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A rule based machine learning approach to the nonlinear multifingered robot gripper problem

by

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Abstract: In this paper, we present a novel method that utilizes the accumulation of knowledge in a rule base for solving the nonlinear frictional gripper problem for both the isotropic and orthotropic cases. The knowledge is discovered and accumulated in a rule base with the aid of a genetic based machine learning mechanism. This machine learning mechanism extracts rules for solving the problem with the help of the Evolutionary Programming (EP) algorithm. The retrievals are done using the nearest-classifier-algorithm. This approach provides online solutions for the problem, and establishes a dynamic and evolving environment that adapts with new and sudden changes on the grip specifications or on the external forces. The resulting grasping forces using the presented method are compared with grasping forces obtained using other methods, such as the Complementarity Problems. The proposed online method could update the needed grasping forces to keep firm grip if the configuration of the forces externally applied to the object is changed. Numerical examples that illustrate the proposed method are presented.

Keywords: robot gripper, Nonlinear Complementarity Problem (NCP), Evolutionary Programming (EP), machine learning, nearest-classifier-algorithm.

1. Introduction

Two important notions in the literature of multifingered robot grippers are *form* and *force closure*. Under a *form closure* grasp any externally applied wrench to the grasped object can be balanced by the grasping forces; whereas under a *force closure* grasp a given external wrench applied on the object can be balanced by the grasping forces. A great deal of research effort has been directed to find a *form* and *force closure* grasp. Most of the work is concentrated on the modelling of the object-gripper system interaction, and then the development of algorithms for generating a secure grasp (see e.g. the excellent survey done by Bicchi, 2000).

The problem of finding *form* and/or *force* closure has been investigated by many researchers (Liu, 1999, 2004; Al-Fahed and Panagiotopoulos, 1992; Markenscoff, Ni and Papadimitriou, 1990; Salisbury and Roth, 1983; Nguyem, 1989; Cutkosky, 1989). All these works consider predefined contact points. Ponce and Faverjon (1995) used the polytope projection method to determine the regions of contact points that yield secure grasp. In Abu-Zitar and Al-Fahed Nuseirat (2001) a rule-based method that determined the optimum grip points is proposed. Markenscoff and Papadimitriou (1989) minimize the worst-case grasping forces needed to balance the externally applied unit force to the grasped object. Mitrich and Canny (1994) first compute the grasps that best counteract pure forces, and then select among these grasps the one which best resists the pure torque. Liu (2004) proposed a heuristic algorithm for searching a *form* closure grasp. In this work the algorithm searches for the contact points that form a *form closure* grasp among many candidate points distributed on the object boundary. Lin, Burdick and Rimon (2000) proposed a quality measure for compliant grasps and fixtures using grasp stiffness matrix. This quality measure is used to select the best fingers' configuration that ensure the secure grasp.

It should be kept in mind that the geometric nonlinearity comes from the unknown kinematic boundary condition; that is, the finger which will be in contact with the object(or the contact region on the object surface) is not known *a priori*, while the material nonlinearity comes from the friction condition. The inequality restriction on the normal contact forces is introduced by the fact that the finger and the object can only push on each other and not pull. These kind of problems are known as unilateral contact problems. Unilateral contact problems with friction have been studied by many researchers (Panagiotopoulos, 1985; Oden and Martins, 1985; Kwak and Lee, 1991). They lead to quasi-variational inequality problems or to nonlinear complementarity problems. In this direction, Al-Fahed Nuseirat and Stavroulakis (2000) suggested a nonlinear complementarity approach (NCP). Han, Trinkle and Li (2000) presented a linear matrix inequality (LMI) approach. Bicchi (1995) proposed an iterative solution of nonlinear ordinary differential equations to solve the problem.

The desired goal is to achieve a stable and firm grip of the grasped object. Hence, the issue of optimizing the grasping forces has been of great importance. Many applications in robotics require a stable grip before any further operations of the robot can be done. The grasp forces have to be exactly balanced against any external wrench. In manipulation the external forces may change their line of action, so the grasping forces need to be adapted in order to keep the object firmly grasped by the gripper. The normal forces have to be within friction cone also. This type of problem may be considered as constraint optimization problem. If the friction cone may be linearized through approximation by a polyhedron then the problem can be formulated as Linear Complementarity Problem (LCP). The yielded LCP could be solved using direct algorithms such as Lemke's algorithm (Al-Fahed, Stavroulakis and Panagiotopoulos, 1992) or it could be solved using Neural Network based methods (Abu-Zitar and Al-Fahed Nuseirat, 2000). This, of course, would be at the expense of accuracy and optimality of solutions (Al-Fahed Nuseirat and Stavroulakis, 2000). The NCP approach, on the other hand, provides a numerically solvable set of equations that lead to better results than that of Linear Complementarity Problem (LCP) (Al-Fahed Nuseirat and Stavroulakis, 2000). However, all these analytical formulation methods and the numerical methods associated with them, have certain degree of accuracy and are affected by the high nonlinearity of the problem. In assembly task applications accuracy in grasping forces is highly required. The need arises for techniques that can generate this type of solutions even at the expense of extra CPU time. This expense is of no importance if the application of the grasping forces is offline, i.e. the solution is not usually required right online for a given grasping task. Moreover, the availability of fast digital computers has diminished the problem of long execution time that usually accompanies the EP based techniques.

The solutions extracted by the EP are instantaneous solutions for some known grip configuration and external force. The problem of providing continuous and online support for the optimum grip in case of new configuration of the external forces and grasp can be overcome with the aid of a rule base that can be used to adapt with the grasping forces. The proposed system, therefore, will be more like a machine learning system that autonomously updates itself to improve its knowledge-base. With adequate initializations during the learning phase, the rule base will reach a point where it can provide an online cover for most of the possible alterations. Once learning reaches saturation, the retrieving shell is used to select the most appropriate rules for any given initial external forces and grasp configuration. The EP is a part of this whole system. Its main job is to extract rules (solutions) that are added to the rule base. The training phase here is time consuming, since the rule base takes a long time to converge to saturation. However, the retrieval phase, and that is the most important, takes little time to find solutions for given grasp configuration. The learning phase involves updating for the rule weights according to the number of times it was selected for retrievals. The weight is an indication of the relative importance of the rule. The selection process involves the usage of some known heuristic technique called the weighted Euclidean distance classifier method (Pao, 1989) to pick the rule whose action will define the values of the grasping forces. A rule base is created for every different grasp configuration that covers a wide range of the potential space of action of the external forces. The retrievals are very efficient and the time delay is much smaller than that needed to find solutions with classical linear programming methods. An efficient search mechanism is used to select the rules during retrievals. The binary search tree method is used to speed up the search for rules in the rule base (Aho, Hopcroft and Ullman, 1994).

The main objective of this work is to present a rule base system to solve a nonlinear gripper problem. The proposed machine learning approach extracts rules to solve the problem with the help of Evolutionary Programming (EP) due to its ability of interfacing with many applications in optimization and machine learning. The obtained results show that the norm of the grasp forces is better than those resulting from the NCP approach. The EP technique considered here is based on algorithms proposed by Fogel (Fogel, 1991). The mechanisms of real life genetics, such as selection and mutation are simulated in EP. The survivability-for-the-fittest is applied to a population of initially randomly generated solutions. The stochastic nature of this technique makes it capable to escape local minima and keep its search toward global solutions. The nonlinearity of the problem is the major motivation for using techniques such as EP. The EP has the ability to handle such circumstances easily. This is due to the richness of solutions EP offers and to the high flexibility it accommodates when dealing with stiff or vague situations.

This paper is organized as follows: in Section 2, the formulation of the problem is presented, followed by the solution of the problem using EP, in Section 3. In Section 4, the proposed Machine Learning system is explored, followed by numerical examples, in Section 5, and finally, in the last section, discussions and conclusions are presented.

2. Problem statement

In this section the formulation of the equilibrium equations and the unilateral contact conditions which arise in the gripper-object system are formulated, followed by the formulation of the constraints introduced by fingers' joints and friction conditions.

2.1. The equilibrium equations and kinematic constraints

For the object of Fig. 1 all external forces and the contact forces should be in equilibrium. The equilibrium equations of the system can be written in the following form:

$$\mathbf{G}\mathbf{r} = \mathbf{P}, \tag{1}$$

where $\mathbf{r} = \{\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_n\}^T$ is the vector of the grasping forces, $\mathbf{r}_i = \{r_{ni}, r_{ti1}, r_{ti2}\}^T$ where r_{ni} is the normal component of the contact forces and r_{ti} are the frictional components (tangential) of the grasping forces, $\mathbf{G} \in \mathcal{R}^{m \times 3n}$ is the equilibrium matrix, and $\mathbf{P} \in \mathcal{R}^m$ is the vector of the external forces applied on the object. The superscript T denotes transpose of matrix or vector.

It should be noted that we choose to neglect kinematical conditions here although we assume that the unilateral contact conditions are true. The goal

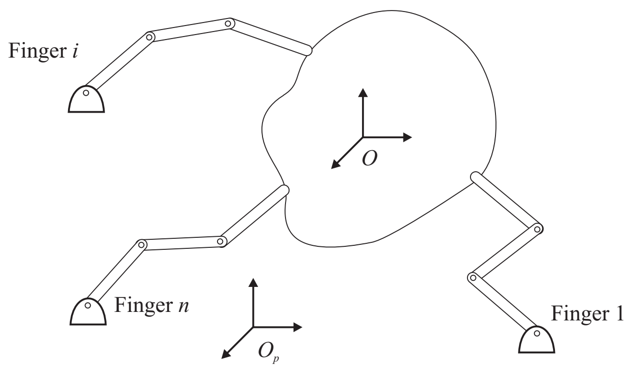

Figure 1. An object in multifingered robot gripper

of our work is mostly to find global optimum values of fingers forces. For further analysis regarding the kinematical conditions see Al-Fahed Nuseirat and Stavroulakis (2000).

The grasp forces also are subject to constraints introduced by fingers' kinematics and design characteristics. These constraints are defined as follows:

$$\begin{aligned} \mathbf{J}_h^T\mathbf{r} &\leq \tau_{max} \\ \mathbf{J}_h^T\mathbf{r} &\geq \tau_{min}\,. \end{aligned} \tag{2}$$

Here

$$\mathbf{J}_h = \text{diag.}[\mathbf{HJ}_1, \mathbf{HJ}_2, \ldots, \mathbf{HJ}_n]$$

where τ_{max} and τ_{min} denote the vectors of the maximum and minimum torques available for the joints of the fingers, $\mathbf{J}_h$ is a $3n \times nk$ global Jacobian matrix, and k is the number of joints in each finger. Moreover, the constraint matrix $\mathbf{H}$ (with dimension equal to 3×6) has the following form

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Rewriting inequalities (2) in a compact form, we obtain

$$\mathbf{J}^T\mathbf{r} \leq \tau \tag{3}$$

where

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_h \\ -\mathbf{J}_h \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_{max} \\ -\tau_{min} \end{bmatrix}.$$

2.2. Friction conditions and the nonlinear optimization problem

In this paper for generality, the following orthotropic friction model is considered. Let the principal orthotropic axes on the tangent plane at the i-th contact point be denoted by 1 and 2 and let r_{ti1}, r_{ti2} be the components of the friction forces along these axes. The corresponding friction coefficients are denoted by μ_{i1} and μ_{i2}. In the frictional model the normal component of the grasping force applied by the i-th finger must satisfy the following relation (Panagiotopoulos, 1985; Michałowski and Mróz, 1978):

$$\gamma_i = |r_{ni}|^2 - \left[(\frac{r_{ti1}}{\mu_{i1}})^2 + (\frac{r_{ti2}}{\mu_{i2}})^2 \right], \qquad \gamma_i \geq 0, \qquad i = 1, 2, \ldots, n \tag{4}$$

where $|*|$ denotes the norm in $\mathcal{R}^3$ and γ_i is the friction cone (domain). The non-slip of the i-th fingertip can be ensured if strict inequality holds in the previous equation. Otherwise there exists a non-negative parameter λ_i (Panagiotopoulos, 1985) such that the slipping values are given by

$$y_{ti1} = -\lambda_i \frac{r_{ti1}}{\mu_{i1}^2} \quad \text{and} \quad y_{ti2} = -\lambda_i \frac{r_{ti2}}{\mu_{i2}^2}. \tag{5}$$

The isotropic friction law is a particular case of the above relation, and it is achieved when $\mu_{i1} = \mu_{i2} = \mu_i$. The friction law can be written in a compact form as follows:

$$\mathbf{B}(\mathbf{r})\mathbf{r} \leq \mathbf{0} \tag{6}$$

where

$$\mathbf{B}(\mathbf{r}) = \text{diag.}[\mathbf{B}(\mathbf{r}_1), \mathbf{B}(\mathbf{r}_2), \ldots, \mathbf{B}(\mathbf{r}_n)]$$

$$\mathbf{B}(\mathbf{r}_i) = \begin{bmatrix} -r_{ni} & \frac{r_{ti1}}{\mu_{i1}^2} & \frac{r_{ti2}}{\mu_{i2}^2} \end{bmatrix}.$$

The optimal grasping forces can be obtained by solving the following nonlinear programming problem:

$$\text{minimize } \frac{1}{2}\mathbf{r}^T\mathbf{r} \tag{7}$$

$$\text{subject to} \quad \begin{aligned} \mathbf{Gr} &= \mathbf{P} \\ \mathbf{Jr} &\leq \tau \\ \mathbf{B}(\mathbf{r})\mathbf{r} &\leq \mathbf{0} \\ \mathbf{Nr} &\leq 0 \end{aligned}$$

where $\mathbf{N} = \text{diag}[\mathbf{N}_1, \mathbf{N}_2, \ldots \mathbf{N}_n]$ and $\mathbf{N}_i = [-1 \ 0 \ 0]$ for $i = 1, 2, \ldots, n$.

3. Solution of the nonlinear gripper problem via the EP

In this section the solution of the Nonlinear Gripper Problem using EP is described. To obtain the objective function used by the EP a nonnegative vectors of slack variables are introduced in order to transform the inequality constraints in (7) to equality, as follows

$$\begin{aligned} &\text{minimize } \frac{1}{2}\mathbf{r}^T\mathbf{r} \\ &\quad\text{subject to} \\ &\qquad \mathbf{Gr} = \mathbf{P} \\ &\qquad \mathbf{Jr} + \mathbf{y} = \tau \\ &\qquad \mathbf{B}(\mathbf{r})\mathbf{r} + \mathbf{z} = \mathbf{0} \\ &\qquad \mathbf{Nr} + \mathbf{u} = \mathbf{0} \end{aligned} \tag{8}$$

where $\mathbf{y} = [y_1^2, y_2^2, \ldots, y_{2nk}^2]^T$, $\mathbf{z} = [z_1^2, z_2^2, \ldots, z_n^2]^T$, and $\mathbf{u} = [u_1^2, u_2^2, \ldots, u_n^2]^T$. The objective function (penalty function) can be obtained as follows:

$$\begin{aligned} E(r) &= k_1 \parallel \mathbf{r} \parallel_2 + k_2 \parallel \mathbf{Gr} - \mathbf{P} \parallel_2 + k_3 \parallel \mathbf{Jr} + \mathbf{y} - \tau \parallel_2 \\ &\quad + k_4 \parallel \mathbf{B}(\mathbf{r})\mathbf{r} + \mathbf{z} \parallel_2 + k_5 \parallel \mathbf{Nr} + \mathbf{u} \parallel_2 \end{aligned} \tag{9}$$

where k_1, k_2, k_3, k_4, and k_5 are weighting constants.

The EP works on a population of strings. The normal components of the the grasping forces compose the elements of each string in the population. Consequently, the length of the string is equal to the the number of fingers, which is assumed to be n. The real values of these elements are initially generated from the uniform random number generator. The EP starts with a fixed population of strings. However, the proposed EP uses an expanding population size strategy. This is closer to what happens in real life. When the population size reaches the maximum allowed limit, half of the members of the population that are the worst elements, according to the objective function measure, are eliminated. This EP has also an adaptive operator for mutation. Mutation is inversely proportional to the fitness of the string. Fitness of the string is inversely proportional to the value of the objective function. The mutation is taken from a normal distribution, whose deviation is also inversely proportional to the fitness of the string being mutated. The equations below show how fitness is calculated for every string in the population, and how the mutation applied on a string is also calculated:

$$\text{Fitness}(\text{string}_j) = 1 - \frac{E(r_{nj})}{\text{Max}\{E(r_{nj})\}_{\text{population}}} \tag{10}$$

where $\text{Max}\{E(r_{nj})\}_{\text{population}}$ means maximum $E(r_{nj})$ in current population. The mutation added on the string associated with r_{nj} (jth normal force) is given by:

$$\text{Mutation}(\text{string}_j) = N(O, CA(\text{strings})) \tag{11}$$

where N denotes the normal distribution function, O means zero mean, C is a weighting constant, and A(strings) is the standard deviation of normal distribution N

$$A(\text{strings}) = \sum_{m=Pop-M}^{Pop} \frac{\text{Fitness}(\text{string}_j)_m}{M}$$

where $\text{Fitness}(\text{string}_j)_m$ is the average fitness of strings of generation m, m is the generation index, Pop is the current generation index, and M is backward steps number in generation index. The flowchart for the proposed EP algorithm is shown in Fig. 2.

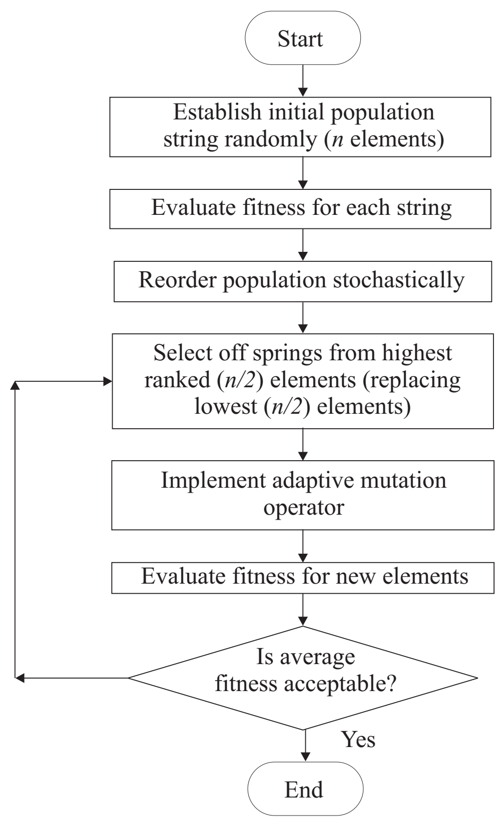

Figure 2. The flowchart of the EP used

4. Machine learning with the aid of EP

The suggested approach for constructing a rule base that provides online solutions for different initial cases is shown in Fig. 3. The EP, as described in the

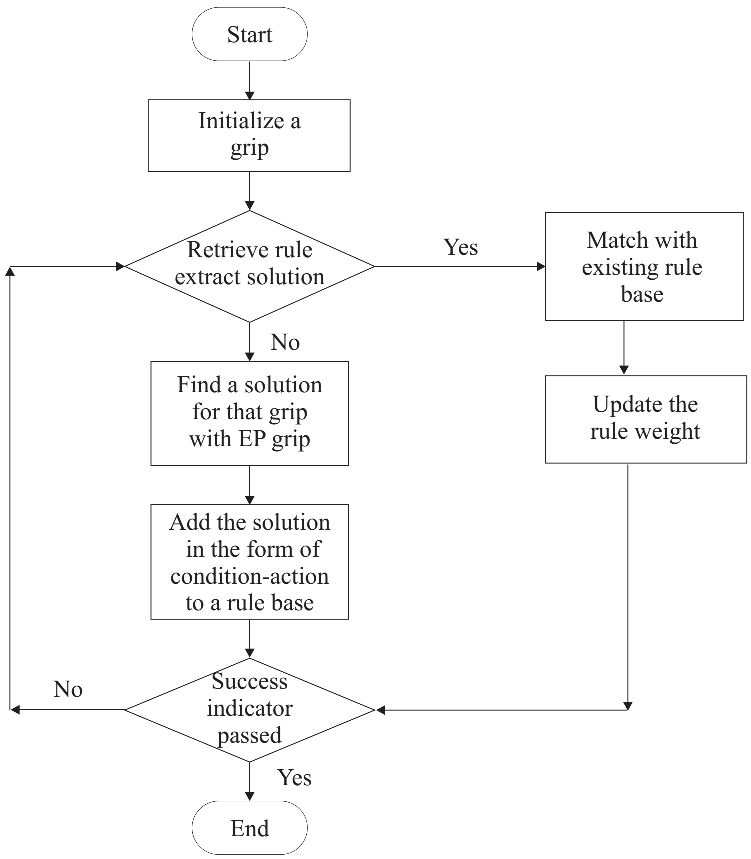

Figure 3. Flowchart of the machine learning system

previous sections plays the major part by extracting solutions for cases with no existing solutions in the rule base. However, if a rule in the rule base with a condition closest to the given initial state exists, then it will be selected and its action will be fired. The weight of the rule is updated to emphasize the relative importance of this rule.

A rule may have the form:

weight: <*condition*>: <*action*>

where all weights initially have similar values and are updated, when selected, with a constant value rp. The condition is a real number that is equal to the externally applied forces. The action is a vector of real numbers in the form r_1, r_2, r_3,..., r_n, where n is the number of fingers.

The selection of rules is done according to the weighted Euclidean distance method as below:

$$d_m(k) = \frac{(P_m - P_m^k)^2}{(w^k)^2}. \tag{12}$$

The P_m stands for the vector of external forces, and P_m^k is the condition of the rule, m is the index of the vector of external forces' initialization, and k is the index of the rule. The weight stands for the weight of the kth rule. This equation measures the weighted closeness (actually: distance) between the external force and the condition of the rule.

The rule in the rule base with the least value of d_m is selected for firing its action directly through the finger tips. However, according to Fig. 3, if the distance d_m was not within ϵ (minimum preset distance) i.e. no matching, the process continues by invoking EP and extracting a solution for that external force and then appending it to the rule base.

5. Numerical examples

In this section numerical examples are provided that illustrate the application of EP in finding minimal grasping forces as well as the application of the machine learning technique applied to find the grasping forces in case of changing the grasp configuration and/or the direction of the externally applied forces. The first two examples illustrate the application of EP to finding the grasping forces for fixed grasp configuration as well as fixed external force. These examples cover both the isotropic friction and the orthotropic friction cases. The three-fingered grasping problem from Al-Fahed Nuseirat and Stavroulakis (2000) is used to demonstrate the effectiveness of the proposed method. The configuration of the example is shown in Fig. 4.

The points of contact with reference to the object coordinate system are

$$r_1 = (0.0, 0.75, 0.75),\ r_2 = (0.75, 0.0, 0.75),\ r_3 = (0.75, 0.75, 0.0)$$

and the normals to the associated contact surfaces are

$$n_1 = (1.0, 0.0, 0.0),\ n_2 = (0.0, 1.0, 0.0),\ n_3 = (0.0, 0.0, 1.0).$$

The externally applied force to the object is assumed to be the object weight acting opposite to the direction of the z-axis and is of magnitude 5. The center of mass is located at point $r_c = (0.5, 0.5, 0.5)$. The friction coefficients are $\mu_1 = \mu_2 = 0.6$. In Table 1 the contact forces and tangential friction forces components are shown. The norm of the normal forces obtained from the EP method was 3.9639 while in Al-Fahed Nuseirat and Stavroulakis (2000) the obtained norm using NCP (Nonlinear Complementarity Problem) approach was

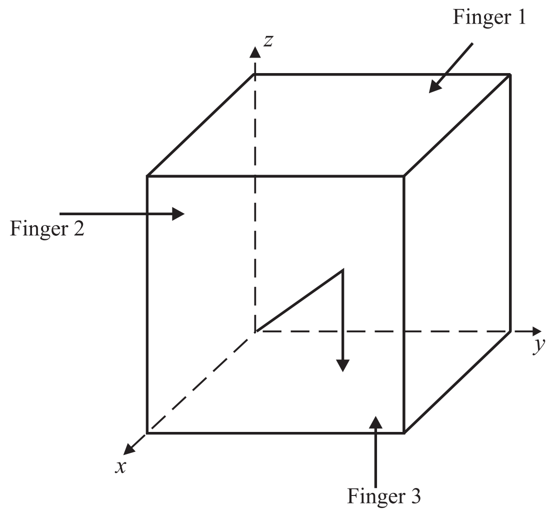

Figure 4. Configuration of the first example

4.0494. The same example was solved using the EP approach, but this time with orthotropic friction coefficients $\mu_1 = (0.5, 0.6, 0.5)$, $\mu_2 = (0.6, 0.5, 0.5)$. The obtained norm of the normal forces was 4.0489 while under the same conditions the obtained norm using NCP approach was 4.1122 (Al-Fahed Nuseirat and Stavroulakis, 2000).

Table 1. Normal contact and friction forces for the example of Fig. 4 using EP

Finger	Contact force	Tangential component	Friction forces
1	1.0496	1	- 0.0958
		2	0.6262
2	1.0597	1	-0.0958
		2	0.6161
3	3.6577	1	-0.9538
		2	-0.9639

As a second example a cube grasped by four fingers (Liu, 1999) was solved using EP approach and LCP (Linear Complementarity Problem) approach (for details about this approach see Al-Fahed and Panagiotopoulos, 1992). The configuration of this example is depicted in Figure 5. The used friction coefficients were $\mu_1 = \mu_2 = 0.6$.

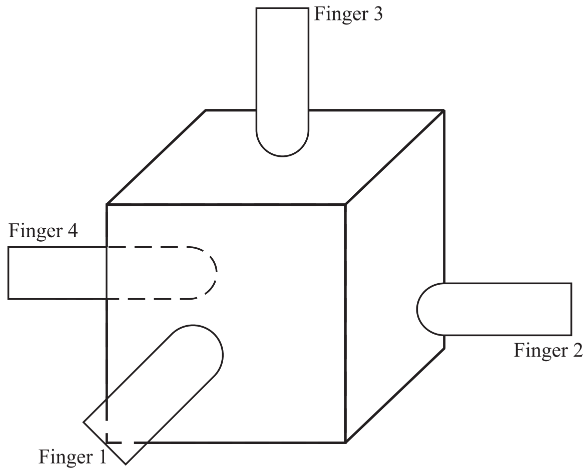

Figure 5. Configuration of the second example

The points of contact with reference to the object coordinate system are $r_1 = (2.0, 0.0, 0.0)$, $r_2 = (0.0, 1.5, 0.0)$, $r_3 = (0.0, 0.0, 2.0)$, $r_4 = (1.2, -2.0, 0.0)$ and the normals to the associated contact surfaces are
$n_1 = (-1.0, 0.0, 0.0)$, $n_2 = (0.0, -1.0, 0.0)$, $n_3 = (0.0, 0.0, -1.0)$,
$n_4 = (0.0, 1.0, 0.0)$.
with external forces $\mathbf{P} = (-0.2, -1.0, -2.0, -0.2, -0.3, -0.2)^T$.

In Tables 2 and 3, the contact forces, tangential friction forces components, obtained using LCP and EP approaches are shown, respectively. The norms of the normal forces, for both the LCP and the EP approaches where 1.7072 and 1.6208.

Table 2. Normal contact and friction forces for the example of Fig. 5 using LCP method

Finger	Contact force	Tangential component	Friction forces
1	0.1741	1	0.0
		2	0.0
2	0.9251	1	-0.2000
		2	-0.5128
3	1.4124	1	-0.1949
		2	-0.2098
4	0.1349	1	-0.0310
		2	0.0748

Table 3. Normal contact and friction forces for the example of Fig. 5 using EP method

Finger	Contact force	Tangential component	Friction forces
1	0.1350	1	-0.0368
		2	-0.0751
2	0.9267	1	-0.2098
		2	-0.5142
3	1.3141	1	-0.2825
		2	-0.1892
4	0.1526	1	0.0076
		2	0.0965

The same simulations for both cubes were repeated, but this time with orthotropic conditions. For the example of Fig. 3 these conditions were $\mu_1 = (0.5, 0.6, 0.5)$, $\mu_2 = (0.6, 0.5, 0.5)$. Table 4 shows the forces values obtained using EP approach. Table 5 shows the results for the same example using the LCP method. The obtained norms of the normal contact forces from both methods were 4.0265 and 4.0489, respectively. The orthotropic conditions used for the example of Fig. 4 were $\mu_1 = (0.5, 0.8, 0.5, 0.8)$, $\mu_2 = (0.8, 0.5, 0.8, 0.5)$. Tables 6 and 7 show force values for both LCP and EP approaches. The norms of the normal forces were 1.7918 and 1.7146, respectively.

Table 4. Normal contact and friction forces for the example of Fig. 4 with orthotropic conditions using EP method

Finger	Contact force	Tangential component	Friction forces
1	1.0899	1	-0.0570
		2	0.6371
2	1.0931	1	-0.0570
		2	0.6340
3	3.7189	1	-1.0329
		2	-1.0360

A third example is given for the purpose of demonstrating our machine learning system. We choose a two-dimensional polygon shaped rigid object grasped by a three-fingered gripper as shown in Fig. 6.

As explained in the previous section, the system used EP in extracting solutions for the different values of external forces. The object edges are assumed to have different friction coefficient for each edge. In Table 8, the external forces, the positions of the contact points, and the resulting grasping forces are shown.

Table 5. Normal contact and friction forces for the example of Fig. 4 with orthotropic conditions using LCP method

Finger	Contact force	Tangential component	Friction forces
1	0.9583	1	0.0
		2	0.7083
2	1.1400	1	0.0
		2	0.5266
3	3.7651	1	-0.9583
		2	-1.1400

Table 6. Normal contact and friction forces for the example of Fig. 5 under orthotropic conditions using EP method

Finger	Contact force	Tangential component	Friction forces
1	0.1572	1	0.0
		2	0.0
2	0.9416	1	-0.1760
		2	-0.4350
3	1.5120	1	-0.1818
		2	-0.1732
4	0.1148	1	-0.0369
		2	0.0530

Table 7. Normal contact and friction forces for the example of Fig. 5 under orthotropic conditions using EP method

Finger	Contact force	Tangential component	Friction forces
1	0.0697	1	-0.0043
		2	-0.0414
2	0.8744	1	-0.1296
		2	-0.4628
3	1.4699	1	-0.2064
		2	-0.2212
4	0.0999	1	-0.0535
		2	0.0259

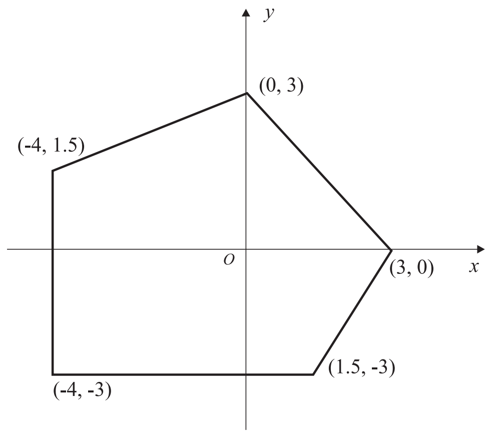

Figure 6. Configuration of the third example

Table 8. Samples of solutions extracted for different cases for the third example

Load **P**= (-0.2232, 0.4744,4.9724)	μ = (0.1,0.4, 0.6)	
Finger and its coordinates	Normal force	Friction forces
1 (x = -4.0000, y = -2.4297)	0.1763	-0.0086
2 (x = -2.8352, y = -3.000)	0.5929	-0.2284
3 (x = 2.9494, y = -0.0999)	0.6956	-0.3556
Load **P**= (0.8769, 0.8235, 4.8531)	μ= (0.1, 0.4, 0.6)	
Finger and its coordinates	Normal force	Friction forces
1 (x = -4.0000, y = -2.5830)	1.1861	0.1171
2 (x = -0.1767, y = -3.0000)	0.0697	0.0117
3 (x = 2.6024, y = -0.7951)	1.5296	-0.0427
Load **P**= (0.0, 0.0, -5.0)	μ = (0.2, 0.3, 0.4)	
Finger and its coordinates	Normal force	Friction forces
1 (x = 2.118, y = 0.8820)	0.3200	0.0334
2 (x = -0.0813, y = 2.9695)	3.1515	-0.2007
3 (x = -3.5164, y = -3.0000)	2.4476	0.9453
Load **P**= (4.3543, 2.3938, 4.9724)	μ = (0.3,0.1, 0.4)	
Finger and its coordinates	Normal force	Friction forces
1 (x = -0.7978, y = 2.7008)	1.1449	0.3160
2 (x = -4.0000, y = -1.5270)	0.6795	0.0580
3 (x = -2.5011, y = -3.000)	0.4495	0.1749

In a manner similar to the explained technique, a rule base is built by appending rules for every finger configuration in the form shown in previous section. Some solutions (finger force values) were feasible for some external forces values, other external forces values had no solutions for the same grip and a new grip (finger configuration) has to be formulated in an attempt to find a solution. The proposed system starts building rules by combining external forces with finger forces in condition/action pairs and for different values of external forces. We assume that there are external forces which are more frequent than others. Hence, finding solutions for these values is given higher priority when building the rule base. A prior knowledge of the nature of external forces would help a lot in building the rule base. However, when an odd value of external force affects the rigid body, the nearest weighted classifier algorithm chooses the most appropriate rule and uses its action. This mechanism would help in deferring the effect of this force momentarily until an exact solution is extracted. Our system has the ability to dynamically update its rule base for every new external force it gets subjected to. The more the system is used, the more knowledge it gains, and the better the quality of solutions it offers.

6. Conclusions and discussions

The goal of the presented work is to provide a system capable of offering quality solutions for different expected values of external forces (loads). The delays and the transitions associated with selecting new fingers' forces to handle changes in the external forces are minimal, especially if the same existing fingers' distributions were used. In some cases when there is no available solution, i.e. no matched rules or no available rules within the minimum distance, the EP extracts a solution and that is considered as a momentary switch to the offline solution. The percentage of offline solutions decreases as the system extracts more and more rules and appends them to the existing ones. Eventually, wider regions in the control space are covered and more tolerant and reliable system is developed. The average number of rules in a rule base for the example in previous section, assuming that the external forces (loads) cover the whole space is around 1500 to 2000 rules. As we mentioned earlier, some regions do not have solutions at all, on the other hand, some regions have larger concentration of rules than other regions. That actually depends on the sampling during training and the priori knowledge we have on expected values of external forces. The search mechanism for the matched rules is done in a manner similar to a binary search tree method where the Euclidean distance between the values of the external forces (load) and the values of the conditions of rules control the access of the branches of the search tree. Only branches that lead to less distances are accessed. In that manner much less search time is spent in selecting the closest-condition rule before its action is fired. The weakness of this method is that it is extremely difficult to provide excellent solutions for all types of grips and all values of external forces. A possible solution for this dilemma is to use the fuzzy logic (Cos, 1994). By using fuzzy logic, wider areas in the control space could be covered by much less rules and the transitions from a state to another state of the system are expected to be smoother. Future work may include ”fuzzifying” the rules in our rule base and implementing them in some standard fuzzy rule system. This system could eventually be integrated with the system we have to end up with a ”Fuzzy Machine Learning System” used for extracting solutions for the gripper problem. The biggest problem we have with this type of systems is the complexity and the larger rule bases that we need, especially if the system is required to find solutions for arbitrary rigid bodies, arbitrary fingers' distributions around the object or grips' configurations. Luckily, the advancement in software and hardware nowadays helps in diminishing this problem.

Another goal of this work is to show the capabilities of evolutionary based methods in finding close to ultimate solutions for the gripper problem. It is worth mentioning that the EP and NCP methods are nonlinear techniques that do not need linearization of the problem through the linearization of the friction law and then finding the solutions. Previous work (Al-Fahed Nuseirat and Stavroulakis, 2000) showed that linearization of the friction law may result in solutions that are quite far from the actual optimal solutions. Many applications are very sensitive to the forces applied by the fingers and may damage the object. The flexibility of the EP and its ability to “dig” deep into the search space of the forces enables it to come up with near optimum solutions. The continuous evolution of the strings pool that is always updated by mutation and reproduction operators is the main reason for the richness of solutions.Al-Fahed Nuseirat and Stavroulakis (2000) solved the NCP using PATH algorithm which is a deterministic nonlinear optimization algorithm (Ferris, Mensier and More, 1997). Deterministic techniques take one direction to an optimal solution. Sometimes, due to the numerical nature of the algorithm and the shape of the search space, the numerical algorithms settle in a region that is not optimal. The deterministic operators of that algorithm can not allow it to skip that region. The Tables 1 through 7 show solid evidence of the better solutions generated by the EP. However, EP is much more computationally costly than the PATH algorithm when solving the NCP. This is a general problem with all evolutionary based methods. Fortunately, many gripper applications are off-line applications. That is, the minimal gripper fingers' forces are calculated only one time and then used for the rest of the time. If the external force varies from time to time, then the rule base is used to find needed adaptation on grasping forces that maintain the firm grip of the object.

Observing the figures in Tables 1 through 7, we find that in the case with the cube of the first example three fingers are enough to grasp the cube, and for both the isotropic and orthotropic cases the EP provided lower values for the normal contact forces by around 2.5%. In the case of second example of the cube with four fingers and for both isotropic and orthotropic cases, figures in the tables show that the EP provided solutions with minimal normal contact forces by around 5% lower than those provided by the NCP algorithm. As mentioned earlier, these savings in minimal forces, although relatively small, can be very useful from the engineering point of view especially if the grasped object is crushable. This, consequently, means more savings in energy and better chance in protecting the grasped body from any possible damage.

Fig. 7 shows the process of evolving solutions by EP. To study this problem we picked the orthotropic case of the second example. The upper graph in the figure shows at each point a minimal norm of total forces (normal and tangential) used by the gripper to grasp the object. The lower graph shows at each point a minimal norm of only normal forces used by the gripper to grasp the object. Each point in both graphs is a possible solution. However, the last point in each graph represents the minimum solution. In general, solutions generated by EP tend to converge faster at the beginning toward the optimum one. As number of iterations increases, EP starts to move slower toward the minimal point until it tends to settle. There, EP hardly moves anywhere else. It is clear from the graphs also that there is constant difference in value between the norm of total forces and the norm of normal forces. Solutions at earlier stages have the same difference between total and normal forces compared to solutions at final stages. This emphasizes that this difference depends on the properties of the grasped body itself and not merely on the values of generated forces themselves.

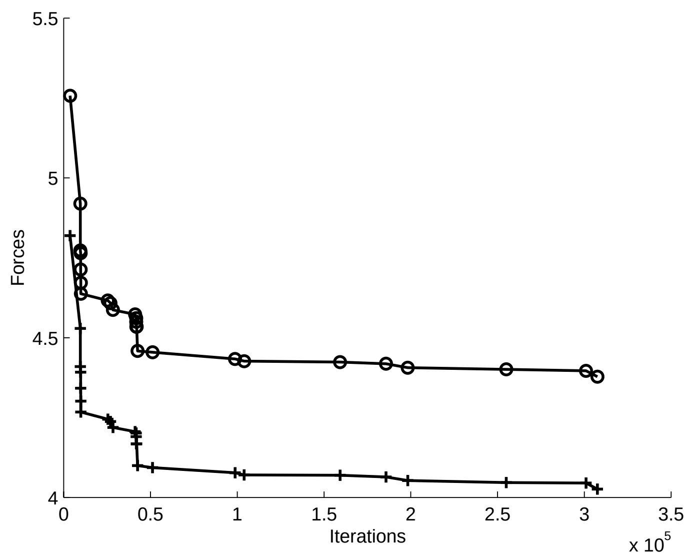

Figure 7. Minimal grasp forces for orthotropic case of second example

A. Analysis and proof that objective function of EP has optimal solution

This appendix is concerned with the solution of the minimization problem stated by equations (8). The Lagrangian function for this problem can be constructed as

$$L(\mathbf{r},\mathbf{y},\mathbf{z},\mathbf{u},\lambda,\eta,\gamma,\varrho) = \frac{1}{2}\mathbf{r}^T\mathbf{r} + \lambda^T(\mathbf{Gr}-\mathbf{P}) + \eta^T(\mathbf{Jr}+\mathbf{y}-\tau) + \gamma^T(\mathbf{B}(\mathbf{r})\mathbf{r}+\mathbf{z}) + \varrho(\mathbf{Nr}+\mathbf{u}) \tag{13}$$

where λ, η , γ, and ϱ are vectors of Lagrangian multipliers.

According to the classical optimization theory (Luenberger, 1984; Murty, 1988), the stationary points of the Lagrangian function can be found by solving the following equations:

$$\frac{\partial L}{\partial \mathbf{r}} = \mathbf{r} + \mathbf{G}^T\lambda + \mathbf{J}^T\eta + (\frac{\partial}{\partial \mathbf{r}}(\mathbf{B}(\mathbf{r})\mathbf{r}))^T\gamma + \mathbf{N}^T\varrho = \mathbf{0} \tag{14}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{Gr} - \mathbf{P} = \mathbf{0} \tag{15}$$

$$\frac{\partial L}{\partial \eta} = \mathbf{Jr} + \mathbf{y} - \tau = \mathbf{0} \tag{16}$$

$$\frac{\partial L}{\partial \gamma} = \mathbf{B}(\mathbf{r})\mathbf{r} + \mathbf{z} = \mathbf{0} \tag{17}$$

$$\frac{\partial L}{\partial \mathbf{y}} = 2\mathbf{x} = \mathbf{0} \tag{18}$$

$$\frac{\partial L}{\partial \mathbf{z}} = 2\mathbf{v} = \mathbf{0} \tag{19}$$

$$\frac{\partial L}{\partial \mathbf{u}} = 2\mathbf{w} = \mathbf{0} \tag{20}$$

where $\mathbf{x} = [\eta_1 y_1, \eta_2 y_2, \ldots, \eta_{2nk} y_{2nk}]^T$, $\mathbf{v} = [\gamma_1 z_1, \gamma_2 z_2, \ldots, \gamma_n z_n]^T$, and $\mathbf{w} = [\varrho_1 u_1, \varrho_2 u_2, \ldots, \varrho_n u_n]^T$.

Equations (14)-(19) are the necessary conditions that ensure the relative minimum of our problem. This relative minimum is global minimum since the cost function is convex (quadratic energy function). Hence, equations (8) have a minimal solution, and that solution is global optimum.

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