bmatrix} 16 \\ 17 \\ 17 \end{bmatrix},$$

$$X(8) = A_2A_1A_2A_1A_2A_1A_2A_1X(0) = \begin{bmatrix} 20 \\ 20 \\ 19 \end{bmatrix},$$

From the evolution of the switched system, we can see that:

1. Initially, the system does not exhibit periodic behavior, since

$$X(2) - X(0) = \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix}, \quad X(3) - X(1) = \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix}.$$

2. A few steps later, the system begins to exhibit periodic behavior

$$\begin{aligned} X(4) - X(2) = X(6) - X(4) = X(8) - X(6) &= \begin{bmatrix} 5 \\ 5 = \\ 5 \end{bmatrix} \\ &= X(2n+2) - X(2n), \quad n \geq 1 \end{aligned}$$

$$X(5) - X(3) = X(7) - X(5) = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} = X(2n+1) - X(2n-1), \quad n \geq 2$$

$$X(5) - X(4) = X(7) - X(6) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = X(2n+1) - X(2n), \quad n \geq 2$$

$$X(6) - X(5) = X(8) - X(7) = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = X(2n+2) - X(2n+1), \quad n \geq 2.$$

This is consistent with the theoretical results established in the previous sections.

Now consider a switched system with a more complex switching rule

$$X(k+1) = \begin{cases} A_1 X(k) & k = 0 \ (\mathrm{mod}\ 3) \\ A_1 X(k) & k = 1 \ (\mathrm{mod}\ 3) \\ A_2 X(k) & k = 2 \ (\mathrm{mod}\ 3) \end{cases}$$

with

$$A_1 = \begin{bmatrix} 1 & 1 & \epsilon \\ \epsilon & 2 & 1 \\ 2 & \epsilon & 3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 4 & \epsilon & 2 \\ 1 & 3 & \epsilon \\ \epsilon & 2 & 1 \end{bmatrix}.$$

It is easy to get

$$A_2 A_1 A_1 = \begin{bmatrix} 7 & 7 & 8 \\ 6 & 7 & 7 \\ 6 & 6 & 7 \end{bmatrix}.$$

It can be verified that they are all irreducible (Cohen et al., 1984, 1985; Cun-

The evolution of the switched system can be easily calculated:

$$X(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad X(1) = A_1X(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad X(2) = A_1X(1) = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix},$$

$$X(3) = A_2X(2) = \begin{bmatrix} 8 \\ 7 \\ 7 \end{bmatrix}, \quad X(4) = A_1X(3) = \begin{bmatrix} 9 \\ 9 \\ 10 \end{bmatrix},$$

$$X(5) = A_1X(4) = \begin{bmatrix} 10 \\ 11 \\ 13 \end{bmatrix}, \quad X(6) = A_2X(5) = \begin{bmatrix} 15 \\ 14 \\ 14 \end{bmatrix},$$

$$X(7) = A_1X(6) = \begin{bmatrix} 16 \\ 16 \\ 17 \end{bmatrix}, \quad X(8) = A_1X(7) = \begin{bmatrix} 17 \\ 18 \\ 20 \end{bmatrix},$$

$$X(9) = A_2X(8) = \begin{bmatrix} 22 \\ 21 \\ 21 \end{bmatrix}, \quad X(10) = A_1X(9) = \begin{bmatrix} 23 \\ 23 \\ 24 \end{bmatrix},$$

...

Again, from the evolution of the switched system, we can see that:

1. Initially, the system does not exhibit periodic behavior, since

$$X(3) - X(0) = \begin{bmatrix} 8 \\ 7 \\ 7 \end{bmatrix}, \quad X(4) - X(1) = \begin{bmatrix} 8 \\ 7 \\ 7 \end{bmatrix}.$$

2. A few steps later, the system begins to exhibit periodic behavior

$$X(5) - X(2) = X(8) - X(5) = \begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix} = X(3(n+1)+2) - X(3n+2), \quad n \geq 0$$

$$X(6) - X(3) = X(9) - X(6) = \begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix} = X(3n+3) - X(3n), \quad n \geq 1$$

$$X(7) - X(4) = X(10) - X(7) = \begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix} = X(3(n+1)+1) - X(3n+1), \quad n \geq 1.$$

This is, again, consistent with the theoretical results established in the previous sections.

6. Future research

Two issues are under investigation:

1. What is the necessary and sufficient condition for the product of some matrices to be irreducible? in some cases, even if each individual matrix is reducible, their product can still be irreducible; for example

$$A = \begin{bmatrix} \epsilon & 1 \\ \epsilon & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} \epsilon & \epsilon \\ 1 & 1 \end{bmatrix}.$$

This issue is important in performance evaluation of the switched discrete event systems.

2. How is the eigenvalue of the product of some matrices related to the the eigenvalue of each individual matrix? The eigenvalue of the product of some matrices represents the asymptotic mean period of the switched system, and thereby plays an important role in performance evaluation.

Yet another interesting research direction is to study the asymptotic behavior of general 2-D discrete-event systems (Roesser, 1975; Kurek, 1985)

$$X(m+1, n+1) = A_1 X(m+1, n) \oplus A_2 X(m, n+1) \oplus A_3 X(m, n)$$

with the boundary condition

$$X(m, 0) = X_{m0}, \quad X(0, n) = X_{0n}, \quad m, n = 0, 1, 2, \ldots$$

Under what conditions does the system exhibit periodic behavior (with respect to m, n) asymptotically? and how to evaluate its asymptotic performance?

A popular model for 2-D systems is the so-called Roesser model (Roesser, 1975):

$$\begin{bmatrix} X^h(i+1, j) \\ X^v(i, j+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X^h(i, j) \\ X^v(i, j) \end{bmatrix}$$

with the boundary condition

$$X^h(0, j) = X^h_j, \quad X^v(i, 0) = X^v_i, \quad i, j = 0, 1, 2, \ldots$$

How the system (in the max-plus algebra sense) evolves asymptotically, and how to evaluate its asymptotic performance are the subjects of current research.

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