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Determining the settings of PI and PID controllers with a convergent method using computer aided design

by

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Abstract: The paper presents a new method applicable in determining the settings of PI and PID controllers for models of controlled systems given by the following transfer functions

$$G_{12}(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$$

and

$$G_{23}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}\,.$$

Using the capabilities of computer operations with symbolic variables, the closed system transfer function is expanded into a chain-type fraction, type V (Bultheel, 1978; Khovanskii, 1956; Halawa and Trzmielak-Stanisławska, 1986; Wall, 1973) and the second order convergent method is used as a simplified model. Denominator of this convergent $M(s)$, is the following polynomial

$$M(s) = s^2 + 2ns + \omega_0^2 = s^2 + 2\xi\omega_0 s + \omega_0^2$$

with coefficients being the functions of controlled system parameters and unknown controller settings. Diagrams of step responses for systems given by second-order transfer functions are well known. Assuming the values of ω_0^2 and $n = \xi\omega_0$, we get a set of non-linear equations used to determine controller settings. The set of equations has been determined and resolved by means of Mathematica language which allows calculations with symbolic variables.

Keywords: system dynamics, selection of controller settings, convergent method.

1. Introduction

The convergent method used to determine PI and PID controller settings will be discussed for two mathematical models of controlled systems, namely

$$G_{12}(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \tag{1}$$

and

$$G_{23}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}\,. \tag{2}$$

In general, responses of these models approximate the higher-order controlled system under identification with the accuracy sufficient for engineering applications. Coefficients of transfer functions (1) and (2) can be found from the controlled system identification procedure using, for instance, the momentum method (Halawa, 1989) or neural networks (Narendra and Pathasarathy, 1990; Phama and Singh, 1993). The method under consideration can be also used for integrating type controlled systems given, for example, by the transfer function

$$G_{02}(s) = \frac{k}{s(Ts+1)}$$

or the transfer function in series with a delay unit given by the transfer function e^{-st_0}. Instead of the transfer function e^{-st_0} we take then the approximation given by its expansion into Padé series (Baker and Graves-Morris, 1981). The introduced method is a novel one. It serves to determine controller settings for the supposed simplified model of controlled system. The method can be used to run computer-aided simulation studies and to determine dynamics of closed-loop control systems. Block diagram of the controlled system under consideration is shown in Fig. 1.

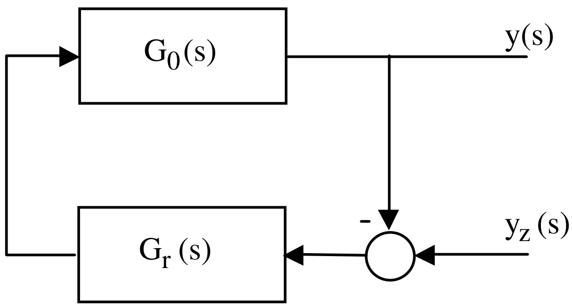

Figure 1. Block diagram of control system

The $G_0(s)$ transfer function depicts the controlled system model. The transfer function of PID controller is the following function

$$G_r(s) = k_p \left(1 + \frac{1}{T_i s} + T_d s\right) . \tag{3}$$

The transfer function of closed control system with the controlled system (1) and controller (3) is given by the formula

$$\begin{aligned} G_{z2}(s) &= \frac{y_2(s)}{y_z(s)} \\ &= \frac{b_1 k_p T_d s^3 + (b_1 k_p + b_0 k_p T_d) s^2 + \left(\frac{b_1 k_p}{T_i} + b_0 k_p\right) s + \frac{b_0 k_p}{T_i}}{(b_1 k_p T_d + 1) s^3 + (b_1 k_p + b_0 k_p T_d + a_1) s^2 + \left(\frac{b_1 k_p}{T_i} + b_0 k_p + a_0\right) s + \frac{b_0 k_p}{T_i}} . \end{aligned} \tag{4}$$

For the model described by the transfer function (2), the transfer function for the closed control system is the rational function

$$G_{z3}(s) = \frac{y_{z3}}{x(s)} = \frac{L_{z3}(s)}{M_{z3}(s)} \tag{5}$$

where

$$\begin{aligned} L_{z3}(s) =& b_2 k_p T_d s^4 + (b_2 k_p + b_1 k_p T_d) s^3 + \left(\frac{b_2 k_p}{T_i} + b_1 k_p + b_0 k_p T_d\right) s^2 + \\ &+ \left(\frac{b_1 k_p}{T_i} + b_0 k_p\right) s + \frac{b_0 k_p}{T_i} \end{aligned}$$

and

$$\begin{aligned} M_{z3}(s) &= (b_2 k_p T_w + 1) s^4 + (b_1 k_p T_w + b_2 k_p + a_2) s^3 + \\ &+ \left(a_1 + \frac{b_2 k_p}{T_i} + b_1 k_p + b_0 k_p T_w\right) s^2 + \left(a_0 + \frac{b_1 k_p}{T_i} + b_0 k_p\right) s + \frac{b_0 k_p}{T_i} . \end{aligned}$$

The transfer function of the closed system with models (1) and (2) is then the rational function of s given as

$$G_z(s) = \frac{c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4}{d_0 + d_1 s + d_2 s^2 + d_3 s^3 + d_4 s^4} . \tag{6}$$

The coefficients of this transfer function are, due to (4) and (5), the functions of coefficients from the controlled system model selected and the controller. The parameters of the controlled system model are given; they are obtained as a result of identification. The settings of the controller shall be selected so as the response of closed control system approximate the selected waveform of response for the second-order system. We shall search for such controller setting which gives step response without overshoot and which stabilizes within a short time. We assume the transfer function describing the second-order model of the form

$$G_2(s) = \frac{y(s)}{x(s)} = \frac{k\omega_0^2}{s^2 + 2ns + \omega_0^2} = \frac{k\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}\,. \tag{7}$$

Fig. 2 illustrates step responses for the system described by the transfer function

$$G_2(s) = \frac{1}{s^2 + 2\xi\, s + 1} \tag{8}$$

for $k = 1,\ \xi = 1$ and several values of ω_0, namely $\omega_0 = 0.5,\ 1,\ 2,\ 4,\ 10$.

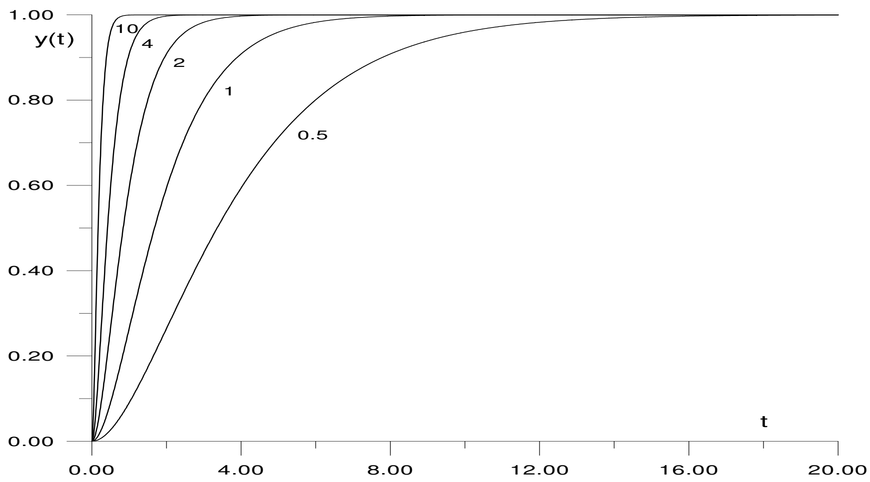

Figure 2. Step responses for the model described by transfer function (8) for $k = 1,\ \xi = 1$ and several values of ω_0

2. Simplified second-order model

As a result of expanding the transfer function of the closed system (the initial transfer function) into a chain fraction and replacing it with the convergent part of this expansion, we are given the simplified system transfer function, whose response waveform is close to the response of the initial closed system to the same forcing function $x(t)$. We assume the convergent $G_{12}(s)$ as a simplified model resulting from expanding the rational function (6) into chain fraction, type V, T or C (Bultheel, 1978; Khovanskii, 1956; Halawa and Trzmielak-Stanisławska, 1986; Wall, 1973), given by the formula

$$G_{12}(s) = \frac{B_1 s + B_0}{s^2 + A_1 s + A_0} \tag{9}$$

where

$$\begin{aligned}
A_0 &= \frac{a_{41} c_0 d_0}{a_{51}}\,, \\
A_1 &= \frac{a_{31}^2 a_{41} + a_{51} c_0 d_0 + a_{41}^2 d_0}{a_{31} a_{51}}\,, \\
B_0 &= \frac{c_0}{d_0} A_0,\ B_1 = \frac{a_{51} c_0^2 + a_{41}^2 c_0}{a_{31} a_{51}} \\
u &= c_0 d_3 - c_3 d_0\,, \\
v &= c_0 d_2 - c_2 d_0\,, \\
a_{31} &= c_0 d_1 - c_1 d_0\,, \\
a_{41} &= a_{31} c_1 - c_0 v\,, \\
a_{51} &= a_{41} v - a_{31}(a_{31} c_2 - u c_0)\,.
\end{aligned} \tag{10}$$

The settings of the PI controller can be calculated by comparing the calculated coefficients, A_0 and A_1, with the prescribed coefficients ω_0^2 and $\xi\omega_0$. As a result, a set of equations is given which is used to calculate the settings k_p and T_i. The settings of the PID controller are derived from the relationships

$$\begin{aligned}
k_p &= f_1(T_d) \\
T_i &= f_2(T_d).
\end{aligned}$$

The value of T_d setting is assumed, while the corresponding positive values of k_p and T_i settings are calculated. Then, the vector of k_p, T_i, T_d settings is selected which, according to computer simulation, gives the most interesting step response waveform for the closed control system.

In order to explain how the simplified model is determined with the chain fraction method (convergent method), we outline here the expansion of rational functions (and the transfer function is a rational function) to the V-type chain fraction.

Expansion into V-type chain fraction

The method of chain (continuous) fractions or the method of convergents is used to determine simplified transfer functions. It consists in replacing the transfer function describing the initial mathematical model (an initial transfer function) with the one of convergents of its expansion into a chain fraction (a simplified transfer function). Many functions may be calculated by expanding them into chain fractions, including such functions, whose expansion into power series is slowly-convergent or even divergent (Danilow, Iwanowa and Isakowa, 1970). Expansions into chain fractions are fast-convergent. They are used in approximate computer calculations. A symbolic notation of chain fraction proposed by Pringsheim will be used in this paper (Danilow, Iwanowa and Isakowa, 1970). In this notation, the chain fraction

$$G(s) = \cfrac{1}{h_1 + \cfrac{1}{\cfrac{h_2}{s} + \cfrac{1}{h_3 + \cfrac{1}{\cfrac{h_4}{s} \ddots}}}} = \cfrac{1}{h_1 + \cfrac{s}{h_2 + \cfrac{s}{h_3 + \cfrac{s}{h_4} \ddots}}}.$$

is written as

$$G(s) = \frac{1|}{|h_1} + \frac{1|}{\left|\frac{h_2}{s}\right.} + \frac{1|}{|h_3} + \frac{1|}{\left|\frac{h_4}{s}\right.} + ... = \frac{1|}{|h_1} + \frac{s|}{|h_2} + \frac{s|}{|h_3} + \frac{s|}{|h_4} \cdots$$

Viskovatov's Algorithm (Khovanskii, 1956)

With this algorithm, the transfer function (a rational function)

$$G(s) = \frac{a_{21} + a_{22}s + a_{23}s^2 + ... + a_{2n}s^{n-1}}{a_{11} + a_{12}s + a_{13}s^2 + ... + a_{1n}s^{n-1}} \tag{11}$$

is expanded into a chain fraction of the form (the V-type fraction)

$$G(s) = \frac{a_{21}}{a_{11}} + \sum_{k=2}^{2n} \frac{a_{k+1,1}s|}{|a_{k,1}}. \tag{12}$$

The algorithm is executed as outlined below. A table is created like Table 1, wherein the first row is filled with successive coefficients of the transfer function's (11) denominator. The first column includes the free term and the next columns – successive terms of the denominator. The second row of Table 1 is, likewise, filled with coefficients of the numerator of the transfer function considered. The elements a_{jk} of this table are calculated from the formula:

$$a_{jk} = a_{j-1,1}\, a_{j-2,k+1} - a_{j-2,1}\, a_{j-1,k+1} = -\begin{vmatrix} a_{j-2,1} & a_{j-2,k+1} \\ a_{j-1,1} & a_{j-1,k+1} \end{vmatrix}, \tag{13}$$

where $j = 3, 4, ..., 2n,\ k = 1, 2, ..., 2n$.

Elements of the first column in Table 1 are elements of chain fraction (12) without the argument of function $G(s)$. In order to get a simplified (convergent) transfer function, whose numerator is of lower order than the denominator, and the denominator is of the second order, it is sufficient to calculate the elements

Table 1. Determination of elements of chain fraction (12)

j \ k	1	2	3	4	5
1	a_{11}	a_{12}	a_{13}	a_{14}	...
2	a_{21}	a_{22}	a_{23}	a_{24}	...
3	a_{31}	a_{32}	a_{33}	a_{34}	...
4	a_{41}	a_{42}	a_{43}	a_{44}	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

from the first column of Table 1 till the element a_{51} inclusive. When the first and second rows of Table 1 are filled with coefficients of transfer function (6) and upon expanding it into V-type chain fraction, we get the relation (10). The simplified transfer function with coefficients (9) is used to determine the settings of controllers. If the term a_{j1}=0, where $j \geq 3$, then the calculation procedure for coefficients a_{jk} will change. The procedure to be used in such a case is outlined in Khovanskii (1956). The paper deals with simplified models, hence the author permits to introduce a small change of coefficient in the numerator of the transfer function if the case of dividing by zero exists. The problem will be explained by the example of expanding the transfer function (31), which describes a closed system of unknown controller settings, into a chain fraction.

Example 2.1

An example of expanding a rational function into a chain fraction will now be analyzed.

Let us expand into a chain fraction the transfer function of numerical coefficients as follows

$$G(s) = \frac{y(s)}{x(s)} = \frac{1}{(s+1)^4} = \frac{1}{s^4 + 4s^3 + 6s^2 + 4s + 1} \, . \tag{14}$$

When the transfer function (14) is expanded into a V-type chain fraction (Bultheel, 1978; Khovanskii, 1956; Halawa and Trzmielak-Stanisławska, 1986; Wall, 1973), we obtain

$$G_{01}(s) = \frac{y_{01}(s)}{x(s)} = \frac{0.25}{s + 0.25} \, . \tag{15}$$

The second- and third-order convergents are the functions

$$G_{12}(s) = \frac{y_{12}(s)}{x(s)} = \frac{-0.2s + 0.3}{s^2 + s + 0.3} \, , \tag{16}$$

$$G_{23}(s) = \frac{y_{23}(s)}{x(s)} = \frac{0.05s^2 - 0.2s + 0.5}{s^3 + 2.25s^2 + 1.8s + 0.5} \, . \tag{17}$$

The chain fractions are fast-convergent (Danilow, Iwanowa and Isakowa, 1970). Fig. 3 illustrates step responses of models described with initial transfer function (14) and simplified transfer functions (16) and (17). The curves of $y(s)$ and $y_{23}(s)$ responses overlap.

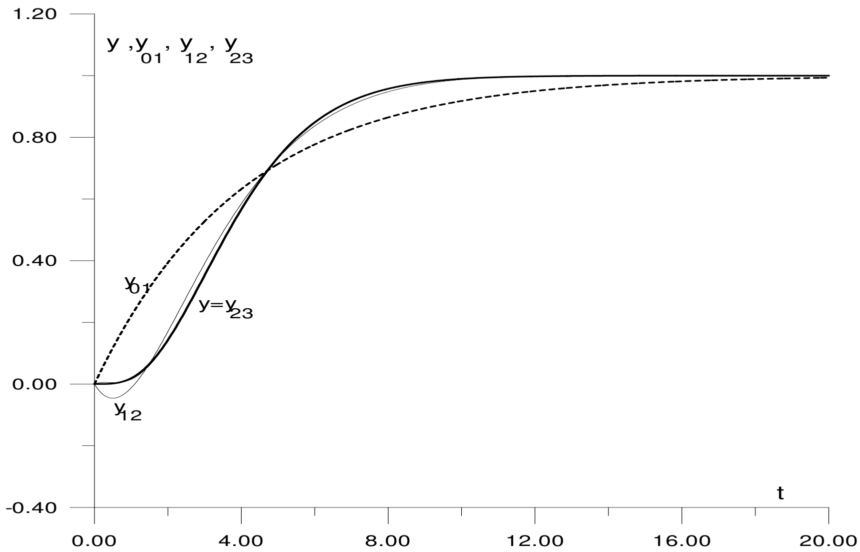

Figure 3. Step responses for models described with transfer functions (14), (16) and (17)

In the case of a PI controller, upon comparing the calculated coefficients A_0 and A_1 with prescribed coefficients ω_0^2 and $\xi\omega_0$, we get a set of equations wherefrom the settings k_p and T_i are calculated. The settings for PID controller are calculated from the relations

$$k_p = f_1(T_d)$$
$$T_i = f_2(T_d).$$

The value of T_d is assumed and relevant positive values of k_p and T_i settings are calculated. Assuming several values of T_d and using computer simulation, selection is made of such vector of k_p, T_i, T_d settings, which provides the better step response for closed control system. Computer simulation is used to get these step response curves.

3. Determination of settings for PI and PID controllers for second-order model of controlled system

As the results from (4) and (6), we have in this case the following:

$$
\begin{aligned}
c_0 &= \frac{b_0 k_p}{T_i},\\
c_1 &= \frac{b_1 k_p}{T_i} + b_0 k_p,\\
c_2 &= b_1 k_p + b_0 k_p T_d,\\
c_3 &= b_1 k_p T_d,\\
d_0 &= c_0,\\
d_1 &= \frac{b_1 k_p}{T_i} + b_0 k_p + a_0,\\
d_2 &= b_1 k_p + b_0 k_p T_d + a_1,\\
d_3 &= b_1 k_p T_d + 1.
\end{aligned}
\tag{18}
$$

Upon comparing the settings (9) with the prescribed values ω_0^2 and $2\xi\omega_0 = 2n$, and while making allowance of (18), we get the following set of equations

$$
\omega_0^2 = \frac{b_0 k_p[-a_1 b_0 + a_0(b_1 + b_0 T_i)]}{T_i[-a_1^2 b_0 + a_0(b_0 + a_1 b_1 + a_1 b_0 T_i) - a_0^2 T_i(b_1 + b_0 T_d)]}\,, \tag{19}
$$

$$
2n = 2\xi\omega_0 = \frac{L_2}{M_2}\,, \tag{20}
$$

where

$$
\begin{aligned}
L_2 = &- a_0 b_1(b_1 k_p + a_0 T_i) + b_0(a_1 - a_0 T_i)(b_1 k_p + a_0 T_i) +\\
&+ b_0^2 k_p[-1 + a_1 T_i + a_0 T_i(T_d - T_i)]\,,
\end{aligned}
$$

and

$$
M_2 = T_i[a_1^2 b_0 - a_0(b_0 + a_1 b_1 + a_1 b_0 T_i) + a_0^2 T_i(b_1 + b_0 T_d)]\,.
$$

When PI controller is considered, we assume the constant T_d in formulae (18-20) to be equal to 0.

As aforementioned, the settings of PID controller are derived by resolving the set of equations for the assumed value of T_d .

Example 3.1

Consider the closed control system with the controlled system given by the transfer function

$$
G_0(s) = \frac{y_0(s)}{x(s)} = \frac{-0.5s + 1}{s^2 + 1.2s + 1} \tag{21}
$$

and a PI controller. The step response for the model described by the transfer function (21) is shown in Fig. 4. The roots of denominator of the transfer function (21) are complex numbers. The model (21) is of the differential type.

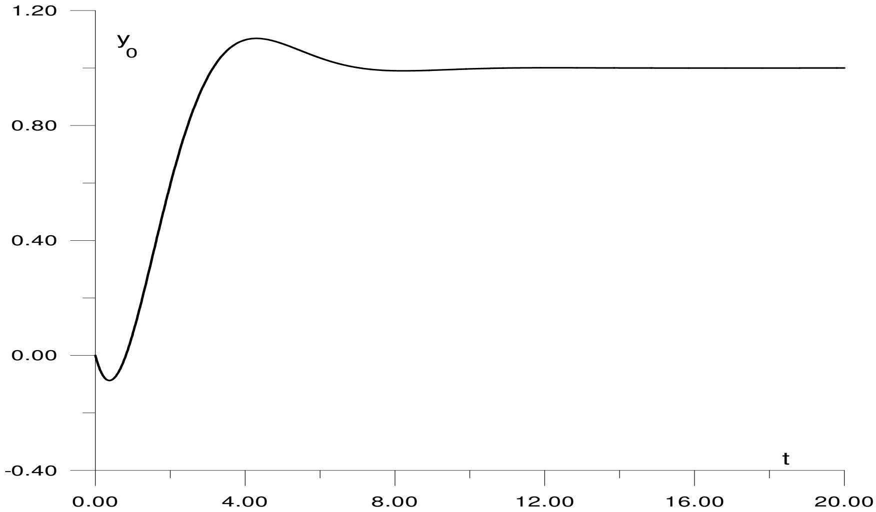

Figure 4. Step response of system given by transfer function (21)

If we want to obtain step response without overshoot, the value of ξ should be sufficiently large (over 1). Assume

$$\omega_0 = 10 \text{ and } n = 120 \tag{22}$$

so that $\xi = 12$. For the model (21) and the assumed values (22), we get the following set of equations

$$\begin{aligned} &170T_i^2 + (-104 - k_p)T_i + 1.7k_p = 0\,, \\ &(407 - k_p)T_i^2 + (-247.9 + 1.7k_p)T_i - 1.85k_p = 0\,. \end{aligned} \tag{23}$$

The non-zero real solution is

$$\begin{aligned} &k_p = 0.1749\,, \\ &T_i = 0.6099\,. \end{aligned} \tag{24}$$

There are also two complex solutions for the set of equations (23), namely

$$\begin{aligned} &K_{p1} = 592.4130 - 0.37774i, \\ &T_{i1} = 2.0480 + 1.3137i, \\ &K_{p2} = 592.4130 + 0.37774i, \\ &T_{i2} = 2.0480 - 1.3137i. \end{aligned}$$

Obviously, these solutions are not considered here. Fig. 5 illustrates the step response for the closed control system with the controlled system given by the transfer function (21) and PI controller with the setting given by (24), i.e. for $\omega_0 = 10$ and $\xi = 12$ (curve 1). The step response for $\omega_0 = 10$ and ξ =6 (curve 2) is also shown. Then, the controller settings are

$$k_p = 0.2663 \text{ and } T_i = 0.6090\,. \tag{25}$$

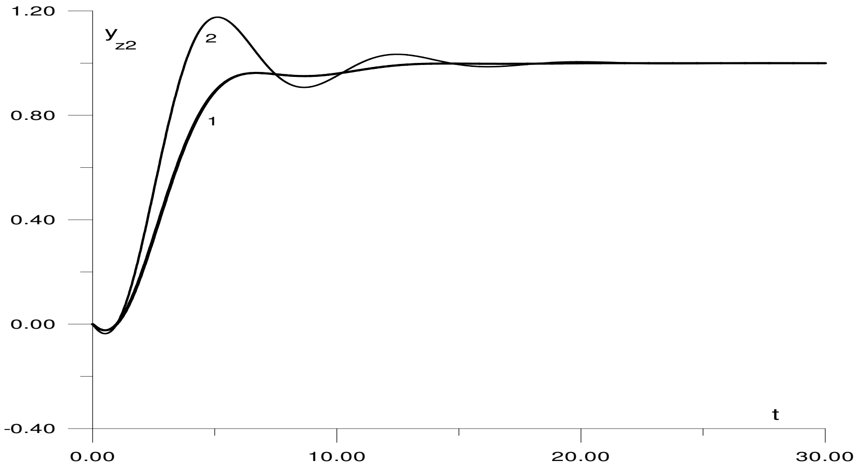

Figure 5. Step responses of the closed control system for PI controller settings given by (24) and (25)

Let us compare also the step responses of the closed system with the ones of the controlled system given by transfer function (18) and PI controllers with settings derived for $\omega_0 = 5$ and $\omega_0 = 10$, and $\xi = 12$. For $\omega_0 = 5$ and $\xi = 12$ we get the solution

$$k_{p1} = 0.1032\,,$$
$$T_{i1} = 0.6074\,. \tag{26}$$

Fig. 6 shows step responses for controller settings given by (24) and (26). Curve 1 corresponds to the settings (24) while curve 2 – to settings (26). The steady state is attained in a shorter time for greater k_p.

The settings of PID controller are determined as the following functions

$$k_p = f_1(T_d), \quad T_i = f_2(T_d).$$

For $\omega_0 = 10$, $\xi = 12$ and $T_d = 0.4$ we get PID controller settings:

$$k_p = 0.2419, \quad T_d = 0.4 \quad \text{and} \quad T_i = 0.7979\,. \tag{27}$$

Fig. 7 shows step responses for controller settings given by (24) and (27). Curve 1 corresponds to the settings 24), while cutve 2 – to settings 27).

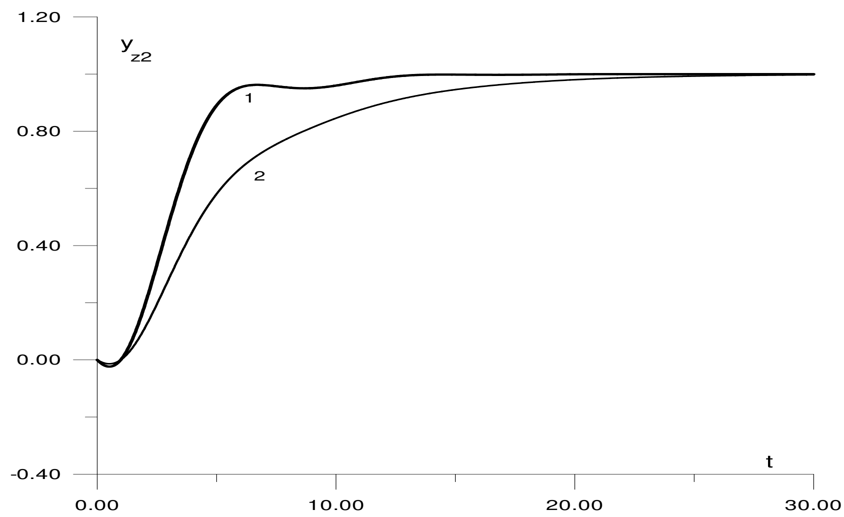

Figure 6. Step responses of the closed control system for PI controller settings given by (24) and (26)

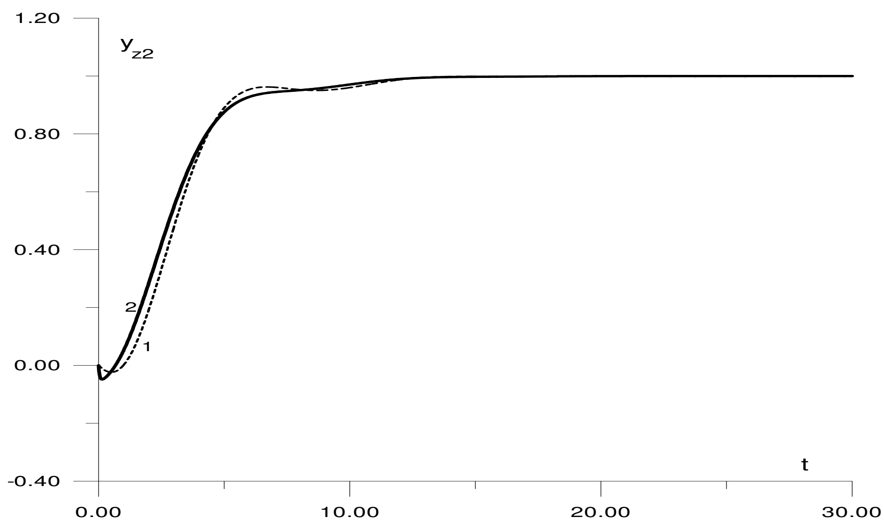

Figure 7. Step responses for the closed control system with PI controller settings equal to (24) and PID controller settings given by (27)

4. Determination of PI and PID controller settings for the third-order model of controlled system

In the case of third-order controlled system (2) with PID controller, according to (5) and (6) we have

$$
\begin{aligned}
c_0 &= \frac{b_0 k_p}{T_i},\\
c_1 &= \frac{b_1 k_p}{T_i} + b_0 k_p,\\
c_2 &= b_1 k_p + \frac{b_2 k_p}{T_i} + b_0 k_p T_d,\\
c_3 &= b_2 k_p + b_1 k_p T_d,\\
c_4 &= b_2 k_p T_d,\\
d_0 &= c_0,\\
d_1 &= a_0 + b_0 k_p + \frac{b_1 k_p}{T_i},\\
d_2 &= a_1 + b_1 k_p + \frac{b_2 k_p}{T_i} + b_0 k_p T_d,\\
d_3 &= a_2 + b_2 k_p + b_1 k_p T_d,\\
d_4 &= 1 + b_2 k_p T_d.
\end{aligned}
\tag{28}
$$

Equations to be used for determining the controller settings are

$$
\omega_0^2 = \frac{L_{w3}}{M_{w3}},
\tag{29}
$$

where

$$
\begin{aligned}
L_{w3} &= b_0 k_p[-a_1 b_0 + a_0(b_1 + b_0 T_i)],\\
M_{w3} &= T_i\left\{-a_1^2 b_0 + a_0 a_1(b_1 + b_0 T_i) - a_0[-a_2 b_0 + a_0(b_2 + b_1 T_i + b_0 T_i T_d)]\right\},
\end{aligned}
$$

and

$$
2n = \frac{L_{n3}}{M_{n3}},
$$

where

$$
\begin{aligned}
L_{n3} &= a_2 b_0^2 k_p - a_1 b_0[b_1 k_p + (a_0 + b_0 k_p)T_i] + a_0[b_1^2 k_p + b_1(a_0 + b_0 k_p)T_i +\\
&\quad + b_0\{-b_2 k_p + T_i[a_0 T_i + b_0 k_p(T_i - T_d)]\}\},\\
M_{n3} &= T_i\left\{-a_1^2 b_0 + a_0 a_1(b_1 + b_0 T_i) - a_0[-a_2 b_0 + a_0(b_2 + b_1 T_i + b_0 T_i T_d)]\right\}.
\end{aligned}
\tag{30}
$$

When we want to find PI controller setting, the constant T_d is prescribed as $T_d = 0$.

Example 4.1

Consider selection of PID controller setting for the controlled system given by the transfer function

$$G_0(s) = \frac{y_3(s)}{x(s)} = \frac{1}{s^3 + 3s^2 + 3s + 1}\,. \tag{31}$$

The step response of the model given by the transfer function (31) is presented in Fig. 8.

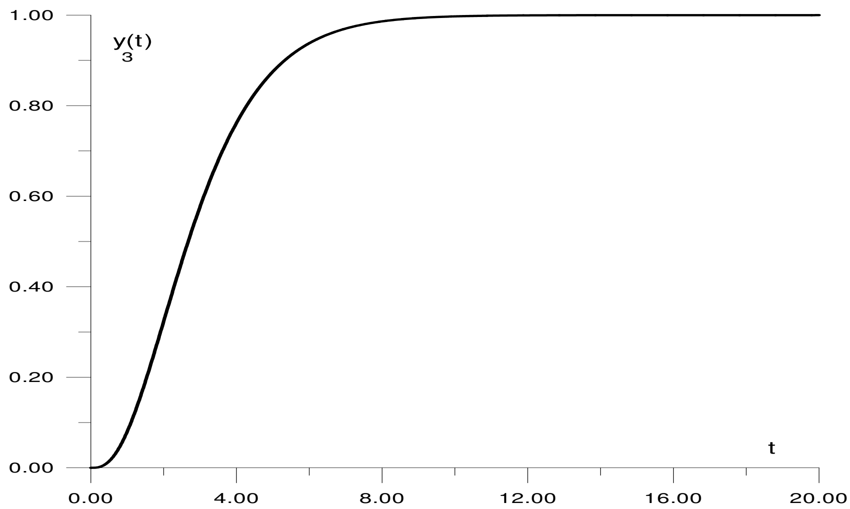

Figure 8. Step response for the model given by transfer function (31)

We are searching for the controller settings, which ensure that the closed system reaches possibly fast the steady state value without any overshoot. For the controlled system model (31) and assumed values ξ =12 i ω_0 =10 we obtain the controller settings as follows

$$k_p = 1.022, \quad T_i = 2.726 \quad \text{and} \quad T_d = 0.800. \tag{32}$$

The step response of the closed control system for controller settings given by (32) is illustrated in Fig. 9.

When Mathematica program is available, we need not calculate the set of equations (29-30).

The program to calculate controller setting by means of Mathematica is quite simple. For the pair of controller setting resulting from calculations, the step response of the closed system shall be verified by means of computer simulation.

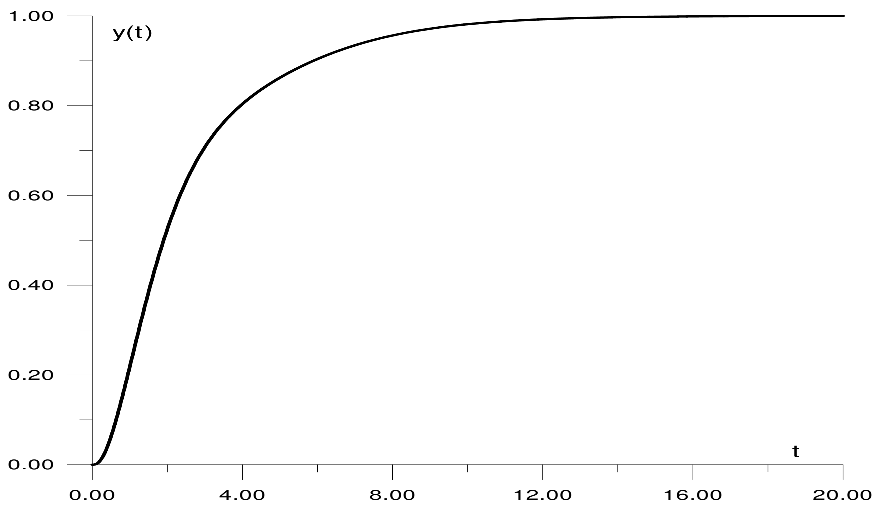

Figure 9. Step response of the closed system with controlled system model (31) and PI controller with settings given by (32)

If the step response for these controller settings is not satisfactory, as – for instance – due to excessive overshoot, then a new vector of controller settings shall be calculated for assumed ω_0 and higher ζ. For the response to reach the steady state earlier, an increased ω_0 shall be used for the assumed ζ.

5. Notes on specific form of transfer function describing the closed system

Sometimes the transfer function of the closed system of the form (6) includes equal coefficients, namely

$$c_0 = d_0 \quad \text{and} \quad c_1 = d_1. \tag{33}$$

Such is the case e.g. for the closed control system with the controlled system given by the transfer function

$$G_0(s) = \frac{b_1 s + b_0}{(s^2 + a_1 s + a_0)s}$$

and with PI controller. The transfer function for the closed system is of the form

$$G_z(s) = \frac{b_1 k_p s^2 + \left(\frac{b_1 k_p}{T_i} + b_0 k_p\right) s + \frac{b_0 k_p}{T_i}}{s^4 + a_1 s^3 + (a_0 + b_1 k_p)s^2 + \left(\frac{b_1 k_p}{T_i} + b_0 k_p\right) s + \frac{b_0 k_p}{T_i}} \quad . \tag{34}$$

The convergent method is also applicable for a static controlled systems. There is a division by zero in the algorithm used to expand the transfer function (34) into V-type chain fraction. The procedure to be used in such case is outlined in Khovanskii (1956). A simpler way will be given which does not complicate operation of this algorithm. The models of controlled systems are the simplified models. Thus, no substantial error will be introduced when we assume

$$c_0 = d_0 \quad \text{and} \quad c_1 = \alpha d_1,$$

where $0.95 \leq \alpha \leq 1$, $\alpha \neq 1$ e.g. $\alpha = 0.98$.

The issue of stability margin (gain and phase margin) plays an important part in the selection of the controller and is discussed in numerous works (Chu and Teng, 1999; Ho et al., 1999, 2001; Zhung and Atherton, 1993). In the method proposed, computer is used to determine the non-oscillatory step responses for the closed-loop control systems. In systems with such responses, the stability margin is maintained.

Conclusions

The method proposed is intended to determine PI and PID controller settings for closed control systems with models of controlled systems described by transfer functions. The results attained are good. The method does not require to specify approximate controller settings which is necessary in calculations using the NCB block of Simulink. When approximate values introduced into Simulink are of too low accuracy for prescribed limitations, no positive results can be reached. The program to calculate controller settings by means of Mathematica is simple and short. The method proposed requires viewing the step response of the closed system for calculated controller settings and, if necessary, repeating the calculations for other values of ω_0 and ξ in case an overshoot exists. It is proposed to calculate controller settings for prescribed ω_0 and several values of ξ. If excessive overshoot occurs, the value of ξ shall be increased.

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