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# Fuzzy adaptive control of a class of MISO nonlinear systems*

by

**I. Lagrat, H. Ouakka and I. Boumhidi**

LESSI, Faculté Des Sciences
B. P 1796 Atlas, 30000 Fez, Morocco
e-mail" i.lagrat@caramail.com

**Abstract:** This paper presents a fuzzy adaptive control of a class of MISO nonlinear systems. The dynamic behaviour of each MISO systems is composed of a nonlinear term, interactions effect between the inputs, and disturbances. In these circumstances, adaptive control becomes very difficult to implement and not always an evident task. Thus, the MISO system is approximated by the Takagi-Sugeno fuzzy model. The advantage of this approximation is beneficial in the sense that it allows for converting the nonlinear problem into a linear one. In this respect, the coupling, nonlinearity and unmodeled dynamics are easily compensated. The identification and the control are conducted at the level of each local linear model based on fuzzy approach. The computational load and the complexity of nonlinear approach are reduced and permit wide applicability. The validity and the performance are tested numerically.

**Keywords:** MISO systems, identification, fuzzy clustering, Takagi-Sugeno fuzzy model, adaptive control, nonlinear system.

## 1. Introduction

Identification of nonlinear multi-variable processes is an important and challenging problem. For nonlinear static and dynamic systems, the conventional techniques of modelling and identification are difficult to implement and sometimes impracticable. However, other techniques, based on fuzzy logic system, can be used for modelling of the complex nonlinear processes (Chen, Chen, 1994). Fuzzy modelling and identification from the input-output process data is shown to be effective for the approximation of nonlinear uncertain dynamic system (Johancen, Foss, 1993). The Takagi-Sugeno model has attracted the attention of many researchers. Accordingly, the most important issue for fuzzy logic system is how to obtain a system design with the guarantee of stability

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and control performance (Tanaka, Wang, 2001). In fact, this model is based on if-then rules, which are characterized by fuzzy antecedent and mathematical functions in the consequent part (Takagi, Sugeno, 1985). The antecedent fuzzy sets divide the input space into a number of fuzzy regions, while the consequent function describes the system behaviour in these regions. The task of fuzzy model construction is to determine both the nonlinear parameters of the membership functions and the linear parameters of the local models (Johancen, Foss, 1993). The human expert, in this regard, is able to formulate the process knowledge in terms of the fuzzy rules. Unfortunately, this can not provide a clear idea of the plant behaviour, because the human expert cannot come up with all the details and might not be able to quantitatively express the observations. However, heuristic approaches, like fuzzy clustering, are applied (Nelles, Fink, Isermann, 2000) to obtain the fuzzy model of nonlinear dynamic using input-output measurement data, e.g., the local linear model tree method and tree construction algorithm (Sugeno, Kang, 1987), or the neuro-fuzzy inference system (Jang, 1993). There are different algorithms that construct fuzzy clusters, such as the c-means algorithm (Bezdek, 1981), the Gath-Geva algorithm (Gath, Geva, 1989), and ultimately the Gustafson-Kessel algorithm (Gustafson, Kessel, 1979), which is the subject of this paper.

The paper presents a fuzzy adaptive control of a class of MISO systems. Each MISO nonlinear system is coupled in the inputs. The considered system is transformed by Takagi-Sugeno approach and fuzzy clustering into linear models (Takagi, Sugeno, 1985). In this transformation, the parameters of the Gaussian membership function and the local linear model are easily obtained. Moreover, the time-variant behaviour of the plant, which is caused by disturbances or aging of components, should be considered in the system model. Therefore, online adaptation of the fuzzy models is required. The local linear parameters of each model in the rule consequents of Takagi-Sugeno fuzzy models are updated by a local recursive weighted least-squares algorithm with a forgetting factor (Trabelsi et al., 2004). The proposed control law is derived from Feng and Chen (2005), and is designed to compensate for the interactions between the inputs. The identification and the control are obtained, independently, for each local model. This strategy reduces the computational burden of the global approach.

The paper is organized as follows: Section 2 contains the presentation of the Takagi-Sugeno fuzzy model. Section 3 describes fuzzy identification of MISO nonlinear system. Section 4 presents the adaptive control law designs. In Section 5, we present a numerical example.

## 2. Takagi-Sugeno fuzzy model of a MISO process

Consider a class of MISO nonlinear system represented as follows:

$$\begin{aligned} y(k+1) = & f(y(k), \ldots, y(k-na+1), u_1(k), \ldots, u_1(k-nb+1), u_2(k), \ldots, \\ & u_2(k-nb+1), \ldots, u_{nu}(k), \ldots, u_{nu}(k-nb+1), \eta(k)) \end{aligned} \tag{1}$$

where $f$ represents the unknown nonlinear function and $\eta(k)$ are the disturbances. We assume that the upper bound of the orders $na$ and $nb$ are known and equal to $n$, and then:

$$\begin{aligned} y(k+1) = & f(y(k),\dots,y(k-n+1),u_1(k),\dots,u_1(k-n+1),u_2(k),\dots, \\ & u_2(k-n+1),\dots,u_{nu}(k),\dots,u_{nu}(k-n+1),\eta(k)) \end{aligned}$$

We define the regression vector $\phi(k)$:

$$\begin{aligned} \phi(k) = & [y(k),\dots,y(k-n+1),\ u_1(k),\dots,u_1(k-n+1),u_2(k),\dots, \\ & u_2(k-n+1),\dots,u_{nu}(k-n+1),\dots,u_{nu}(k-n+1),\eta(k)] \end{aligned} \tag{2}$$

Then, equation (1) can be written as:

$$y(k+1) = f(\phi(k)). \tag{3}$$

The function $f(\phi(k))$ is approximated by Takagi-Sugeno fuzzy models, which are characterized by the linear function rule consequents as in Takagi and Sugeno (1985). The Takagi-Sugeno MISO rules are estimated from the system input-output data (Babuska, 1998). The base rule contains $M$ rules of the following form:

$$\begin{aligned} & R_j :\ \textit{if}\ \ y(k)\ \ \textit{is}\ \ \Omega_{j1}\ \ \textit{and}\ \ u_{nu}(k-n+1)\ \ \textit{is}\ \ \Omega_{jnu}\ \ \textit{then} \\ & y_j(k+1) = \sum_{r=1}^{n} a_{jr} y_j(k-r+1) + \sum_{r=1}^{n} b_{jr} u(k-r+1) \\ & \qquad + \sum_{l=1}^{nu}\sum_{r=1}^{n} b_{jlr} u_l(k-r+1) + c_j \quad j = 1,2,\dots,M. \end{aligned} \tag{4}$$

The number $M$ of rules is determined by testing many values according to the error criterion as given in Trabelsi et al. (2004). The antecedent fuzzy sets $\Omega_{ji}$ are defined on the universe of discourse of input $i$; $c_j$ is the offset and the linear parameters $\theta_j$ in the rule consequents are given by:

$$\theta_j = [\,a_{j1},\dots,a_{jn},b_{j1},\dots,b_{jn},\dots,b_{jnu1},\dots,b_{jnu\,n},c_j]. \tag{5}$$

The membership function is chosen as Gaussian with centre $v_j$ and standard deviation $\sigma_j$.

We take the product as the AND operator. The output of the fuzzy system with $M$ rules is aggregated as:

$$y(k+1) = \frac{\sum\limits_{j=1}^{M} \mu_j(\phi(k)) \cdot y_j(k+1)}{\sum\limits_{j=1}^{M} \mu_j(\phi(k))} \tag{6}$$

or:

$$y(k+1) = \sum_{j=1}^{M} y_j(k+1) \cdot \Phi_j(\phi(k), v_j, \sigma_j) \tag{7}$$

where $\Phi_j(\phi(k), v_j, \sigma_j)$ denotes the normalized validity function such that $\sum_{j=1}^{M} \Phi_j(\phi(k), v_j, \sigma_j) = 1$ for all the premise inputs $\phi(k)$.

This normalization is achieved by:

$$\Phi_j(\phi(k), v_j, \sigma_j) = \frac{\mu_j(\phi(k))}{\sum_{j=1}^{M} \mu_j(\phi(k))} \tag{8}$$

with $\mu_j$ being the degree of fulfillment of the rule $j$:

$$\mu_j(\phi(k)) = \exp(-\frac{1}{2}(\frac{(y(k)-v_{j1})^2}{\sigma_{j1}^2}))\cdots\exp(-\frac{1}{2}(\frac{(u_{nu}(k-n+1)-v_{ju_{nu}})^2}{\sigma_{ju_{nu}}^2})). \tag{9}$$

Once the structure is fixed, the MISO parameters are estimated independently by fuzzy clustering (Babuska, Verbruggen, 1995).

## 3. Fuzzy identification

Generally, the approximation by a Takagi-Sugeno fuzzy model enables wide applicability in identification, modelling and control (Babuska, 1998; Takagi, Sugeno, 1985). In fact, fuzzy clustering facilitates automatic generation of Takagi-Sugeno rules and their antecedent parameters. The identification procedure consists of two distinct steps, Trabelsi et al. (2004). The first step is off-line identification, where nonlinear parameters of the Gaussian membership function $(v_j, \sigma_j)$ and linear parameters of the local models $\theta_j$ are determined by the fuzzy clustering algorithm (Babuska, 1998). In the second step, the rule consequents are locally adapted on-line by a recursive least-squares algorithm (Nelles, Fink, Isermann, 2000).

### 3.1. Off-line identification of the fuzzy model

The previous section has shown how the consequent part of Takagi-Sugeno models can be identified by weighted least-squares method. This would not occur unless the antecedent membership functions are given. The Gustafson-Kessel algorithm is used to identify the Takagi-Sugeno models. The available data samples are collected in matrix $Z = [\phi^T y]$, formed by concatenating the regression matrix and the output vector. Through clustering, the data $Z$ are partitioned into $N_c$ clusters. The result is a fuzzy partition matrix $U = [u_{ij}]_{N_c \times N}$, whose elements $u_{ij} \in [0,1]$ represent the degree of membership in cluster $i$, a prototype matrix $V_j = [v_{j1}, \ldots, v_{jnu}]$, and the set of cluster covariance matrices $F_j = [F_{j1}, \ldots, F_{jnu}]$. Once the triplet $(U_j, V_j, F_j)$ determined, the parameters of the rule premises $(v_{ji}, \sigma_{ji})$ and the consequent parameters $\theta_j$ are computed. For more details see Babuska and Verbruggen (1996). Afterwards, the antecedent membership functions of the cluster parameters are determined. The Gaussian functions are used to represent the fuzzy sets, $\omega_{ji}$:

$$\omega_{ji}(\phi(k)) = \exp(-\frac{1}{2}(\frac{(\phi_i(k) - v_{ji})^2}{\sigma_{ji}^2}))\,. \tag{10}$$

The consequent parameters $\theta_j$ in each rule are estimated separately by weighted least-squares method and by minimizing the following criterion (Babuska, Verbruggen, 1996):

$$\min_{\theta_j} \frac{1}{N} \left[y - \xi\theta_j\right]^T Q_j \left[y - \xi\theta_j\right] \tag{11}$$

where $\xi = [\phi \ 1]$ is the regression extended by a unitary column and $Q_j$ is a matrix containing the values of validity functions $\Phi_j$ of each $jth$ local model.

The weighting matrix is:

$$Q_j = diag\left(\Phi_j(\phi(1), v_j, \sigma_j), \ldots, \Phi_j(\phi(N), v_j, \sigma_j)\right). \tag{12}$$

Then, $\theta_j$ is obtained as the weighted least-squares solution:

$$\theta_j = (\xi^T Q_j \xi)^{-1} \xi^T Q_j y. \tag{13}$$

### 3.2. On-line adaptation of fuzzy model

In the on-line phase, the rule consequents (13) are adapted by the recursive weighted least-squares (RWLS) algorithm with a forgetting factor. This approach is used to estimate the parameters of each local linear model. For the $jth$ local linear model, we can compute new parameter estimates $\hat{\theta}_j(k)$ as in (14):

$$\hat{\theta}_j(k) = \hat{\theta}_j(k-1) + \delta_j(k)\left(y(k) - \xi^T(k)\hat{\theta}_j(k-1)\right) \tag{14}$$

$$\delta_j(k) = \frac{P_j(k-1)\xi(k)}{\xi^T(k)P_j(k-1)\xi(k) + \lambda/\Phi_j(\phi(k), v_j, \sigma_j)} \tag{15}$$

$$P_j(k) = \frac{1}{\lambda}\left[\mathrm{I} - \delta_j(k)\xi^T(k)\right] P_j(k-1). \tag{16}$$

Here, $\lambda$is a forgetting factor and $\Phi_j(\phi(k), v_j, \sigma_j)$ provide the weights of actual data. Then, $P_j$ is the matrix of adaptation gain.

## 4. Fuzzy adaptive control design

In this section, we will shed light on the adaptive control design for MISO nonlinear system using the Takagi-Sugeno fuzzy models as shown in Feng and Chen (2005). In this respect, we propose a control law, which can easily control a MISO nonlinear system without any restrictive conditions on nonlinearity. The strategy of this control is based on the fuzzy approach, which transforms the nonlinear problem to a linear one, and compensates for the unmodeled dynamics and nonlinearity effects. The structure of the proposed control law takes into account the coupling between the inputs and the disturbances, and permits to ensure stability, performance and wide applicability.

The controller rule has the same antecedents and fuzzy sets as in plant rules. The proposed local adaptive control law is given by:

$$\begin{aligned}
&R_j : \textit{if } y(k) \textit{ is } Omega_{j1} \textit{ and } \ldots \textit{ and } u_{nu}(k-n+1) \textit{ is } \Omega_{jnu} \textit{ then}\\
&u_j(k) = \frac{1}{\hat{b}_{j\hat{1}}}[-\sum_{r=1}^{n} \hat{a}_{jr} y_j(k-r+1) - \sum_{r=2}^{n} \hat{b}_{jr} u(k-r+1)\\
&\quad -\sum_{l=1}^{nu}\sum_{r=1}^{n} \hat{b}_{jlr} u_l(k-r+1) - \hat{c}_j + y_m(k+1) - \alpha_1 e(k) - \cdots\\
&\quad -\alpha_n e(k-n+1)], \quad j = 1, 2, \ldots, M. \qquad (17)
\end{aligned}$$

The global control law is obtained as:

$$u(k) = \sum_{j=1}^{M} \Phi_j(\phi(k), v_j, \sigma_j)\, u_j(k). \qquad (18)$$

More precisely, the objective of the fuzzy adaptive control is to find an adaptive control law, which guarantees that the output of the MISO systems can track a given bounded reference signal. The tracking error is given by:

$$e = y - y_m. \qquad (19)$$

With $\{\alpha_i\}$ being the coefficients of the Hurwitz polynomial:

$$\alpha(z) = z^n + \alpha_1 z^{n-1} + \cdots + \alpha_{n-1} z + \alpha_n \qquad (20)$$

we pass over to

THEOREM 1 *For the fuzzy dynamic model of the system (7), if the adaptive control law is chosen as (17), or (18), using the adaptation algorithm (14), then the closed-loop system is stable in the sense that the output and all inputs are bounded for all the time. The output tracking error* $e = y - y_m$ *will approach to the zero as time goes to infinity.*

*Proof.* Substituting the control law (18) into the fuzzy dynamic model (7) leads to the following closed-loop system:

$$\begin{aligned} &e(k+1) + \alpha_1 e(k) + \cdots + \alpha_{n-1} e(k-n+2) + \alpha_n e(k-n+1) \\ &= H(k) - \hat{H}(k) + \sum_{j=1}^{M} \Phi_j(\phi(k), v_j, \sigma_j)(b_{j1} - \hat{b}_{j1}) u_j(k) \end{aligned} \tag{21}$$

where

$$\begin{aligned} H(k) &= \sum_{r=1}^{n} a_{jr} y_j(k-r+1) + \sum_{r=2}^{n} b_{jr} u(k-r+1) + \sum_{l=1}^{nu} \sum_{r=1}^{n} b_{jlr} u_l(k-r+1) + c_j \\ \hat{H}(k) &= \sum_{r=1}^{n} \hat{a}_{jr} y_j(k-r+1) + \sum_{r=2}^{n} \hat{b}_{jr} u(k-r+1) + \sum_{l=1}^{nu} \sum_{r=1}^{n} \hat{b}_{jlr} u_l(k-r+1) + \hat{c}_j. \end{aligned} \tag{22}$$

By defining:

$$x_e(k) = [e(k-n+1) \;\; e(k-n+2) \;\; \cdots \;\; e(k)]^T \tag{23}$$

the above closed-loop system can be expressed in state-space form, as:

$$x_e(k+1) = Ax_e(k) + B\Big\{H(k) - \hat{H}(k) + \sum_{j=1}^{M} \Phi_j(\phi(k), v_j, \sigma_j)(b_{j1} - \hat{b}_{j1}) u_j(k)\Big\}. \tag{24}$$

That is:

$$\begin{aligned} x_e(k+1) =& Ax_e(k) + B\Big\{(H(k) + \sum_{j=1}^{M} \Phi_j(\phi(k), v_j, \sigma_j) b_{j1} u_j(k)) \\ & - (\hat{H}(k) + \sum_{j=1}^{M} \Phi_j(\phi(k), v_j, \sigma_j) \hat{b}_{j1} u_j(k))\Big\} \end{aligned} \tag{25}$$

where

$$\begin{aligned} y(k+1) =& H(k) + \sum_{j=1}^{M} \Phi_j(\phi(k), v_j, \sigma_j) b_{j1} u_j(k) \\ \hat{y}(k+1) =& \hat{H}(k) + \sum_{j=1}^{M_i} \Phi_j(\phi(k), v_j, \sigma_j) \hat{b}_{j1} u_j(k). \end{aligned} \tag{26}$$

We consider:

$$e(k+1) = y(k+1) - \hat{y}(k+1) \tag{27}$$

and so:

$$x_e(k+1) = Ax_e(k) + Be(k+1) \tag{28}$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ -\alpha_n & -\alpha_{n-1} & \cdots & -\alpha_2 & -\alpha_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \tag{29}$$

It should be noted that the matrix $A$ has all its eigenvalues located inside the unit circle of the z-plane, and so stability is ensured. ■

## 5. Simulation example

Consider a MIMO system (Weng, Lang, 1990) having a polynomial nonlinearity described as:

$$A(q^{-1})y(t) = B_d(q^{-1})Z(t) + \eta(t) \tag{30}$$

where $q^{-1}$ is the back shift operator.

$$A(q^{-1}) = diag[A_i(q^{-1})]\,,\ A_i(q^{-1}) = \sum_{r=0}^{na(i)} a_{ir}q^{-r},\ a_{i0} = 1$$

$$B_d(q^{-1}) = [q^{-dij}B_{ij}(q^{-1})],\ B_{ij}(q^{-1}) = \sum_{r=0}^{nb(i,j)} b_{ijr}q^{-r},\ b_{ij0} \neq 0$$

$d_{ij}$ is time delay between the input $u_j(t)$and the output $y_i(t)$, $y(t) \in R^n$, $Z(t) \in R^n$ and $\eta(t) \in R^n$are respectively the output, nonlinear input and the disturbances vectors. As in Jinxing and Shijium (1989) the nonlinearity is assumed to be represented as follows:

$$Z_i(t) = f_{i0} + f_{i1}u_i(t) + \ldots + f_{ip}u_i^{p_i-1}(t). \tag{31}$$

The MIMO system is represented through two MISO nonlinear systems as follows:

$$A_1(q^{-1})y_1(t) = q^{-d_{11}}B_{11}(q^{-1})Z_1(t) + q^{-d_{12}}B_{12}(q^{-1})Z_2(t) + \eta_1(t)$$
$$A_2(q^{-1})y_2(t) = q^{-d_{22}}B_{22}(q^{-1})Z_2(t) + q^{-d_{21}}B_{21}(q^{-1})Z_1(t) + \eta_2(t)$$

with

$$A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}^T = \begin{bmatrix} 1 + 0.35q^{-1} + 0.15q^{-2} \\ 1 + 0.72q^{-1} + 0.05q^{-2} \end{bmatrix},$$

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 1 + 0.85q^{-1} & 2 + 1.25q^{-1} \\ 0.12 + 0.65q^{-1} & 1.65 + 0.23q^{-1} \end{bmatrix}$$

$$d_{11} = 1,\ d_{12} = 1,\ d_{21} = 1,\ d_{22} = 1$$

$$Z_1(t) = 0.5u_1(t) + 0.25u_1^2(t), \quad Z_2(t) = u_2(t) + 0.83u_2^2(t)$$

$$\eta_1(t) = 0.1rand(1, 600), \quad \eta_2(t) = 0.12rand(1, 600)$$

The inputs are $u_1$ and $u_2$. The outputs are $y_1$ and $y_2$, with $\eta_1$ and $\eta_2$ representing disturbances. A prototype matrix and covariance matrix are determined by the Gustafson-Kessel algorithm (Babuska, 1998):

$$V_1 = \begin{bmatrix} 0.0235 & 0.0720 & 0.0023 & 0.0022 & 0.0041 & 0.0042 \\ 0.0424 & 0.0422 & 0.0022 & 0.0022 & 0.0040 & 0.0040 \\ 0.0846 & 0.0688 & 0.0022 & 0.0022 & 0.0041 & 0.0039 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 0.0713 & 0.0269 & 8.0335 & 8.4517 & 4.8071 & 4.9064 & 0.0408 \\ 0.1299 & 0.1224 & 6.4028 & 6.8515 & 4.1058 & 3.7856 & 0.0311 \\ 0.3344 & 0.4947 & 12.9454 & 13.1246 & 7.9306 & 8.1032 & 0.0614 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 0.0313 & 0.0457 & 0.0023 & 0.0023 & 0.0041 & 0.0042 \\ 0.0378 & 0.0377 & 0.0022 & 0.0022 & 0.0040 & 0.0040 \\ 0.0722 & 0.0883 & 0.0022 & 0.0022 & 0.0040 & 0.0039 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 0.1878 & 0.1397 & 13.1520 & 11.7753 & 6.1561 & 6.7180 & 0.0669 \\ 0.1956 & 0.1556 & 11.1458 & 10.6673 & 5.6689 & 5.8682 & 0.0579 \\ 0.4412 & 0.2936 & 23.6297 & 25.8100 & 14.0950 & 13.6570 & 0.0970 \end{bmatrix}$$

The consequent parameters of each rule of Takagi-Sugeno fuzzy model are computed from equation (13) and adapted by using RLS algorithm with forgetting factor ($\lambda = 0.45$), Trabelsi et al. (2004).

For the rule $i$:

$$\begin{aligned} R_i : y_{i1}(k+1) &= a_{i111}(k)y_1(k) + a_{i112}(k)y_1(k-1) + b_{i111}(k)u_1(k) \\ &\quad + b_{i112}(k)u_1(k-1) + b_{i121}(k)u_2(k) + b_{i122}(k)u_2(k-1) + c_{i111} \\ y_{i2}(k+1) &= a_{i211}(k)y_2(k) + a_{i212}(k)y_2(k-1) + b_{i211}(k)u_2(k) \\ &\quad + b_{i212}(k)u_2(k-1) + b_{i221}(k)u_1(k) + b_{i122}(k)u_1(k-1) + c_{i211} \end{aligned}$$

Fig. 1 shows the evolutions of parameters of the first output $y_1$ for three rules. Fig. 2 shows the evolutions of parameters of the second output $y_2$ for three rules.

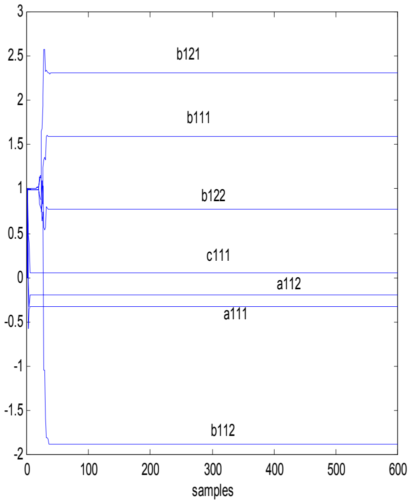
In Figs. 1 and 2 we can notice that the linear parameters vary until the sample 80 and 100, respectively for the output $y_1$ and $y_2$. Afterwards, they are practically constant.

The reference signals $y_{1m}(k)$ and $y_{2m}(k)$ are square waves with period 100.

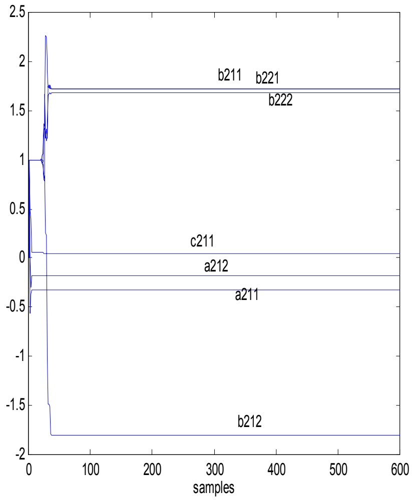
As shown in Figs. 3 and 4, exact tracking is obtained using the proposed adaptive fuzzy control. The corresponding control laws are presented in Figs. 5 and 6.

## 6. Conclusion

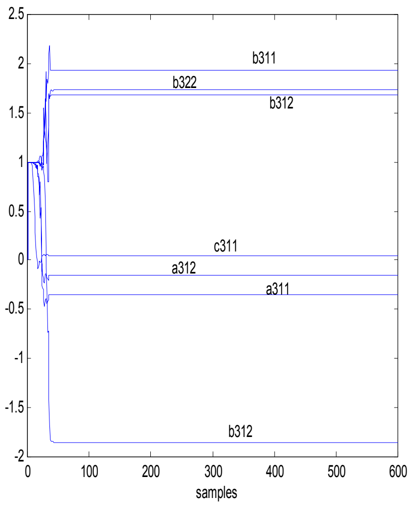
The approach presented provides a solution to the problem of robust control of MISO nonlinear systems. For each MISO system a local fuzzy adaptive




evolution of parameters of rule (1)

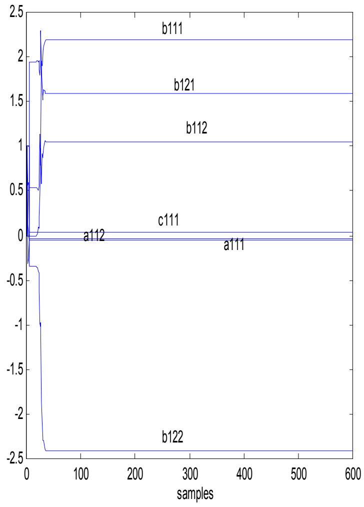



evolution of parameters of rule (2)

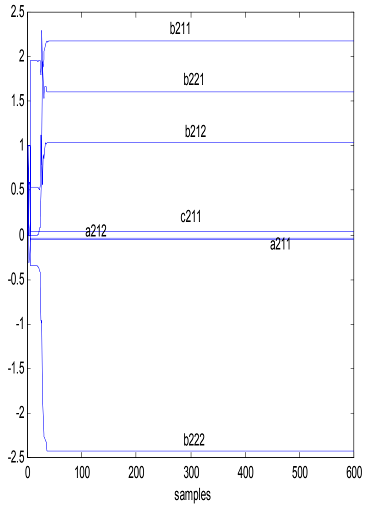



evolution of parameters of rule (3)

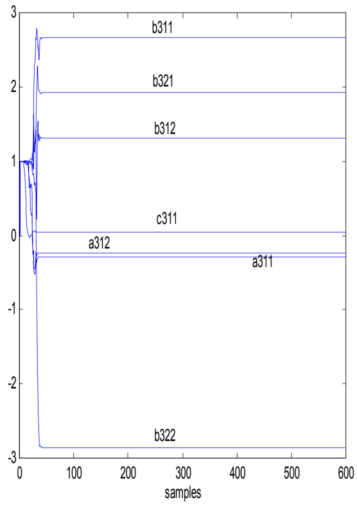
Table 1. Evolution of parameters of the first output $y_1(t)$




evolution of parameters of rule (1)




evolution of parameters of rule (2)




evolution of parameters of rule (3)

Table 2. Evolution of parameters of the second output $y_2(t)$

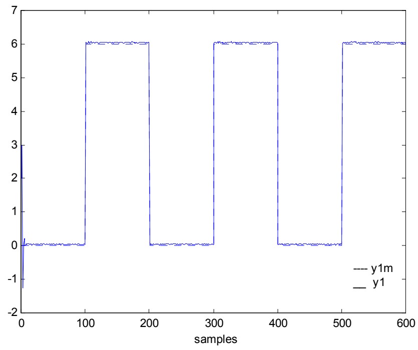



Figure 3. The behaviour of $y_1(k)$ and $y_{1m}(k)$

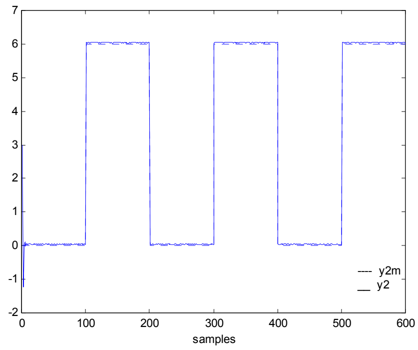



Figure 4. The behaviour of $y_2(k)$ and $y_{2m}(k)$

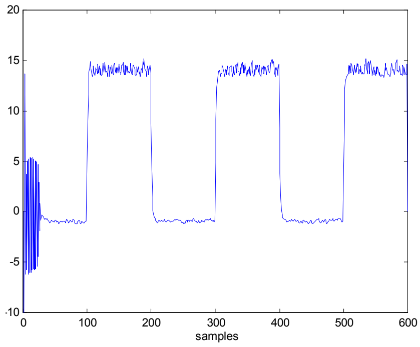



Figure 5. The behaviour of the control $u_1(k)$

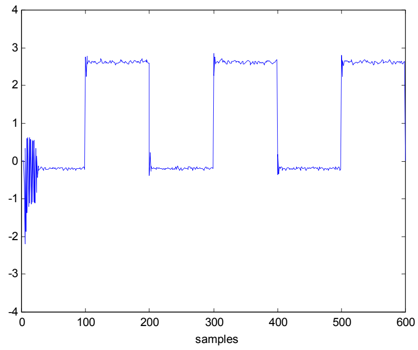



Figure 6. The behaviour of the control $u_2(k)$

control is given. The assumed unknown nonlinearity, coupling between inputs, unmodeled dynamics and disturbances are approximated by the Takagi-Sugeno fuzzy model. The latter allows the conversion of the nonlinear problem into a linear one. This approximation leads to both solving the nonlinearity problem and to releasing the decoupling through the control synthesis. It is shown on a numerical example that the proposed approach is a robust scheme and can deal with a large class of MISO nonlinear systems.

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