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Book review:

NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

by

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The book contains an expanded version of lecture notes delivered by the authors at the Advanced School on *Numerical Solutions of Partial Differential Equations: New Trends and Applications.* The School was held in Barcelona in November 2007. The book consists of three parts.

The first part of the book, written by Silvia Bertoluzza and Silvia Falletta, is devoted to new wavelets based approaches in the numerical solution of partial differential equations (PDEs). The notion of a wavelet and its fundamental properties are recalled. One of these properties is good simultaneous space and frequency localization. The norm equivalence theorems for Sobolev spaces $H^s(\Omega)$ and their duals in Lipschitz domains $\Omega \subset R^n$, $n \geq 1$, and s taking negative or positive fractional values within the bounded segment, and for Besov spaces $B_q^{s,p}(\Omega)$, $0 < p, q < +\infty$ in terms of l^2 and l^p norms of the sequence of wavelets coefficients, respectively, are provided. Wavelet preconditioning, nonlinear wavelet methods for the solution of PDEs and wavelet stabilization of unstable boundary value problems such as Stokes problem are presented and discussed.

The second part, written by Giovanni Ruso, provides an overview of the finite-volume and/or finite-difference shock-capturing schemes for hyperbolic systems of conservation and balance laws. First, basic numerical schemes for hyperbolic systems of conservation laws including three point schemes, second order methods and upwind schemes, are recalled. Moreover, the entropy inequality, ensuring the uniqueness of weak solutions to PDEs, stability conditions and the selection of numerical flux function in the numerical schemes are discussed. Higher order finite-volume schemes with essentially non-oscillatory (ENO) or weighted ENO (WENO) reconstruction approach and central scheme based methods are also presented. Finally, different numerical schemes including finite-difference method with implicit-explicit Runge-Kutta schemes are presented and analyzed. Numerical examples illustrating the behavior of this method, including the solution of PDEs governing gas dynamics (Boltzmann equation), shallow water flow and vehicular traffic flow are provided and discussed.

The last part of the book, written by Chi-Wang Shu, is a general introduction to the discontinuous Galerkin methods for solving time-dependent convection-dominated PDEs. Discontinuous Galerkin methods are a class of the finite element methods using completely discontinuous basis functions. This implies their flexibility, which is not shared by the typical finite element methods. This flexibility consists in the allowance of arbitrary triangulation with hanging nodes, complete freedom in changing the polynomial degrees in each element independent of those of the neighbors and extremely local data structure resulting in high parallel efficiency.

The list of PDEs approximated and analyzed using discontinuous Galerkin approach includes, among others, hyperbolic systems of conservation laws, convection-diffusion equations, Korteweg-de Vries (KdV) equations and other nonlinear dispersive wave equations. Discontinuous Galerkin approximation of these boundary value problems is formulated. Numerical fluxes constituting the essential part of numerical schemes are introduced and discussed. These schemes are shown to satisfy cell entropy inequality, implying their L^2 stability. Assuming the existence of smooth solutions to these boundary value problems, L^2 error estimates are also provided. As an example of excellent numerical performance of the Runge-Kutta discontinuous Galerkin scheme, numerical solutions to $1D$ and $2D$ transport equation using piecewise linear and piecewise polynomial of degree 6 functions are presented and discussed.

The book in a clear and concise way describes the new trends in numerical techniques for solving PDEs, especially hyperbolic equations of conservative or balance laws. The latest results and techniques in the field of PDEs approximation, numerical schemes for hyperbolic equations as well as their stability and convergence, are reported. Since the book presents different approaches to numerical solution of PDEs, therefore it requires from the reader the basic knowledge of partial differential equation theory, finite-dimensional approximation methods such as finite element or finite volume methods and their convergence, Sobolev and Besov spaces theory, error estimates. The book is well written and organized at a difficulty level that precisely meets the target audiences' needs. Each chapter contains an explanatory introduction. The avoidance of many technical details of numerical schemes and proofs or their parts which can be found in literature help the readers understand the text as well as makes the material more accessible to a wider cross-disciplinary audience. Each part of the book is accompanied by the long list of references allowing the reader to continue in depth studies on the presented topics.

PhD students as well as engineers and researchers in the field of applied mathematics or scientific computing and interested graduate students will find this book an excellent resource to rapid introduction into the field of modern numerical methods for solving PDEs. The book may also serve as a textbook for graduate – level courses in numerical methods for PDEs. The advanced numerical techniques for solving PDEs presented in the book may also be useful for experienced researchers and practitioners both from academia or industry.

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