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An approximate economically optimal inspection interval for production processes with finite run length*

by

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Abstract: Increased competition on the global market forces producers to follow policies leading to finite production runs. This situation requires implementation of a new type of inspection procedures with the aim to improve or sustain production quality levels. One of the most important aspects in the design of inspection processes is the specification of inspection intervals. This paper provides a simple procedure to determine an approximate optimal inspection interval h for a given inspection plan, characterized by its probability of type-I error α and probability of type-II error β, for processes with finite run length.

Keywords: inspection plan, inspection interval, just-in-time production, finite run length.

1. Introduction

Quality is the most important decision factor, however, the occurrence of assignable or random causes results in variation of the quality characteristics of interest. Thus, it is desirable to inspect the output at different stages of a production process in order to correct it and/or to assure its quality. The inspection is usually done by periodically drawing random samples from the process. However, other type of inspections could also be applied.

The process of designing an inspection procedure consists, mainly, of two stages:

1) specification of the inspection (sampling) plan to be performed at the end of a given inspection interval, and
2) determination of the inspection (sampling) interval for a given inspection plan.

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In this paper we focus on the second stage of this process. There are many approaches to determination of the inspection interval. However, the economic approach has attracted many researchers, who proposed many models and algorithms to determine optimal inspection intervals. Further information on this subject can be found in Bather (1963), Chiu and Wetherill (1975), Duncan (1956, 1978), Gibra (1970), Ladany (1973), Lorenzen and Vance (1986), Montgomery et al. (1975), Montgomery (1980, 1982), Panagos et al. (1985), Saniga (1989). The general economic model for the optimization of statistical process control can be found in Keats et al. (1997). The most recent review of the problems of the optimal design of control charts is given in Ho and Case (1994).

Nowadays, industry faces rapid changes in user requirements, which force firms to follow the "just-in-time" policy that allows them to produce a smaller number of items in response to customers' immediate request. This environment leads to frequent setups of the process, causing shorter (finite) production runs. The new circumstances require methods to determine inspection intervals different from those for infinite production runs. Some research has been done to solve this problem, and the most interesting results have been presented in the paper by Del Castillo et al. (1996). The problem was also considered in the papers by Quesenberry (1991), Crowder (1992), Del Castillo and Montgomery (1994, 1995), who discussed methods for the calculation of a sampling interval in the context of the design of control charts. Unfortunately, most of the proposed algorithms are too complicated to be used at a production line. In this paper we present an easy to compute procedure that solves the problem of the determination of the optimal economic inspection interval, h, for a process of finite length. Our approach is based on the results presented in a seminal work of von Collani (1986, 1989). As the objective function we propose the loss per unit produced. The calculation of this characteristic requires detailed information about the process behavior and knowledge of statistical properties of the inspection procedure. Moreover, we assume that some economic quantities, like the gain from the correctly operating process and some other cost parameters are also known.

The paper is organized as follows: In the second section we introduce the mathematical model of the inspection process when the production period is finite, i.e. in the situation of finite production runs. In the third section we present an economic model that describes the consequences of the implementation of the inspection procedures. This model is used in the fourth section for the optimization of the inspection interval. Finally, in Section 5, we discuss the results obtained and present their possible generalizations.

2. The mathematical model of the process

The process under investigation is assumed to have a finite length of t items, and constant production rate of ν items per hour. When the process is described by a real-valued variable we assume that the process starts in a stable state of control (in-control State I) centered at the target value μ_0. We also assume that its variability is known, and described by the standard deviation σ. Moreover, let us assume that the process can go out of control, and its deterioration takes the form of a shift of a known magnitude ($\pm\delta\sigma$) in the process mean. The deterioration shifts the process from the in-control State I to the out-of-control State II, characterized by its mean value, either $\mu_1 = \mu_0 - \delta\sigma$ with probability $P(\mu = \mu_1)$ or $\mu_2 = \mu_0 + \delta\sigma$ with probability $P(\mu = \mu_2)$, where $\delta > 0$ is the shift size, while the variance remains unchanged. In a general case, we assume that there exists a precise description of both states of the process, and the out-of-control state is unique.

Let us formulate some further assumptions.

1) The states of the process are recognized by inspection only.
2) The process is not self-correcting, that is, once a transition to State II has occurred, the process remains there until some corrective actions are taken in order to return the process to State I.
3) The duration of State I is a random variable, T^*, which is exponentially distributed with a known parameter λ.

The inspection procedure consists in the determination of the state of the process, e.g. by drawing periodically samples of known size at intervals of h hours. According to the inspection results, appropriate actions should be taken in order to bring the process back to State I in case of its deterioration. However, due to the randomness of the results of the process inspections there is a possibility for two erroneous signals:

a) a false alarm which occurs with known probability α when the inspection gives a signal that the process is in state II when it is not (type-I error);
b) a non-detection of an existing shift, which occurs with a known probability β, i.e. indication that the process is in state I whereas it is in state II (type-II error).

Now, let us introduce several random variables that will be used for the formulation of the objective function. Let U_j be the number of inspections performed during state I of the j^{th} cycle. A random event $\{U_j = i\}$ means that during the j^{th} cycle the transition from the in-control State I to the out-of-control State II occurs in the time interval $(ih, (i+1)h)$, and is equivalent to a random event $\{ih < T^* < (i+1)h\}$. The probability mass function of U_j for the exponentially distributed time T^* is given by

$$P(U_j = i) = (1 - e^{-\lambda h})(e^{-\lambda h})^i,\ i = 0, 1, 2, \ldots \quad j = 1, 2, \ldots \tag{1}$$

It is easy to show that U_j has the expectation

$$E(U_j) = \mu_U = \frac{1 - (1 - e^{-\lambda h})}{1 - e^{-\lambda h}} = \frac{e^{-\lambda h}}{1 - e^{-\lambda h}} = \frac{1}{e^{\lambda h} - 1} \tag{2}$$

and the variance

$$\sigma_U^2 = \frac{1-(1-e^{-\lambda h})}{(1-e^{-\lambda h})^2} = \frac{e^{-\lambda h}}{(1-e^{-\lambda h})^2} = \frac{e^{\lambda h}}{(e^{\lambda h}-1)^2} \quad . \tag{3}$$

Now, let us introduce a random variable V_j that describes the number of inspections in the j^{th} cycle drawn during the out-of-control State II. The random event $\{V_j = k\}$ means that the shift is detected after the k^{th} inspection, i.e. that k inspections are required to detect a shift which had occurred during the j^{th} cycle. Since the probability of not detecting the existing shift is equal to β, the probability of its detection is equal to $1-\beta$. Thus, the probability mass function of V_j is given by the following formula

$$P(V_j = k) = (1-\beta)\beta^{k-1},\ k = 1, 2, \ldots; \quad j = 1, 2, \ldots \tag{4}$$

Hence, the expectation and the variance of the number of inspections performed in State II of each cycle are given by

$$E(V_j) = \mu_V = \frac{1}{1-\beta}, \tag{5}$$

and

$$\sigma_V^2 = \frac{\beta}{(1-\beta)^2} \quad , \tag{6}$$

respectively. Now, let us denote by F_j the number of false alarms during the j^{th} cycle. Since false alarms occur only as the result of inspections in state I, and since each inspection in this state triggers a false alarm with probability α, the number of false alarms is described by the binomial distribution with parameters (U_j, α), where U_j is a random variable described previously. Since the $U_j^{'}$s are i.i.d. we can easily find that the expectation of the conditional random variable $F_j|U_j$ is equal to $E(F_j|U_j) = E(F|U) = \alpha U$, and its variance is equal to $V(F_j|U_j) = V(F|U) = \alpha(1-\alpha)U$. Consequently, the unconditional expected number of false alarms observed in each cycle is given by

$$E(F_j) = \mu_F = \frac{\alpha}{e^{\lambda h}-1},\ \ j = 1, 2, \ldots, \tag{7}$$

and its variance is given by

$$\sigma_F^2 = V(F_j) = V(F) = V[E(F|U)] + E[V(F|U)] = V(\alpha U) + E[\alpha(1-\alpha)U] =$$
$$= \alpha^2\sigma_U^2 + \alpha(1-\alpha)\mu_U = \frac{\alpha^2 e^{\lambda h}}{(e^{\lambda h}-1)^2} + \frac{\alpha(1-\alpha)}{e^{\lambda h}-1} \tag{8}$$

The next random variable W_j, $j = 1, 2, \ldots$ represents the number of inspections of the process during its j^{th} cycle. It is easy to notice that $W_j = U_j + V_j$ for all

$j = 1, 2, \ldots$. Assuming the independence of U_j and V_j for all $j = 1, 2, \ldots$, the sequence of random variables $\{W_j\}_{j=1}^{\infty}$ are i.i.d. with the expectation

$$\mu_W = \mu_U + \mu_V = \frac{1}{e^{\lambda h} - 1} + \frac{1}{1-\beta} = \frac{e^{\lambda h} - \beta}{(1-\beta)(e^{\lambda h} - 1)} = \frac{1 + B(e^{\lambda h} - 1)}{e^{\lambda h} - 1}, \quad (9)$$

and variance

$$\sigma_W^2 = \sigma_U^2 + \sigma_V^2 = \frac{e^{\lambda h}}{(e^{\lambda h} - 1)^2} + \frac{\beta}{(1-\beta)^2} = \frac{e^{\lambda h}}{(e^{\lambda h} - 1)^2} + B(B-1), \quad (10)$$

where

$$B = \frac{1}{1-\beta}\,. \quad (11)$$

Let S_k be the number of inspections of the process up to its k^{th} renewal, i.e., S_k gives the time of the k^{th} renewal in terms of the number of inspections. Hence,

$$S_k = \sum_{j=1}^{k} W_j, \quad k = 1, 2, \ldots. \quad (12)$$

Since $\{W_j\}_{j=1}^{\infty}$ are i.i.d., then the above sequence of random variables defines an ordinary renewal process.

Further, as we have assumed that inspections are performed every h hours, then for a process with a run of t consecutive items at the production rate of ν items per hour, the expected number of inspection performed during the run is $t/\upsilon h$ samples. Now, let N_t denote the number of renewal cycles completed within a production run of t items (or $t/\upsilon h$ samples). To analyze this random variable let us use the approach proposed by Blackwell (1977) and Yang (1983) who utilized the basic results of Cox (1962). Cox (1962) has shown that the approximate expected value of the number of renewals $N_{t'}$ in the time interval $(0, t')$ can be found from the following expression

$$E(N_{t'}) = \frac{t'}{\mu} + \frac{\sigma^2 - \mu^2}{2\mu^2} + o(1). \quad (13)$$

Using (13) along with the previous results we arrive at the following approximation for the expected number of the renewal cycles completed during the production run of length t:

$$E(N_t) \approx \frac{t}{\upsilon h \mu_W} + \frac{\sigma_W^2 - \mu_W^2}{2\mu_W^2}. \quad (14)$$

Let F_t be a random variable representing the number of false alarms observed during the production run of length t. It is defined by

$$F_t = \sum_{j=1}^{N_t} F_j. \quad (15)$$

Thus, from the well known Wald's equation we find the expected number of false alarms for the whole production run from the following formula

$$E(F_t) = E\left(\sum_{j=1}^{N_t} F_j\right) = E(F_j)E(N_t) \approx \mu_F\left(\frac{t}{\upsilon h \mu_W} + \frac{\sigma_W^2 - \mu_W^2}{2\mu_W^2}\right). \quad (16)$$

This result will be used for the evaluation of economic consequences of the inspection procedure.

3. The economic consequences (costs and profits) of the inspection procedure

It is rather obvious that an item produced during the in-control State I of production process is on average more profitable than that produced in the out-of-control State II. Thus, it is better to have the process run in state I, and whenever an alarm is observed, some investigations should be conducted and upon their results the appropriate corrective actions must be taken to put the process in the state of control again. The production process can be considered as a series of renewal cycles, each cycle consisting of the in-control State I period, the out-of-control State II period, and the idle time period necessary for taking renewal actions. In any renewal cycle there are two types of actions associated with the application of an inspection procedure, namely, the inspection actions and the renewal actions.

The inspection actions consist of all actions that are responsible for detecting a shift. They consist of periodical inspection and testing, as well as investigations of false alarms. The economic consequences of these actions are represented by their respective costs. Let a_1^* be the cost of a single inspection. Thus, the expected cost of inspections for the whole run is

$$S_t = a_1^* \frac{t}{\upsilon h}. \quad (17)$$

Let a_2^* be the cost of investigating a false alarm (which might include the cost of stopping the process during its investigation). Hence, the expected cost for false alarms for the whole run is given by

$$A_t = a_2^* E(F_t) = a_2^* \mu_F\left(\frac{t}{\upsilon h \mu_W} + \frac{\sigma_W^2 - \mu_W^2}{2\mu_W^2}\right). \quad (18)$$

The renewal actions consist of all duties undertaken in order to bring the process from State II to State I. The economic consequences of these actions are two-fold:

- negative ones, that are represented by the costs of the renewal actions, a_3^*, which might include the cost of the possible shutdown of the process while repairing, and

– positive ones, that are represented by the benefit from the transition to the in-control State I.

Suppose that g_1 and g_2 are the expected profits from an item produced in State I and State II, respectively. Thus, $g_1 - g_2$ ($\geqslant 0$) is the gain per unit from the transition from the out-of-control State II to the in-control State I. Since the expected duration of State I is $1/\lambda$ hours, and the production rate is υ items per hour, then the expected gain per cycle due to inspection is equal to $\upsilon(g_1 - g_2)/\lambda$. Let b^* be the expected net benefit per renewal, i.e., the difference between the expected gain and the expected renewal cost per cycle. Thus, we have

$$b^* = \frac{g_1 - g_2}{\lambda}\upsilon - a_3^*, \tag{19}$$

and the expected gain from the transitions to state I is given by

$$G_t = b^* E(N_t) \approx b^* \left(\frac{t}{\upsilon h \mu_W} + \frac{\sigma_W^2 - \mu_W^2}{2\mu_W^2} \right). \tag{20}$$

However, if we do not use any inspection procedure the process remains in State II until the end of the production run. Thus, the expected gain from producing in State II for the whole run is tg_2. The economic consequences of the inspection procedures are used in the next section for finding an optimal inspection interval.

4. Optimization of the inspection interval

Let $L(t)$ be the expected loss incurred in the run. From the considerations presented in the previous sections we can find that $L(t)$ is given by the following formula

$$L(t) = a_1^* \frac{t}{\upsilon h} + a_2^* \mu_F E\left(N_t\right) - b^* E\left(N_t\right) - tg_2\,. \tag{21}$$

The expected loss per unit produced, expressed as a function of h for a given production run t, is given now by

$$\begin{aligned} L(h|t) &= \frac{L(t)}{t} = \frac{1}{t}\left\{ a_1^* \frac{t}{\upsilon h} + a_2^* \mu_F E N_t - b^* E N_t - t g_2 \right\} = \\ &= \frac{a_1^*}{\upsilon h} - \frac{b^* - a_2^* \mu_F}{\upsilon h \mu_W} - \frac{1}{2t}\left(\frac{b^* - a_2^* \mu_F}{\mu_W}\right)\left(\frac{\sigma_W^2 - \mu_W^2}{\mu_W}\right) - g_2\,. \end{aligned} \tag{22}$$

This function has to be minimized in order to determine the optimal inspection interval h for a process with a finite run t.

To reduce the number of the input parameters of the objective function we can follow von Collani (1986, 1989). The following expression gives the time-standardized loss function

$$S(y|r) = \left(L\left(\frac{y}{\lambda} | t = \frac{r\upsilon}{\lambda}\right) + g_2 \right) \frac{\upsilon}{a_2^* \lambda}\ . \tag{23}$$

Let $a_1 = \dfrac{a_1^*}{a_2^*}$, $b = \dfrac{b^*}{a_2^*}$, $y = \lambda h$, $r = \frac{\lambda t}{\upsilon}$, $A = \dfrac{1}{\alpha}$, and $B = \dfrac{1}{1-\beta}$.

Hence,

$$S(y|r) = \frac{1}{y}\left\{a_1 - \frac{b(e^y-1)-\alpha}{e^y-b}(1-\beta)\right\} + \frac{1}{2r}\left[\frac{b(e^y-1)-\alpha}{e^y-\beta}(1-\beta)\right]\left[\frac{e^y+\beta}{e^y-\beta}\right], \tag{24}$$

or, equivalently,

$$S(y|r) = \frac{1}{y}\left\{a_1 - \frac{b(e^y-1)-\frac{1}{A}}{1+B(e^y-1)}\right\} + \frac{1}{2r}\left[\frac{b(e^y-1)-\frac{1}{A}}{1+B(e^y-1)}\right]\left[\frac{1-B(e^y+1)}{1-B(e^y-1)}\right]. \tag{25}$$

The transformation of the objective function (22) to the form of (23) has reduced the complexity of the optimization problem because of the following reasons:

(1) the transformed objective function $S(y|r)$ depends only on two cost parameters, namely, a_1 and b, instead of four parameters in the original objective function $L(h|t)$ given by (22);

(2) the time-standardized objective function $S\,(y|r)$ depends on the process parameters $\frac{1}{\lambda}$, υ, and t only through a new variable r.

The loss function $L(h|t)$ attains its minimum at h^* iff the time-standardized loss function $S(y|r)$ attains its minimum at $y^* = \lambda h^*$. Thus, it is sufficient to optimize the time-standardized loss function $S(y|r)$ given by (25) in order to determine the optimal standardized inspection interval y^*, and thus the optimal inspection interval h^*.

The optimal standardized inspection interval y^* can be found by solving the following equation

$$\frac{d}{dy}S(y|r) = 0. \tag{26}$$

After some calculations we present (26) in the following form

$$-\left\{a_1 - \frac{b(e^y-1)-\frac{1}{A}}{1+B(e^y-1)}\right\}\frac{1}{y^2} - \frac{\left(b+\frac{B}{A}\right)e^y}{\left[1+B(e^y-1)\right]^2}\frac{1}{y} - -\frac{1}{2r}\left\{\frac{2b(B-1)e^y}{\left[1+B(e^y-1)\right]^2}\right.$$
$$\left. -\frac{2(B-1)\left(b+\frac{B}{A}\right)e^y}{\left[1+B(e^y-1)\right]^3} + \left(b+\frac{B}{A}\right)\frac{\left[1-B(e^y+1)\right]e^y}{\left[1+B(e^y-1)\right]^3}\right\} = 0 \tag{27}$$

Let us introduce the following notation

$$C = \frac{b-Ba_1}{b+\frac{B}{A}}, \tag{28}$$

$$D = \frac{1}{r}\left[\frac{bB(B-1)}{b+\frac{B}{A}}\right], \tag{29}$$

$$E = \frac{B}{2r}. \tag{30}$$

After some mathematical transformations we obtain the following compact version of (27):

$$\{1+B\left[e^{y}(1+y)-1\right]\}\left[1+B(e^{y}-1)\right]-C\left[1+B(e^{y}-1)\right]^{3}+ \\ +D\left[1+B(e^{y}-1)\right]y^{2}e^{y}+E\left(3-3B-Be^{y}\right)y^{2}e^{y}=0 \tag{31}$$

or, equivalently,

$$\frac{1+B\left[e^{y}(1+y)-1\right]}{\left[1+B(e^{y}-1)\right]^{2}}+\frac{Dy^{2}e^{y}}{\left[1+B(e^{y}-1)\right]^{2}}+\frac{E(3-3B-Be^{y})y^{2}e^{y}}{\left[1+B(e^{y}-1)\right]^{3}}=C. \tag{32}$$

The solution of any of the above equations determines the optimal standardized inspection interval y^* for monitoring a process with a finite production run. The solution of these equations requires a numerical procedure. Moreover, the impact of the input parameters on the optimal length of the sampling interval is not visible. Therefore, there is a practical need to obtain an approximate closed formula for the optimal inspection interval.

To find the approximately optimal inspection interval $\hat{y}$ we expand the left hand side of the equation (32) around $y = 0$, and neglect all terms of order higher than two. This expansion seems to be reasonable if the length of the inspection interval h is small in comparison to the expected time to deterioration $1/\lambda$. After some transformations we arrive at the following equation:

$$1+\frac{1}{2}\left\{B(1-2B)+2\left[D+(3-4B)E\right]\right\}y^{2}\approx C. \tag{33}$$

Hence, the approximately optimal standardized inspection interval is given by the following simple formula

$$\hat{y}\approx\sqrt{\frac{2(C-1)}{(1-2B)B+2D+2(3-4B)E)}}=\sqrt{\frac{2r(1-\beta)^{2}(a_{1}+\alpha)}{(1+r)(1+\beta)\left[b(1-\beta)+\alpha\right]+2\alpha\beta}}. \tag{34}$$

Once the approximately optimal standardized inspection interval $\hat{y}$ is obtained, the approximately optimal inspection interval $\hat{h}$ is given by

$$\hat{h}=\frac{\hat{y}}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{2r(1-\beta)^{2}(a_{1}+\alpha)}{(1+r)(1+\beta)\left[b(1-\beta)+\alpha\right]+2\alpha\beta}}. \tag{35}$$

5. Discussion

The approximate inspection interval $\hat{y}$ depends on a, β, a_1, b, and r. Thus, it is desirable to investigate the effects of these parameters on $\hat{y}$, and on its accuracy as well. The exact optimal standardized inspection interval y^* has been computed by the minimization of the objective function $S(y|r)$ given by (25) with

Table 1.

α	β	a_1	b	r	y^*	$\hat{y}$	$S(y^*\|r)$	$S(\hat{y}\|r)$
0.01	0.01	0.1	10	10	0.1475	0.1399	-8.479	-8.477
				50	0.1533	0.1453	-8.5354	-8.5334
				500	0.1546	0.1466	-8.5483	-8.5464
			50	10	0.0642	0.0626	-46.537	-46.5361
				50	0.0667	0.0650	-46.6653	-46.6644
				500	0.0672	0.0656	-46.6948	-46.694
			500	10	0.02	0.0198	-488.94	-488.94
				50	0.0208	0.0206	-489.35	-489.35
				500	0.0209	0.0207	-489.444	-489.444
		1	10	10	0.5071	0.4240	-5.6284	-5.5741
				50	0.5268	0.4404	-5.7909	-5.7384
				500	0.5314	0.4443	-5.8281	-5.7761
			50	10	0.2041	0.1897	-39.7362	-39.7114
				50	0.212	0.1970	-40.1169	-40.0929
				500	0.2138	0.1988	-40.2043	-40.1806
			500	10	0.0613	0.0600	-466.713	-466.705
				50	0.0638	0.0623	-467.946	-467.939
				500	0.0643	0.0629	-468.23	-468.223
		5	10	10	1	0.9444	-0.9613	-0.8267
				50	1	0.9808	-1.2142	-1.168
				500	1	0.9895	-1.2711	-1.246
			50	10	0.5048	0.4225	-28.1884	-27.9199
				50	0.5244	0.4388	-28.9989	-28.7396
				500	0.5292	0.4427	-29.1845	-28.9277
			500	10	0.1407	0.1336	-426.869	-426.782
				50	0.146	0.1388	-429.58	-429.496
				500	0.1472	0.1400	-430.204	-430.12

respect to y using a standard minimization routine. The approximately optimal standardized inspection interval $\hat{y}$ has been computed from (32). Typical results the comparison of the exact and approximate solutions for various values of α, β, a_1 and r based on extensive computations are presented in Deeb and Hryniewicz (2004). Tables 1 and 2 contain typical results of those comparisons. Note, that in order to compare the exact and the approximate solutions, we present not only their values, but the values of the objective function of the respective cases as well.

From the analysis of Tables 1–2, and other comparisons given in Deeb and Hryniewicz (2004) we arrive at the following conclusions

a) The approximately optimal inspection interval $\hat{y}$ is always shorter than the optimal inspection interval y^* for all considered values of α, β, a_1, b, and r.

b) The smaller the value of the inspection cost a_1, the better the approxima-

Table 2.

α	β	a_1	b	r	y^*	$\hat{y}$	$S(y^*\|r)$	$S(\hat{y}\|r)$
0.05	0.1	0.1	10	10	0.1612	0.149	-8.1209	-8.1155
				50	0.1674	0.1547	-8.1895	-8.1844
				500	0.1687	0.1561	-8.2053	-8.2003
			50	10	0.0691	0.0668	-45.6414	-45.6389
				50	0.0717	0.0693	-45.8018	-45.7995
				500	0.0724	0.07	-45.8387	-45.8364
			500	10	0.0213	0.0211	-485.933	-485.932
				50	0.0222	0.0219	-486.453	-486.452
				500	0.0223	0.0221	-486.573	-486.572
		1	10	10	0.4959	0.3941	-5.2706	-5.1781
				50	0.5136	0.4093	-5.4406	-5.353
				500	0.5179	0.4129	-5.4794	-5.3931
			50	10	0.1942	0.1766	-38.7051	-38.6607
				50	0.2013	0.1834	-39.118	-39.0761
				500	0.2032	0.1851	-39.2127	-39.1714
			500	10	0.0575	0.0559	-463.016	-463.002
				50	0.0597	0.058	-464.381	-464.367
				500	0.0602	0.0586	-464.695	-464.681
		5	10	10	1	0.8643	-0.5722	-0.188
				50	1	0.8976	-0.8258	-0.5409
				500	1	0.9056	-0.8828	-0.6215
			50	10	0.4852	0.3874	-26.5856	-26.1475
				50	0.5027	0.4023	-27.4284	-27.0137
				500	0.5068	0.4059	-27.6209	-27.212
			500	10	0.1306	0.1226	-420.221	-420.07
				50	0.1357	0.1273	-423.15	-423.008
				500	0.1369	0.1284	-423.823	-423.683

tion.

c) The value of r has almost no effect on the accuracy of the approximation.

d) The benefit per cycle b has a dominant effect on the accuracy of the approximation procedure. The larger the value of b, the better the approximation.

e) The probabilities of false decisions α and β have a minor effect on the accuracy of the approximation. However, by increasing their values we obtain a slight improvement of the approximation.

Further analysis reveals that from a practical point of view there is no difference between the approximate and the exact values of the standardized inspection interval. Moreover, even if such a difference exists then the difference between the corresponding losses is negligible.

The existence of the closed formula for the approximately optimal inspection interval allows us to formulate some practical observations:

a) Longer inspection intervals correspond to smaller expected shifts of the process mean.
b) Any change in the inspection cost produces a change in the same direction for the optimal inspection interval.
c) The benefit from the inspection affects the interval between inspections in such a way that any change of the benefit b results in the change of the optimal inspection interval in the opposite direction.
d) Any change in the probability of false alarms α produces the change of the optimal inspection interval in the same direction.
e) Changes in β produce changes in the optimal inspection interval in the opposite direction.
f) Increase of the cost of a false alarm results in an increase of the inspection interval.
g) Changing the renewal cost changes the interval between inspections in the same direction.
h) Small values of the ratio of the production run length to the length of the in-control period, r, have minor effect on the optimal inspection interval.
i) Changes of the mean number of occurrences of the assignable cause in a time unit changes the inspection interval in the same direction. The same conclusion holds for the production run length.
j) Changing the production rate changes the inspection interval in the opposite direction.

In the model considered we have assumed that the time between consecutive disorders of the process is described by the exponentially distributed random variable. A possible generalization of the model can be obtained using the approach proposed by Hryniewicz (1992). Another generalization can be obtained when we assume that the search for the assignable cause may not be perfect, as it was proposed in Hryniewicz (1996). When the inspection procedures, e.g. particular control charts, are specified, there is also a possibility to look for the optimal values of their parameters.

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