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The MOORA method and its application to privatization in a transition economy

by

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Abstract: A new method is proposed for multi-objective optimization with discrete alternatives: MOORA (Multi-Objective Optimization on the basis of Ratio Analysis). This method refers to a matrix of responses of alternatives to objectives, to which ratios are applied. A well established other method for multi-objective optimization is used for comparison, namely the reference point method. Later on, it is demonstrated that this is the best choice among the different competing methods. In MOORA the set of ratios has the square roots of the sum of squared responses as denominators. These ratios, as dimensionless, seem to be the best choice among different ratios. These dimensionless ratios, situated between zero and one, are added in the case of maximization or subtracted in case of minimization. Finally, all alternatives are ranked, according to the obtained ratios. Eventually, to give more importance to an objective, an objective can be replaced by different sub-objectives or a coefficient of importance can be specified. An example on privatization in a transition economy illustrates the application of the method. If application is situated originally in a "welfare" economy, centered on production, MOORA becomes even more significant in a "wellbeing economy", where consumer sovereignty is assumed.

Keywords: multi-objective optimization, alternative measurement, discrete alternatives, ratio analysis, sub-objectives, reference point method.

1. A new method: the MOORA method

1.1. Some assumptions

The MOORA Method (Multi-Objective Optimization on the basis of Ratio Analysis) is based on a set of different assumptions.

The assumption of cardinal numbers

Cardinal numbers, as defined by direct or by alternative measurement or as dimensionless numbers, are only involved. Sometimes direct measurement, being too difficult, is substituted by *Alternative Measurement,* such as for pollution abatement, quality and individual choice. Take the example of alternative measurement for pollution. Air pollution is difficult to measure directly. If air pollution causes cancer, what is the effect over many years and is cancer related to the pollution in question? Therefore, pollution abatement costs, for instance the facility installation costs in a factory meant to diminish the emission of dangerous gas and dust, represent an alternative measurement for pollution. The cost of complete isolation of houses, the drop in prices of these houses or the amortization of the last models of airplanes would mean an alternative measurement for noise pollution for the neighbors of airfields.

Dimensionless numbers have no specific unit of measurement, but are obtained for instance by deduction, multiplication or division.

Nominal scales, such as *excellent*, *good*, *fair*, *bad*, are transformed to cardinal numbers by alternative measurement or by dimensionless numbers (Brauers, 2004, p. 99).

In addition, the use of *Ordinal numbers* presents only limited possibilities, whereas "obviously, a cardinal utility implies an ordinal preference but not vice versa" (Arrow, 1974).

The assumption of discrete choices

The discrete case consists of a number of well-defined and possible alternatives (projects, design). On the contrary, the continuous case generates alternatives from a continuous set of options during the process itself.

The assumption of attributes

In order to define objectives better we have to focus on the notion of *Attribute*. Keeney and Raiffa (1993) present the example of the objective "reduce sulfur dioxide emissions" to be measured by the attribute "tons of sulfur dioxide emitted per year". It signifies that an objective and a correspondent attribute always go together. Therefore, when the text mentions "objective", the corresponding attribute is meant as well.

1.2. Definition of the MOORA method

The method starts with a matrix of responses of different alternatives to different objectives:

$$(x_{ij}) \tag{1}$$

where x_{ij} is the response of alternative j to objective i, $i = 1, 2, \ldots, n$ are the objectives, $j = 1, 2, \ldots, m$ are the alternatives.

MOORA refers to a ratio system in which each response of an alternative on an objective is compared to a denominator, which is representative for all alternatives concerning that objective. For this denominator the square root of the sum of squares of each alternative per objective is chosen (Van Delft and Nijkamp, 1977):

$$_Nx_{ij} = \frac{x_{ij}}{\sqrt{\sum\limits_{j=1}^{m} x_{ij}^2}} \tag{2}$$

with:

x_{ij} = response of alternative j to objective i, $j = 1, 2, \ldots, m$; m the number of alternatives, $i = 1, 2, \ldots, n$; n the number of objectives,

$_Nx_{ij}$ = a dimensionless number representing the normalized response of alternative j to objective i; these normalized responses of the alternatives to the objectives belong to the interval $[0; 1]$.

For optimization, these responses are added in case of maximization and subtracted in case of minimization:

$$_Ny_j = \sum_{i=1}^{i=g} {_Nx_{ij}} - \sum_{i=g+1}^{i=n} {_Nx_{ij}} \tag{3}$$

with:

$i = 1, 2, \ldots, g$ for the objectives to be maximized, $i = g + 1, g + 2, \ldots, n$ for the objectives to be minimized,

$_Ny_j$ = the normalized assessment of alternative j with respect to all objectives.

In this formula linearity concerns dimensionless measures in the interval $[0; 1]$. An ordinal ranking of the $_Ny_j$ shows the final preference.

1.3. Introduction of ratios in a reference point theory

The reference point theory starts from the already normalized ratios as defined in the MOORA method, namely formula (2).

Next, the reference point theory chooses for maximization a reference point, which has as co-ordinates the highest co-ordinate per objective of all the candidate alternatives. For minimization, the lowest co-ordinate is chosen.

In order to measure the distance between the alternatives and the reference point, the Tchebycheff Min-Max metric is chosen (Karlin and Studden, 1966, p. 280):

$$\min_{(j)} \{\max_{(i)} |r_i - {}_N x_{ij}|\} \tag{4}$$

where:

$i = 1, 2, \ldots, n$ are the objectives, $j = 1, 2, ..., m$ are the alternatives,

r_i = the i^{th} co-ordinate of the maximal objective reference point; each co-ordinate of the reference point is selected as the highest corresponding co-ordinate of the alternatives,

${}_N x_{ij}$ = the normalized objective i of alternative j.

A simulation exercise on privatization illustrates the application of the MOORA and reference point methods.

2. Privatization in a transition economy

In a transition economy the previous collectivistic economies of Central and Eastern Europe or Asia are transformed into controlled market economies (Brauers, 1987). In such a transition economy privatization will take place. Privatization means that government services or state enterprises are turned over to private ownership. It is not the point here to give an overview of privatization (see, e.g., Parker and Saal, 2003). The aim is rather to look for optimization in privatization processes and in particular for transition economies.

The purpose of privatization is mainly to increase effectiveness, while the government would like to maximize its revenue at this occasion. Most of the public enterprises show a shortage in investment whereas maintenance of a reasonable employment level in the new private firm is also strongly desirable. It means that privatization bids have to face multiple objectives and that different parties, which are called different "stakeholders", are interested in the issue. Traditionally, joint consideration of all these objectives is done case by case in a rather subjective way. Consequently, there is a need for a more general and non-subjective, not to say scientific, method which can compare several bids for privatization, optimizing multiple objectives. Two kinds of objectives are considered, these of the management side, like maximization of the IRR, increase in productivity and minimization of the payback period and those of the macro-economic side like maximization of investments, of employment, of value added and of the influence on the current balance of payments.

1. Maximization of the Internal Rate of Return (IRR). The Internal Rate of Return is the interest rate for which the discounted cash flow (net profits plus depreciation) equals the discounted investments over the planning period under consideration.

2. Minimization of the payback period. The payback period is the number of years in which the discounted cash flow equals the discounted investments over the planning period under consideration.
3. The maximization of the increase in productivity. Productivity growth is mostly considered as unfavorable for the workers. Employers would force them to work faster or annoy them with all kind of experiments such as time and motion studies. However, productivity growth is more than that. It is the increase of the ratio of value added to employment that measures the growth of productivity. This growth may come from the value added side in constant prices by an effort of the workers but also by an increase in investments, by a better organization and management or by higher marketing opportunities. At the denominator side, the increase may come from a decrease in employment, contrary to the macro-economic objective of increasing employment.
4. The macro-economic objectives cover the maximization of investments, of employment, of value added and of the influence on the current account of the balance of payments, as indicated in Table 1.

According to both methods, MOORA and the reference point, project A is chosen above projects B and C. This result is communicated to the government under consideration, which will start the negotiations for the takeover price. The government would like to get the highest price, but on the contrary, the eventual private partner would try the lowest possible bid. It is a typical situation, in which two opponents have opposite points of view (Brauers, 2004, p. 91).

It may happen that the government is not impressed by the differences between projects A, B and C, whereas previously other projects failed to fulfill different hard constraints, either as a threshold (a lower bound) or as a ceiling (an upper limit).

Brauers called previous filtering followed by indifference “the Indifference Method” as applied for instance for national defense (Brauers, 1977; Despontin et al., 1983, p. 20; Brauers, 2004, pp. 139-145). In the indifference method, given the indifference towards the other objectives, the final decision is made on basis of cost considerations. For privatization an appeal is made for a public auction whereby different candidates after passing the filtering stage undergo the cost test. In the 1991 drive for privatization, the Brazilian government was indifferent about new investments, the effective way of working of the new private firm and about employment (Brauers, 1991). In this way for the Brazilian government the selling price of the firm remained the only objective to be maximized. The bottom price for the public auction was then the economic value of capital for production i.e. the sum of the discounted expected cash flows known also from project management (for this definition, see Brauers, 1990). The economic value of capital for production of a firm was estimated by two separate consulting firms. In case of strong differences in estimation, a third consulting firm was approached.

Table 1. A simulation for privatization by MOORA and by the reference point method based on ratios

1a - Matrix of responses of alternatives to objectives: (x_{ij})

Projects	**IRR (in %) MAX**	**Payback period (in years) MIN**	**Increase of productivity (in %) MAX**	**New investment value (10^9 €) MAX**	**New employment (job number) MAX**	**V.A. (discounted) (10^6 €) MAX**	**Balance of payments (10^6 €) MAX**
Project A	12	5	4	4.5	750	700	150
Project B	10	7	4	3	800	600	200
Project C	10	8	1	2.5	1,000	900	225

1b - Sum of squares and their square roots

Projects							
Project A	144	25	16	20.25	562500	490000	22500
Project B	100	49	16	9	640000	360000	40000
Project C	100	64	1	6.25	1000000	810000	50625
Sum of squares	344	138	33	35.5	2202500	1660000	113125
Square roots	18.54723699	11.7473401	5.74456265	5.9581876	1484.08221	1288.4099	336.3406012

1c - Objectives divided by their square roots and MOORA

Projects								**Totals**	**Rank**
Project A	0.646996639	0.425628	0.69631062	0.7552632	0.50536284	0.54331	0.445976488	3.16759	**1**
Project B	0.539163866	0.595880	0.69631062	0.5035088	0.539054	0.4656903	0.594635317	2.74248	**2**
Project C	0.539163866	0.681005	0.17407766	0.4195907	0.67381712	0.6985355	0.668964732	2.49314	**3**

1d - Reference point method with ratios: co-ordinates of the reference point equal to the maximal objective values

r_i	0.646996639	0.425628	0.69631062	0.7552632	0.67381712	0.69854	0.668964732

1e - Reference point method: deviations from the reference point

Projects								**Max**	**Rank min**
Project A	0	0	0	0	0.16845428	0.15523	0.222988244	0.22299	**1**
Project B	0.107832773	0.170251	0	0.2517544	0.134763	0.23285	0.074329415	0.25174	**2**
Project C	0.107832773	0.255377	0.52223297	0.3356725	0	0	0	0.52223	**3**

In MOORA (see, e.g. Tables 1b and 1c), the choice of the square roots of the sum of the squared responses as denominators may look rather arbitrary (Peldschus, Zavadskas and Vaigauskas, 1983; Zavadskas, 2000; Zavadskas et al., 2002, 2003). Therefore, the search for alternative denominators will be an important topic of this research.

3. Why not use other ratio systems in the MOORA method?

Let us remind that the denominator $\sqrt{\sum_{j=1}^{m} x_{ij}^2}$ was chosen in the MOORA formula (2). Possibilities of use of other ratio systems are discussed, which does not mean that the following description is exhaustive.

3.1. Voogd (1983) ratios

$$ {}_N x_{ij} = \frac{x_{ij}}{\sum_{j=1}^{m} x_{ij}} . \tag{5} $$

The normalized responses of the alternatives on the objectives belong to the interval $[0;1]$. Allen (1951) used already this formula, but Voogd applied it for multiobjective evaluation.

For optimization these responses are added in case of maximization and subtracted in case of minimization (see (3)).

Nevertheless, the Voogd ratios and their totals are smaller than those in the square roots method. Voogd ratios are less complicated than those of the square roots method. However, they will not necessarily lead to the same results. If they lead to the same ranking, it would form an additional control on MOORA. An example in Appendix A shows that the Voogd ratios, as applied to the matrix of responses on objectives of Table 1, result in the same ranking.

3.2. Schärlig (1985) ratios

With the Schärlig ratios, one of the alternatives is taken as a reference. This mechanical approach is comparable with the formula of Schärlig, which multiplies all these ratios (Schärlig, 1985, p. 76).

A problem arises if one of the objectives is missing in an alternative and this alternative is used as a basis. The result is that some ratios obtained are undefined, because the denominator is zero. Therefore, an alternative has to be chosen as a basis with none of its objectives equal to zero.

Worse even, the results depend essentially on the choice of the alternative to be the basis. Consequently, ratio analysis, with one of the alternatives taken as a reference, produces no univocal outcome (see simulations in Brauers, 2004, p. 297).

3.3. Weitendorf (1976) ratios

Weitendorf compares the responses with the interval Maximum-Minimum in the following way:
– if ${}_N x_{ij}$ should be maximized:

$$ {}_N x_{ij} = \frac{x_{ij} - x_i^-}{x_i^+ - x_i^-} \, , \tag{6} $$

– if ${}_N x_{ij}$ should be minimized:

$$ {}_N x_{ij} = \frac{x_i^+ - x_{ij}}{x_i^+ - x_i^-} \tag{7} $$

with: x_i^+ representing the maximum value and x_i^- the minimum value of objective i. The normalized responses belong to the interval $[0;1]$.

This method, which seems interesting at the first glance, has to be rejected on the following grounds:

1. The Min-Max metric cannot be applied, as all co-ordinates of the reference point are equal to one, which makes a ranking impossible. Following sub-Tables 2e and 2f show an example.
2. With only the maximum and the minimum per objective of all alternatives the composition of the whole series of objectives is not taken into consideration, i.e. are not considered:
   – the spread as measured by the standard deviation; this spread can be different for several series though with the same maxima and minima;
   – the median and the quartiles can be different for several series though with the same maxima and minima.

Given these remarks, a simulation is made with Weitendorf ratios for the same matrix of responses of alternatives on objectives as in Table 1. In this way Table 2, with the application of the Weitendorf ratios, shows other results compared to the square roots ratios of Table 1.

Moreover, thousands and thousands other matrices of responses of alternatives on objectives, with the same outcomes of formulae (6) and (7), as given in Tables 2b and 2c, will lead to the same ranking. Even the same results could be obtained. For example, Table 3 shows the same ranking and even the same results for project C as in Table 2, though with another matrix of responses as a starting point and having the same relations to their maxima and minima.

If MOORA uses the matrix of responses of alternatives to objectives, $\boldsymbol{(x_{ij})}$ of Table 3, the outcome is entirely different, namely: B**P**C**P**A (**P** means "preferred to").

3.4. Van Delft and Nijkamp (1977) ratios of maximum value

In the method of maximum value, the objectives per alternative are divided by the maximum or the minimum value of that objective, which are found in one

Table 2. Multiple objective optimization with Weitendorf ratios

2a - Matrix of responses of alternatives to objectives: (x_{ij})

Projects	**IRR (in %) MAX**	**Payback period (in years) MIN**	**Increase of productivity (in %) MAX**	**New investment value (10^9 €) MAX**	**New employment (job number) MAX**	**V.A. (discounted) (10^6 €) MAX**	**Balance of payments (10^6 €) MAX**
Project A	12	5	4	4.5	750	700	150
Project B	12	7	4	3	800	600	200
Project C	10	8	1	2.5	1,000	900	225

2b – Responses minus minimum for maximization or maximum minus responses for minimization

Projects							
Project A	2	3	3	2	0	100	0
Project B	2	1	3	0.5	50	0	50
Project C	0	0	0	0	250	300	75

2c – For the denominator: maximum minus minimum

	2	3	3	2	250	300	75

2d - Data from 2b divided by 2c and additive method with Weitendorf ratios

								Totals	**Rank**
Project A	1	1	1	1	0	0.33333	0	2.33333	**3**
Project B	1	0.333333	1	0.25	0.20	0	0.66666667	2.7833	**2**
Project C	0	0	0	0	1	1	1	3.0000	**1**

2e - Reference point method with ratios: co-ordinates of the reference point equal to the maximal objective values

r_i	1	1	1	1	1	1	1

2f - Reference point method: deviations from the reference point

Projects								**Max**	**Rank min**
Project A	0	0	0	0	1	0.66667	1	1	**1**
Project B	0	0.666667	0	0.75	0.80	1	0.33333333	1	**1**
Project C	1	1	1	1	0	0.00000	0	1	**1**

Table 3. Multiple objective optimization with Weitendorf ratios (second trial)

3a - Matrix of responses of alternatives to objectives: (x_{ij})

Projects	IRR (in %) MAX	Payback period (in years) MIN	Increase of productivity (in %) MAX	New investment value (10^9 €) MAX	New employment (job number) MAX	V.A. (discounted) (10^6 €) MAX	Balance of payments (10^6 €) MAX
Project A	14	6	4	7	900	1,200	-100
Project B	14	8	3	5.5	1,100	1,000	60
Project C	12	10	3	5	1,150	1,500	60

3b – Responses minus minimum for maximization or maximum minus responses for minimization

Projects							
Project A	2	4	1	2	0	200	0
Project B	2	2	0	0.5	200	0	160
Project C	0	0	0	0	250	500	160

3c – For the denominator: maximum minus minimum

	2	4	1	2	250	500	160

3d - Data 3b divided by 3c and additive method with Weitendorf ratios

								Totals	**Rank**
Project A	1	1	1	1	0	0.40	0	2.4000	**3**
Project B	1	0.50	0	0.25	0.80	0	1	2.5500	**2**
Project C	0	0	0	0	1	1	1	3.0000	**1**

3e - Reference point method with ratios: co-ordinates of the reference point equal to the maximal objective values

r_i	1	1	1	1	1	1	1

3f - Reference point method: deviations from the reference point

Projects								**Max**	**Rank min**
Project A	0	0	0	0	1	0.60	1	1	**1**
Project B	0	0.50	1	0.75	0.20	1	0	1	**1**
Project C	1	1	1	1	0	0	0	1	**1**

of the alternatives:

$$_{N}x_{ij} = \frac{x_{ij}}{x_i^+} \tag{8}$$

with: x_i^+ being the maximum or minimum x_{ij} depending on whether a maximum or a minimum of an objective is strived for.

As only maxima, minima and the responses are involved, the same comments on the spread, the median and quartiles, as for the Weitendorf ratios, are also of application here.

A fundamental problem arises for minimization. The ideal situation for minimization occurs when zero is attained. This could mean dividing by zero. If at that moment the numerator is not zero the fraction is undefined. Even if in that case to an alternative a symbolic number 0.001 were assigned, the result would be negatively biased for the other alternatives. It could even solely determine the final ranking of the alternatives, which is not correct (see an example in Brauers, 2004, p. 298). Anyway, the ratios can deviate largely from the interval [0; 1]. In this way one of the advantages of the ratio system disappears, namely that the relation of the ratios to one another can only differ by one at the utmost.

Once again, with the application of the reference point method, all coordinates of the maximal objective reference point are equal to one. Indeed, the maximal criterion values are either the maximum value divided by itself or the minimum value divided by itself.

3.5. Jüttler (1966) ratios

For normalization, it is also possible to use Jüttler's ratios:

$$_{N}x_{ij} = \frac{x_j^+ - x_{ij}}{x_i^+}\,. \tag{9}$$

As only maxima, minima and the responses are involved, the same comments on the spread, the median and the quartiles, as mentioned earlier, are also of application here.

If x_i^+ represents a minimum it can have a zero value in the denominator. At that moment, the same objections can be made as against the van Delft and Nijkamp method of maximum value.

3.6. Stopp (1975) ratios

If maximum x_{ij} is sought:

$$_{N}x_{ij} = \frac{100x_{ij}}{x_i^+}\,. \tag{10}$$

If minimum x_{ij} is sought:

$$_N x_{ij} = \frac{100 x_i^-}{x_{ij}}. \tag{11}$$

These normalized values are expressed in percentages.

As maxima and minima are used, the same objections as against the Weitendorf ratios are valid here.

Hwang and Yoon (1981, p.100) mention the same formulae, but without percentages.

3.7. Körth (1969 a, b) ratios

$$_N x_{ij} = 1 - \left| \frac{x_i^+ - x_{ij}}{x_i^+} \right|. \tag{12}$$

The same objections as against the van Delft and Nijkamp method of maximum value and against the Weitendorf ratios are also valid here, as the maximum value is used.

3.8. Peldschus et al. (1983) and Peldschus (1986) ratios for nonlinear normalization

If minimum x_{ij} is sought:

$$_N x_{ij} = \left(\frac{x_i^-}{x_{ij}} \right)^3. \tag{13}$$

If maximum x_{ij} is sought:

$$_N x_{ij} = \left(\frac{x_{ij}}{x_i^+} \right)^2. \tag{14}$$

Once again, as only maxima and minima are used, the same objections as against the Weitendorf ratios are valid here.

4. The importance given to an objective

With the retained methods, MOORA and possibly the Voogd sum ratios, one objective cannot be very much more important than another one, as all their ratios are smaller than one. On the contrary, it may turn out to be necessary to stress that some objectives are more important than others.

In order to give explicit importance to an objective, the dimensionless numbers, obtained in MOORA are multiplied by coefficients. The attribution of sub-objectives represents another solution. Take the example of a purchase of fighter planes (Brauers, 2002). For economics the objectives are threefold: price, employment and balance of payments, but there is also the military effectiveness. In order to give more importance to the military aspect, effectiveness is broken down to, for instance, the maximum speed, the power of the engines and the maximum range of the plane.

The attribution method is more refined than the coefficient method, as the attribution method succeeds in characterizing an objective better. For instance, the coefficient for employment in Table 1c for Project A after the coefficient method, namely twice 0.50536284, is changed in the attribution method into two separate numbers characterizing the direct and indirect employment separately.

Concerning an effective ratio system, the choice is not difficult to make. A ratio system in which each response to an alternative to an objective is divided by the square root of the sum of squares of each alternative per objective and eventually the Voogd sum ratios are representative for the comparison between alternatives and objectives. On the basis of these ratios, MOORA results in a ranking between the alternatives.

As a second method, reference point theory with the min-max metric was taken as the most representative choice. Was this a correct choice?

5. Is the min-max metric the first choice for reference point theory?

5.1. The choice of the reference point

The reference point theory is a very respectable theory going back to forerunners such as Tchebycheff (1821-1894) and Minkowski (1864-1909) (see Karlin and Studden, 1966, and Minkowski, 1896, 1911). The choice of a reference point and the distance to the reference point are essential for reference point theory.

Preference is given to a reference point possessing as co-ordinates the dominating co-ordinates per objective of the candidate alternatives, which is designated as the Maximal Objective Reference Point. On the contrary, the *Utopian* Objective Vector gives higher values to the co-ordinates of the reference point than the maximal objective vector. The *Aspiration* Objective Vector moderates the aspirations of the stakeholders by choosing smaller co-ordinates than in the maximal objective vector.

5.2. How to measure the distance between the discrete points of the alternatives and the reference point?

The Minkowski metric as a discrepancy measure brings the most general synthesis (Minkowski, 1896; 1911; Pogorelov, 1978):

$$\min M_j = \left\{ \sum_{i=1}^{i=n} (r_i - {}_N x_{ij})^{\alpha} \right\}^{1/\alpha} \tag{15}$$

with: M_j = Minkowski metric for alternative j,

r_i = the i^{th} co-ordinate of the reference point,

${}_N x_{ij}$ = the normalized attribute i of alternative j.

The Minkowski metric represents the basis of what is designated in the literature as Goal Programming. From the Minkowski formula, the different forms of goal programming are deducted depending on the values of α.

With the rectangular distance metric ($\alpha = 1$), the results are very unsatisfactory. In the case of two attributes, suppose e.g., a reference point (100;100) then the points (100;0), (0;100), (50;50), (60;40), (40;60), (30;70), and (70;30) all show the same rectangular distance and they all belong to the same line: $x + y = 100$. Ipso facto, a midway solution like (50;50) takes the same ranking as the extreme positions (100;0) and (0;100). In addition, the points: (30;30), (20;40), (40;20), (50;10), (25;35), (0;60) and (60;0), all belonging to the line: $x + y = 60$, show the same rectangular distance to a reference point (50;40), which is not defendable. Even worse, theoretically for each line an infinite number of points will result in the same ranking. The outcome does not change if weights are given to the attributes (as done by Tamiz and Jones, 1996).

With $\alpha = 2$, radii of concentric circles, with the reference point as central point, will represent the Euclidean distance metric. This distance metric applied for two attributes is similar to linear distance. Applying the Euclidean distance metric to the example above gives a very unusual outcome. The midway solution (50;50) is ranked first with symmetry in ranking for the extreme positions: (100;0) and (0;100); the same for (60;40) and (40;60) and for (30;70) and (70;30). Once again, numerous solutions are available.

With three attributes, radii of concentric spheres with the reference point as center represent the Euclidean metric. This, once again, results in numerous solutions. By analogy, one could imagine that for more than three attributes, corresponding conclusions can be drawn.

With $\alpha = 3$, negative results are possible if some co-ordinates of the alternatives exceed the corresponding co-ordinate of the reference point.

The same remarks as made above would apply for $\alpha > 3$, with the exception of $\alpha \to \infty$. In this special case of the Minkowski metric only one distance per point, namely the largest one, is kept in the running. The Minkowski metric becomes the Tchebycheff min-max metric with formula (4).

Also here it is possible that exceptionally more than one solution is obtained, but not as general as in the previous cases. Take Table 1 as an example. Two equal numbers can be obtained, either with the same r_i plus the same deviation, or with a different r_i, but with the same deviation in Table 1e. With nine decimals as in Table 1, it is even less probable. Finally, the two equal optima have to be characterized as the maximum deviation for all objectives.

In the field of reference point theory a method called "TOPSIS" raises high interest of the practitioners.

5.3. Is TOPSIS a better choice for the reference point theory?

In fact, TOPSIS is a reference point method, which was launched later than the traditional reference point techniques (Hwang and Yoon, 1981).

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is "based upon the concept that the chosen alternative should have the shortest distance from the ideal solution" (Hwang and Yoon, 1981, p. 128), which is in fact the aim of every reference point method. The distinction lies in the definition of the distance and how the co-ordinates of the reference point are defined. Moreover, an objective can ask for a maximum or for a minimum attainment. The choice of the distance function and how to handle maxima and minima make TOPSIS a target for comments.

In TOPSIS, the Euclidean distance is chosen to define the shortest distance. The Euclidean distance was criticized above.

After normalization and eventually attributing weights, TOPSIS proposes two kinds of reference points, a positive and a negative one. The positive reference point has as co-ordinates the highest corresponding co-ordinates of the alternatives (the lowest in the case of a minimum). The negative reference point has as co-ordinates the lowest corresponding co-ordinates of the alternatives (the highest in the case of a minimum). With regard to these two kinds of reference points, Euclidean distances are calculated. Consequently, each alternative will have two outcomes. Let us call them: ${}_Ny_{j+}$ and ${}_Ny_{j-}$.

In order to come to one solution, TOPSIS proposes the following formula, which is rather arbitrarily chosen (Hwang and Yoon, 1981, pp. 128-134):

$$ {}_Ny_j = \frac{{}_Ny_{j-}}{{}_Ny_{j+} + {}_N y_{j-}} \tag{16} $$

with: $j = 1, 2, ..., m$; m the number of alternatives.

In addition, Opricovic and Tzeng (2004, p. 450) conclude that the relative importance of the two outcomes is not considered, although it could be a major concern in decision making.

6. The "wellbeing" economy

Until now what we call "welfare economy" was taken into consideration, assuming that a higher production will automatically lead to general welfare for everybody. This production approach puts productivity central, giving the opportunity to raise the general wage level. This may not correspond to the general wellbeing of the population. It is possible that the push for higher productivity results in stress or sickness for the workers. Maybe they prefer more quietness, more leisure in a rather "wellbeing" approach.

In the wellbeing economy consumer sovereignty is put central. Consumer sovereignty means that the economic law of marginal decreasing utility is fully respected. The law of marginal decreasing utility is the best visualized by drawing indifference curves, which consequently, have the hollow on the inside. Suppose an individual has the choice between a higher salary and more leisure time. Fig. 1 illustrates this situation.

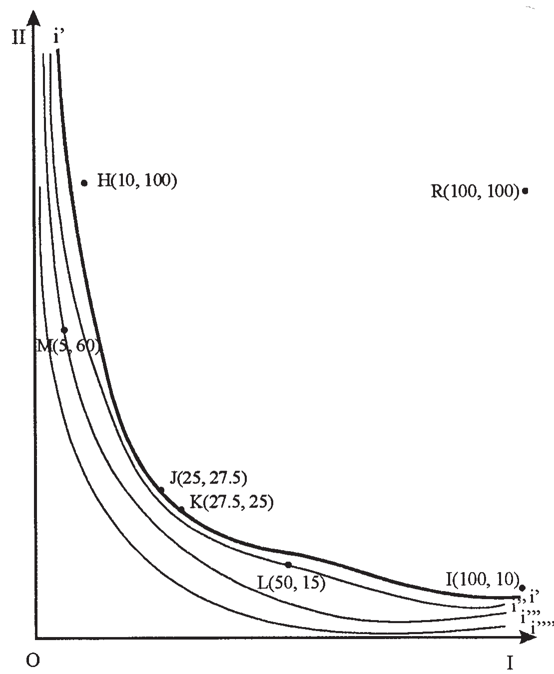

Figure 1. Indifference curves with convex and non-convex zones

Different indifference curves i', i'', i''' and i'''' are possible, depending of the budget possibilities of the individual. Anyway, the highest possible indifference curve has to be chosen. Suppose objective I represents leisure time and objective II salary. Indifference curves show plenty of midway solutions but also extreme situations: a very high salary with very little leisure time or a very low salary with plenty of leisure time. In other words, indifference curves demonstrate the economic law of marginal utility.

Indifference curves delimit a non-convex set of points after the Minkowski (1896, 35) definition of convexity: "*a convex set has the characteristic that all points on a line between any two points of that set have to belong to that set*" (for more details on indifference curve analysis see Vickrey, 1964).

Incompatibility arises between the weighted linearity of the different objectives and the economic law of decreasing marginal utility, as visualized by indifference curve analysis. Is an optimum reachable with weighted linearity of the different objectives? In the two objectives example, as the highest possible indifference curve delimits a non-convex set of points, which is not allowed of being reached, any straight line has to remain partly or totally under that curve. A straight line tangent to an indifference curve will automatically produce only a single common point with that indifference curve.

Indifference curves remaining theoretical constructions, there is no certainty that even this common point with the straight line is located in the convex zone. In addition, piece-wise linearity will not help, as here also the different contact points with an indifference curve are not known.

Does the danger not exist that with MOORA the non-convex zone is reached? Once again, there is no certainty given the theoretical position of indifference curves. Nevertheless, the averaging process in the ratio system of the alternatives for each objective rules out the extreme situations.

In the reference point theory, a reference point will tend to pull the coordinates of the alternatives into the non-convex zone, as the corresponding distances have to be minimized. The min-max metric, however, ending with a minimum presents a guaranty for not entering the forbidden zone. Choosing an aspiration point instead of a maximal objective reference point will present an additional guarantee for not entering that zone. Wierzbicki (1984) calls such change a quasi-satisficing behavior.

Why is it so difficult to determine the shape of indifference curves? First, the leisure-salary example needs the setting up of social indifference curves. A stratified and representative sample of the population will provide information on the relation between leisure and salary. Sensitivity analysis will study the very sensitive part of the variation pattern between the two objectives. In that way, different points are obtained. If many points are assembled, they can be aggregated into one or more indifference curves.

Given these difficulties, the question arises if hypothetical indifference curves can be assumed and if the different methods of reference point theory can be helpful for that purpose.

In fact, we consider two stages: one where the indifference curves are known and one where they are not known.

1$^{\text{st}}$ stage: the indifference curves are known

Indifference curve i' represents the highest possible indifference curve, consequently points H(10,100) and I(100,10) are not attainable being in the non-convex zone. The reference point after the maximum objective value R(100,100) lies in the non-convex zone too, a constant danger for reference points after the maximum objective value, less the case for a reference point with the aspiration objective value.

points J and K are on indeffirence curve i'
point L on the next lower indifference curve i''
point M on the next lower one i'''
Consequently: J **I** K **P** L **P** M   **I** means indifferent
P means preffered to

H and I are out.
This sequence we call the *Real Ranking*.

2nd stage: the indifference curves are unknown

A said earlier, indifference curves are very difficult to determine. Therefore the question is raised if other multi-objective methods can produce the same outcomes as the real ranking without knowing the shape of the indifference curves.

The min-max metric with MOORA or with total coefficients is the only method of reference point theory in which the extreme and non-convex points H and I are not first ranked. This ought to be the normal situation as the economic law of decreasing marginal utility stresses midway solutions. All the other methods: rectangular distance metric, rectangular distance metric with weights and Euclidean distance metric (used e.g. in TOPSIS, Hwang and Yoon, 1981, p. 132) rank the nonconvex points H and I on the first places. Even for the remaining rankings, they are not correct as shown in Table 4. Indeed, J and K instead of being the first ranked are ranked very low. Only the min-max metric with MOORA or total coefficients shows the same ranking as the real ranking. It is true that a distinction is made between the two points K and J, although both are situated on the same indifference curve.

Table 4. Comparison of the rankings from the different methods of reference point theory with the real outcome[a]

	Real ranking	Min-max with MOORA coeff.	Min-max with total coeff.	Rectangular	Rectangular with weight 2 for leisure	Rectangular with weight 2 for salary	Euclidean
J	1	2	2	3	3	2	3
L	3	3	3	1	1	2	1
K	1	1	1	3	2	4	3
M	4	4	4	1	4	1	1

[a] Alternatives H and I are not considered as belonging to the nonconvex zone.
Details of computation are available from the authors.

How is it possible to translate these considerations in simulations on privatization inside a "wellbeing" economy?

7. Privatization in a "wellbeing" economy in transition

In our simulations shown in Table 5 for a "wellbeing" economy in transition, salary and leisure time replace productivity of the "welfare" economy of Table 1. In this way, in a stakeholder society, the aspirations of the working class are considered better.

Productivity pressure is considered as a main reason for stress of the active population in industrialized countries. Economies in transition should not import this stress from the industrialized countries, even not to replace the shortcomings they show now. For instance, in the previously centrally planned economies of East Germany and Ukraine the state enterprises were in bad shape with very poor management, low productivity and an urgent need for new investments, being not ready to face competition in a free market economy (Brauers, 1995, p. 369 and 1994). In 1991 Brauers presented a privatization model to "Treuhand", the official body responsible for privatization in former East Germany. A member of this important body stated that privatization was mainly done on pragmatic and political and not on strategic grounds. In fact the employment and productivity problems were not sufficiently considered, creating in that way a huge wave of unemployment and later much social unrest in former East Germany.

In Table 5, salary in Euro per working hour will be the unit for salary. Minimization of weekly hours on the job (shop time) is assumed to measure maximization of leisure time.

Project B, being the first ranked according to the reference point theory, presents a typical midway solution. Indeed, in Table 5e, project A ranks first for four objectives and also project C ranks first for four objectives with no first ranking for project B. A midway solution will have no chance in a weighted linearity of the different objectives (this is proven by Brauers, 2004, pp. 153-156). As demonstrated earlier, in the last paragraph before Table 4, it seems to be also the case for the rectangular metrics and for the Euclidean one.

Consequently, the method of correlation of ranks, consisting of agregating ranks, has to be rejected too. Such a rank correlation was introduced first by psychologists such as Spearman (1904, 1906, 1910) and later taken over by statisticians (Kendall, 1948, and Mueller, 1970, pp. 267-294). By application, A and C would dominate B, having each four dominating positions.

Table 5 shows a difference in ranking between MOORA and the reference point theory. At this occasion, it is perhaps better to give preference to reference point theory with Project B. Indeed, reference point theory may correspond better with the midway solutions, which are preferred by the economic law of decreasing marginal utility. In this way, reference point theory fulfills its controlling function.

It is always possible to keep productivity as an objective besides salary size and leisure time. Multi-objective optimization has to consider these contradictory objectives simultaneously.

8. General conclusions

A new method for multiple objectives optimization called MOORA was presented. MOORA is a ratio system in which each response of an alternative per objective is compared to the square root of the sum of squares of the responses. Out of a selection of ratio systems, it is proved that MOORA outranks the other ones. Other multiple objective methods are criticized like the weighted

Table 5. A multiple objective optimization simulation for privatization in a "wellbeing" economy

5a - Matrix of responses of alternatives to objectives: (x_{ij})

Projects	**IRR (in %) MAX**	**Payback period (in years) MIN**	**New investments (10^9 €) MAX**	**Salary per hour (in €) MAX**	**Shop time (in weekly hours)) MIN**	**New employment (job number) MAX**	**V.A. discounted (10^6 €) MAX**	**Balance of payments (10^6 €) MAX**
Project A	12	5	4.5	30	50	750	700	150
Project B	10	7	3	25	35	800	600	200
Project C	10	8	2.5	10	30	1,000	900	225
Totals	32	20	10	65	115	2,550	2200	575

5b - Sums of squares and their square roots

Projects								
Project A	144	25	20.25	900	2500	562500	490000	22500
Project B	100	49	9	625	1225	640000	360000	40000
Project C	100	64	6.25	100	900	1000000	810000	50625
Sum od squares	344	138	35.5	1625	4625	2202500	1660000	113125
Square root	18.547237	11.7473401	5.9581876	40.311289	68.007353	1484.08221	1288.4099	336.3406012

5c - Attributes divided by their square roots and MOORA

									Sum	**Rank**
Project A	0.6469966	0.425628	0.7552632	0.7442084	0.7352146	0.50536284	0.54331	0.445976488	2.4802701	**1**
Project B	0.5391639	0.595880	0.5035088	0.6201737	0.5146502	0.539054	0.4656903	0.594635317	2.151696	**2**
Project C	0.5391639	0.681005	0.4195907	0.2480695	0.4411288	0.67381712	0.6985355	0.668964732	2.126007	**3**

5d - Reference point method with ratios: co-ordinates of the reference point equal to the maximal objective values

r_i	0.6469966	0.425628	0.7552632	0.7442084	0.4411288	0.67381712	0.69854	0.668964732

5e - Reference point method: deviations from the reference point

Projects									**Max**	**Rank min**
Project A	0	0	0	0	0.2940858	0.16845428	0.15523	0.222988244	0.2940858	**2**
Project B	0.10783277	0.170251	0.2517544	0.1240347	0.0735215	0.134763	0.23285	0.074329415	0.2517544	**1**
Project C	0.10783277	0.255377	0.3356725	0.4961389	0	0	0	0	0.4961389	**3**

linearity of different objectives. In this linear method, multiple objectives are replaced by one super objective and priority is given to the extreme alternative solutions, whereas an intermediate alternative does not rank first. Even rectangular and Euclidean distance metrics seem to give priority to extreme alternative solutions.

A utility problem with different independent objectives and with different alternative solutions has to be resolved. The notion of utility has always been a crucial point in decision making. For us, this boils down to four problems: the choice of units per objective, the normalization, the optimization and the importance, which is given to an objective. MOORA tries to satisfy all these preliminary conditions. In this way, this ratio development can be a full-fledged method for multiple objective optimization.

MOORA is assisted by a second method, the reference point method with a maximal objective reference point, which can control and certify the outcomes of MOORA. In the course of research, it is found that also ratios with the sum of responses of alternatives per objective as denominator, can serve as a control method. For both MOORA and the reference point method, square roots ratios are the most suitable.

With the square roots ratios of MOORA and the sum ratios, one objective cannot be very much bigger than another one, as their ratios are all smaller than one. On the contrary, it may be necessary that some objectives are considered as more important than others. The use of coefficients of importance is the traditional answer. The breakdown of objectives into sub-objectives represents another solution.

In reference point theory, preference is given to the Tchebycheff min-max metric with the maximum objective reference point. This reference point per objective possesses as co-ordinates the dominating co-ordinates of the candidate alternatives. For minimization, the lowest co-ordinates are chosen, which is more logical than in the TOPSIS method. Indeed, the TOPSIS method arrives finally at two kinds of reference points, a maximum and a minimum one, making the co-ordination of the sets of points extremely difficult.

Thus, the following steps are foreseen in MOORA:

1. For MOORA square roots ratios are chosen as the best choice.
2. Explicit importance is introduced for an objective by coefficients or by replacing the objective by different sub-objectives.
3. The ratios per alternative are added for the objectives to be maximized. The ratios per alternative for the objectives to be minimized are subtracted. The general total per alternative will compete in a ranking of all alternatives.
4. The ranking is set up.
5. Reference point theory with the min-max metric on the one hand and sum ratios on the other will be used as control instruments.
6. If these steps are taken in the production sphere, they can be repeated in

the consumption sphere. Here, the economic law of decreasing marginal utility, as visualized by indifference curve analysis puts additional limits on the different methods. In a two objectives example, as any possible indifference curve delimits from above a non-convex set of points, any straight line has to remain partly or totally under that curve, which limits the possibilities for the weighted linearity of different objectives.

MOORA is illustrated for privatization in a transition economy. First, a "welfare" economy forms the framework, centered on production and on market competition with productivity growth as an objective. Promotion of productivity could mean putting too much stress on the employees as well in an industrialized as in a country in transition. Secondly, inside a stakeholders' mentality, a "wellbeing" economy will take better care of the wellbeing of the employees. In a simulation exercise, the employees will have the choice between a higher salary and more leisure time. Even contradictory objectives could be considered simultaneously, such as increase in productivity, in salary and in leisure time.

From the simulations some fundamental conclusions can be drawn.

1. Alternatives with midway attributes can rank first in MOORA, which is not possible with weighted linearity of the different objectives. It seems also difficult for the rectangular distance metric, the Euclidean distance metric and the method of correlation of ranks.
2. Consideration of contradictory objectives is possible.
3. Even if the simulations are theoretical constructions, it can be concluded that MOORA is operational and ready for practical use when data are available from desk research.

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APPENDIX A

Multiple objective optimization with total ratios (Voogd)

Aa - Matrix of responses of alternatives to objectives: (x_{ij})

Projects	IRR (in %) MAX	Payback period (in years) MIN	Increase of productivity (in %) MAX	New investment value (10^9 €) MAX	New employment (job number) MAX	V.A. (discounted) (10^6 €) MAX	Balance of payments (10^6 €) MAX
Project A	12	5	4	4.5	750	700	150
Project B	10	7	4	3	800	600	200
Project C	10	8	1	2.5	1,000	900	225
Totals	32	20	9	10	2,550	2200	575

Ab - Objectives divided by their total and additive method with ratios

Projects								Totals	Rank
Project A	0.3750	0.2500	0.4444	0.4500	0.2941	0.318182	0.261	1.8926	**1**
Project B	0.3125	0.3500	0.4444	0.3000	0.3137	0.272727	0.347826	1.6412	**2**
Project C	0.3125	0.4000	0.1111	0.2500	0.3922	0.4091	0.391304	1.4662	**3**

Ac - Choice of the co-ordinates of the reference point

r_i	0.375	0.2500	0.4444	0.4500	0.3922	0.4091	0.391

Ad - Reference point method: deviations from the reference point

Projects								Max	Rank min
Project A	0	0	0	0	0.0980	0.0909	0.1304	0.1304	**1**
Project B	0.0625	0.1	0	0.15	0.0784	0.1364	0.0435	0.1500	**2**
Project C	0.0625	0.15	0.3333	0.2	0	0	0	0.3333	**3**