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.... I think that it is a relatively good approximation to truth (...) that mathematical ideas originate in empirics, although the genealogy is sometimes long and obscure. But, once they are so conceived, the subject begins to live a peculiar life of its own and is better compared to a creative one, governed by almost entirely aesthetical motivations, than to anything else and, in particular, to an empirical science. There is, however, a further point which, I believe, needs stressing. As a mathematical discipline travels far from empirical sources, or still more, if it is a second and third generation only indirectly inspired by ideas coming from 'reality', it is beset with very great dangers (...) that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganized mass of details and complexities.(...) At the inception the style is usually classical; when it shows signs of becoming baroque, then the danger signal is up. (John von Neumann, The Mathematician, in: *The work of the mind*, R.B. Heywood, ed., University of Chicago, 1947, **1** (1), 180–196, also *Collected Works*, vol. I, Pergamon Press, 1961, 1–9)

Interview with Stefan Rolewicz and Danuta Przeworska-Rolewicz

Professor Rolewicz, you were a winner of the First Polish Mathematical Olympiad after World War II. Was it that what made you study mathematics?

> **S.R.**: I was supposed to become a mechanical engineer and work for a company producing pumps, co-owned by my father. However, several years after the war ended it became evident that running a private enterprise is impossible and eventually my father's business was nationalized. My choice of mathematics was influenced by this olympiad. It was Kazimierz Kuratowski and Wacław Sierpiński who made me decide to enroll at the Faculty of Mathematics and Science of Warsaw University.

Professor Przeworska-Rolewicz, what made you choose mathematics as well? Your family tradition is rooted in humanities.

> **D.P.-R.**: Indeed, no one in the family was in the sciences. There were physicians, artists, both of my parents were archeologists. I intended to be an archaeologist and take advantage of my parents' large collection of books. As a matter of fact, I enrolled at the archaeological studies. But at the very last moment I changed my mind – after having accompanied

my mother at an archaeological conference. For that reason I had very little time to prepare for the entrance examinations, but I succeeded. That is how it all began.

Polish mathematics suffered severe losses during World War II. Is it possible to speak about the continuity of the Polish Mathematical School after the war?

S.R.: Definitely yes. However, the war caused a generation gap and most of our teachers were two generations older. At the time of my studies our Faculty had only three lecturers with degrees in mathematics obtained during the German occupation from an underground university: Krzysztof Maurin, Helena Rasiowa and Roman Sikorski.

Who taught mathematics at Warsaw University at that time?

S.R. and D.P.-R.: We had excellent lecturers, including Karol Borsuk, Stanisław Mazur and Andrzej Mostowski. Stanisław Mazur taught Calculus. Helena Rasiowa conducted laboratories for this class, and later, when Mazur moved to the Polish Academy of Sciences, she took over the lecture as well. Stefan Kulczycki's course, "Elementary mathematics from the advanced perspective", was devoted to the history of mathematical notions. For first two years, Stefan Pieńkowski and Leonard Sosnowski gave lectures in physics. There were also laboratories in physics.

S.R.: I believe that the decision to abolish teaching physics to mathematics students was a great mistake. Inspirations in mathematics come from the outside and physics is a source of such external inspirations. Mathematics is a tool. The fact that I had attended laboratories in physics and measurements helped me later on to collaborate with engineers. In the 19^{th} century, strong links existed between mathematics, physics and engineering and, in consequence, the level of knowledge of mathematics among engineers was high. To some extent, rapid development of topology and foundations of mathematics—to which Polish Mathematical School contributed so significantly—is responsible for weakening of those links.

Professor Rolewicz, what were the topics of Stanisław Mazur's seminar at that time? What were your main topics of interest?

S.R.: The main topic was the theory of F-spaces (linear complete metric spaces) and B_0-spaces (locally convex metric spaces). My doctoral dissertation concerned this subject and its main result was published in 1957 in the Bulletin of the Polish Academy of Sciences. Several years later I learned that during the war, in Japan, Tosio Aoki had studied similar questions (though he did not continue to work in this area). Since then one of the results of my doctoral research is known as the Aoki–Rolewicz theorem.

My research of that time was summarized in the book *Metric Linear Spaces* published by Polish Scientific Publishers in the series *Monografie Matematyczne* in 1972 and by Reidel in 1985. Its eighth chapter contains the main results of my second (habilitation) dissertation.
Quite often I present questions and open problems. For instance, in 1969 in *Studia Mathematica* I proved the existence of dense orbits in infinite-dimensional spaces and I formulated some open problems. These issues were later intensively studied and to this day many researchers deal with hypercyclic vectors (i.e. vectors whose orbits are dense).

D.P.-R.: Birkhoff's classical theorem on entire functions is in fact a theorem about orbits.

S.R.: In turn, the paper *On convex sets containing only points of support* published in *Commentationes Mathematicae* in 1978 appeared to be related to foundations of mathematics. The paper addresses the question of existence of a set with the property that each its point is a support point and provides the negative answer to this question in separable Hilbert spaces. In non-separable Hilbert spaces I constructed a set having this property, by making use of Zermelo's theorem (that is, the axiom of choice). Only recently it turned out that the existence of such a set is equivalent to the axiom of choice.

Stanisław Mazur's second dissertation, *On convex sets and convex functions in linear spaces* (1936), concerned convex analysis in locally convex linear topological spaces. Is the *paraconvex analysis* you are dealing with a development of those ideas?

S.R.: No, paraconvex analysis is another story. The underlying concept in paraconvex analysis is that of Φ-convexity, where Φ is a class of real-valued functions, invariant with respect to addition of constants and defined on a set X. A real-valued function f is Φ-convex if it is the supremum of a subfamily $\Phi_0 \subset \Phi$, i.e. $f(x) = \sup\{\phi(x) : \phi \in \Phi_0,\ \phi \leq f\}$. Elements of Φ-convex analysis are presented in the book *Foundations of Mathematical Optimization* co-authored by Diethard Pallaschke and published by Kluwer in 1997.
Various function classes Φ are of interest, e.g. the class of all real-valued functions, the class of all continuous functions, and the class $\Phi_\alpha = \{\phi(\cdot) - \alpha(d_X(\cdot, x_1)) : \phi \in \Phi, x_1 \in X\}$, where $\alpha : [0, +\infty) \to [0, +\infty)$ is continuous and nondecreasing with $\alpha(0) = 0$ and $\alpha(t) > 0$ for $t > 0$. If Φ is a class of affine functions, then Φ_α-convex functions coincide with the $\alpha(\cdot)$-paraconvex functions. For $\alpha(t) = t^2$, we obtain paraconvex functions. My main point of interest is the differentiability of $\alpha(\cdot)$-paraconvex functions. The crucial observation is that Φ-convex analysis can be built up on the monotonicity of the subdifferential rather than linearity. Hence the subtitle of the book with Diethard Pallaschke is: *Convex analysis without linearity.*

Recently you are developing convex analysis on manifolds.

S.R.: The starting point is the fact that if f is a locally strongly paraconvex real-valued function and σ is a map whose differential is uniformly continuous in norm, then the composition $f(\sigma(\cdot))$ is locally strongly paraconvex. This enables defining the convexity of a function on a manifold in a manner independent of a chart.

In the early twentieth century, the Polish Mathematical School introduced collaborated research to mathematics. At that time it was a novelty but today it is a standard. Do you consider collaborated research to be important for progress in mathematics?

S.R.: Definitely so, this leads to a synergy. A collaborated work is inspiring and prompts new questions. I already had joint works with Czesław Bessaga and Aleksander Pełczyński during my doctoral studies, later also with Czesław Ryll-Nardzewski, Boris Mityagin, Wiesław Żelazko and others.

S.R. and D.P.-R.: Another issue is dissemination of results. At conferences, the participants were sometimes surprised that we discussed new and unpublished results. But for us it was quite natural—that's what we had been taught.

What are the areas of scientific interests you share?

D.P.-R.: The main area of our common interest is linear algebra in infinite-dimensional spaces. We have joint works on perturbations of equations with transformed argument, both linear and nonlinear, and we wrote a book *Equations in Linear Spaces* published by Polish Scientific Publishers in 1968.

In 1987 in *Colloquium Mathematicum* we proved that in the space $C_c[0,1]$ of complex-valued continuous functions on the interval $[0,1]$, equipped with the sup norm, the only continuous Volterra right inverse to the operator $\frac{d}{dt}$ is the operator $\int_a^t$, $0 \leq a \leq 1$ (an operator A is a Volterra operator if the operator $A - \lambda I$ is invertible for every scalar λ). Our joint paper *On integrals with values in a complete linear metric space* published in *Studia Mathematica* in 1966, was related to the ideas of Witold Pogorzelski, my thesis supervisor. Namely, in 1953 Mazur and Orlicz gave a definition of the Riemann integral for a function $x(t)$ defined on a Euclidean space and taking values in a complete linear metric space X. We showed that the space X is locally convex if and only if every continuous function $x(t)$ is Riemann integrable. Moreover, we showed that if the space is not locally convex, then the Bochner–Lebesgue integral cannot be defined in a natural way.

Professor Rolewicz, what triggered your interest in control theory and optimization?

S.R.: The Warsaw Mathematical School was focused on pure theoretical research. One of my last conversations with Kazimierz Kuratowski concerned that matter. In Kuratowski's opinion, theory should be developed as far as possible, because 'every theory will find its application in the future'.

In the academic year 1960/1961 I attended Gelfand's seminar at Moscow University. Israil Moiseevich Gelfand, an outstanding personality, at his seminar insisted that each presentation of a new idea is limited to 15-20 minutes because —in his opinion— that is the time the audience is able to concentrate on a new topic. Obviously, the discussion of this new topic would continue at consecutive meetings. At that time the seminar had a few dozens of participants, including biologists and neurologists; Gelfand himself was interested in neural networks. Together with Wiesław Żelazko we described our stay in Soviet Union in the journal *Wiadomości Matematyczne*. After my return to Poland, I started to work in applied mathematics.

Which centres in Warsaw were active in applied mathematics at that time?

S.R.: In 1962 the Institute of Automatics of Polish Academy of Sciences was created with a mandate to conduct research in control theory and cybernetics. There I taught functional analysis for engineers. Together with my wife we started a seminar *Mathematical methods in technology* at the Institute of Mathematics of the Academy. We also collaborated with Władysław Findeisen, who headed the Institute of Automatics of the Warsaw University of Technology. For several years I was a member of the Committee for Automatics and Robotics of the Academy. At the Faculty of Cybernetics of the Military Engineering Academy (WAT) I taught Calculus in Banach spaces, Measure theory, Optimization.

D.P.-R.: Our seminar was attended by engineers and people from technical milieus. At that time Stanisław Piasecki suggested that we start teaching at the Faculty of Cybernetics of WAT. Sylwester Kaliski, Rector of WAT, supported the idea although at the beginning he was not fully convinced. I taught algebraic methods in analysis. Students who learned these methods were able to grasp differential equations very quickly. In addition, they were more successful in the course of their studies. If not for the tragic death of Sylwester Kaliski, we would continue our collaboration with WAT.

Together with the Institute of Automatics of the Warsaw University of Technology I organized a series of conferences, *Functional differential systems and related topics*, devoted to equations with transformed argument.

At that time your research, Professor Rolewicz, focused on control theory.

S.R.: Yes, the idea was to apply methods of functional analysis in control theory. A linear system was then interpreted as a linear operator.

The drop property, in turn, is related to optimization and well-posedness.

S.R.: The starting point was a result of Daneš, asserting that if we have a closed set A and a closed ball B with $\mathrm{dist}(A, B) > 0$, then there exists a point $x \in A$ such that $\mathrm{conv}\{x, B\} \cap A = \{x\}$. The question was to what extent the assumption $\mathrm{dist}(A, B) > 0$ is essential and what can be said in the case $\mathrm{dist}(A, B) = 0$. This led to the definition of a space with drop property. In a couple of papers with Denka Kutzarova we proved the drop property for any two closed subsets, one of which is convex.

Your seminar, *Mathematical methods in technology*, has recently changed its name to *Mathematical methods in technology and economics.* Where, in your opinion, does mathematics find new applications today?

S.R. and D.P.-R.: The change of name does not mean that earlier we considered applications in economics as less important. However, in general, the economists from former communist countries did not feel the need and did not have understanding for collaboration with mathematicians. On the other hand, our collaboration with engineers is not always that smooth. The reason is that engineers tend to adopt certain assumptions for granted and do not formulate them explicitly—to which mathematicians instinctively object. The 'language' of engineers is somewhat different and requires clarification from the mathematical point of view.

Today, of course, nobody objects to the usefulness of mathematics in describing economical phenomena. The scope of applications widens constantly. John von Neumann's views, formulated 60 years ago, have turned out to be true and are still relevant today:

...In modern empirical sciences it has become more and more a major criterion of success whether they have become accessible to the mathematical method or to the near-mathematical method of physics. Indeed, throughout the natural science an unbroken chain of successive pseudomorphoses, all of them pressing towards mathematics, and almost identified with the idea of scientific progress, has become more and more evident. (John von Neumann, The Mathematician, op.cit.)

Interview by Ewa Bednarczuk, July 2007

Translated by Jerzy Trzeciak