A few words about history of extremal problems at Moscow University[*]

by

Vladimir M. Tikhomirov and Vasily B. Demidovich

Department of Mechanics and Mathematics, Moscow State University
Vorobyevy Gory, Moscow 119899, Russia
e-mail: vmtikh@googlemail.com

The development of theory of extremal problems at Moscow University in the 20th century took place in three stages.

At the beginning of the century a rather narrow mathematical direction (in differential geometry) was developing in Moscow. The greatest Moscow mathematician, the real founder of the Moscow Mathematical School — Dmitry Egorov (1869 – 1931) — was not only a prominent specialist in differential geometry, but had wide interests in mathematical analysis and function theory. Egorov was a lecturer at many courses in analysis and geometry. In particular, he was a lecturer of the course of the "Calculus of Variations". He started to give this course at the beginning of the century and continued to do this till the end of the third decade of the 20th century (when he was arrested and soon after that he died in prison).

Egorov published a lithographical edition of a textbook, devoted to his course entitled "An abstract of lectures in the Calculus of Variations, which were read at the Imperial Moscow University in 1912".

Egorov's book was influenced by investigations of Weierstrass, which were reflected in the well known text-book of Weierstrass' student O. Bolza (1857 – 1942) "Vorlesungen über Variationsreschnung", Berlin, Teubner, 1909. Egorov uses many notations and pictures from the Bolza's book and he repeats the same layout of the course (the first variation, the fundamental lemma of the Calculus of Variations, Euler equation, the isoperimetrical problem, problems with higher derivatives, field theory, sufficient conditions).

In 1905 Egorov found an outstanding student: this was Nikolai Lusin (1883 – 1950). Suddenly, within the course of seven years (from 1915 – 1922) a great pleiad of eminent students of Egorov and Lusin appeared. Among them were

---

P. Alexandrov, N. Bary, A. Khinchin, A. Kolmogorov, M. Lavrent'ev, L. Lyusternik, D. Men'shov, P. Novikov, I. Petrovskii, L. Shnirel'man and others. Lavrent'ev (1900 – 1980) and Lyusternik (1899 – 1981) had deep interest in different aspects of the theory of extrema. The second period of optimization theory in Moscow University is mostly connected with their names.

Lavrent'ev and Lyusternik reformed the course of the Calculus of Variations. They were lecturers of this course from the 1930s till the 1940s. They had planned to write a textbook for this course and realized one-half of their plan. The first volume entitled "Elements of the Calculus of Variations" was published in 1935 in two books. The second volume devoted to the Calculus of Variations in the Large was not written.

The book of Lavrent'ev and Lyusternik has many distinctions from books of Bolza and Egorov. The authors included in their course the ideas of infinite dimensional analysis. This was a very unexpected thing because foundations of infinite dimensional functional analysis were established in the Polish mathematical school and the total presentation of the theory (Banach's book on linear operators) appeared only in 1931-32.

In 1933 two copies of Banach books appeared in Moscow. One of them was presented to A. Plessner by the author, the other was sent to Kolmogorov for reference. Plessner sent his book to the University library. Two PhD students of A. Kolmogorov studied this book during the period 1933-35. They were I. Gelfand and S. Nikolskii. They defended their dissertations devoted to the problems of functional analysis in the same day of 1935. Both of them used functional analysis in their investigations of extremal problems. In 1934 Lyusternik proved a fundamental result of the general theory of extremal problems. He opened an infinite-dimensional version of the Lagrange multiplier rule. But the general style of the Lavrent'ev and Lyusternik's book was traditional.

L. Lyusternik continued to give the course of the Calculus of Variations in the 1940s and the beginning of 1950s. In 1950, a reduced short version of the book of Lavrent'ev and Lyusternik was published. In 1955, I. Gelfand was a lecturer of the course on the Calculus of Variations. On the basis of his lectures a textbook "The Calculus of Variations" was written by I. Gelfand in collaboration with S. Fomin (1917 – 1975). It contains many interesting approaches, but it is also written in the style which can be traced back to the book of O. Bolza.

A new, third period of the theory of extrema in Moscow University was connected with the creation of the Optimal Control theory.

In the middle of the 1950s L. Pontryagin (1908 – 1988) opened a seminar devoted to problems of control. He engaged his former students V. Boltyanskii, R. Gamkrelidze and E. Mishchenko to study the new subject. Pontryagin introduced a new class of optimal problems, which includes problems of the Calculus of Variations and covers many questions of engineering and control. In 1961 the book of four authors devoted to the theory of Optimal Control was published. It induced a great interest in this theory in the whole world.

Among mathematicians, who tried to comprehend the new aspects of the theory of optimization were A. Dubovitskii (1923 – 2007) and A. Milyutin (1925 – 2001). They made an attempt to look at some different chapters of optimization theory from a uniform point of view. This ideology was actively perceived by I. Girsanov (1934 – 1967). He was a talented mathematician who tragically perished in the mountains at the beginning of his creative career. During the academic year 1963/64 Girsanov gave a course, entitled "Lectures on the mathematical theory of extremal problems". (In 1970, B. Polyak, Girsanov's friend and colleague, published a book based on the lectures of Girsanov).

Under the influence of Milyutin and Girsanov the first author of this report opened in 1964 a seminar devoted to the theory of optimization. This seminar is still functioning till now. (Sometimes this seminar was directed by a group of mathematicians). Among participants of this seminar (some of them were co-leaders of the seminar) at different times were V. Alexeev. A. Arutyunov, E. Avakov, A. Buslaev, V. Demidovich, A. Dmitruk, A. Furikov, E. Galeev, A. Ioffe, A. Kochurov, S. Konyagin, A. Kurzhanskii, G. Magaril-Il'yaev, A. Milyutin, K. Osipenko, N. Osmolovskii, E. Polovinkin, B. Polyak, V. Protassov, M. Zelikin and others.

In 1974, the book by A. Ioffe and V. Tikhomirov "Theory of extremal problems" was published. Some principles, ideas and results of this thirty five years old book are explained in the paper by V.M. Tikhomirov, "Lagrange principle and necessary conditions", included in the second part of this issue.