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Takagi-Sugeno fuzzy control scheme for electrohydraulic active suspensions*

by

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Abstract: The paper presents a new control strategy for active vehicle suspensions using electrohydraulic actuators based on Takagi-Sugeno (T-S) fuzzy modelling technique. As the electrohydraulic actuator dynamics is highly nonlinear, the T-S fuzzy modelling technique using the idea of "sector nonlinearity" is applied to exactly represent the nonlinear dynamics of electrohydraulic actuator in a defined region at first. Then, by means of parallel distributed compensation (PDC) scheme and Lyapunov method, a fuzzy H_∞ controller is designed for the T-S fuzzy model to optimise the suspension ride comfort performance, considering actuator input voltage saturation problem. The sufficient conditions for the existence of such a controller are derived in terms of linear matrix inequalities (LMIs). The advantage of this new control strategy for electrohydraulic active suspensions is that it directly aims at optimising suspension performance with guaranteeing the closed-loop system stability. Thus, two-loop control strategy, where the inner loop is used to make the electrohydraulic actuator tracking a desired force (pressure, or displacement, etc.), is not necessary. In addition, the controller is simple in structure compared to the adaptive control algorithms. A numerical example is used to validate the effectiveness of the proposed approach. It is confirmed by the simulations that the designed controller can achieve better performance than the active suspension with optimal skyhook damper.

Keywords: vehicle active suspension, electrohydraulic actuator dynamics, Takagi-Sugeno fuzzy modelling, actuator saturation.

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1. Introduction

Active suspensions play an important role in modern vehicles in improving suspension performance (Hrovat, 1997; Williams, 1997). It is noted that the inputs to the active suspensions are usually given as actuator forces. To realise the desired forces in real world applications, the actuators that fit into the suspension packaging space and satisfy the practical power and bandwidth requirements should be chosen. Electromagnetic actuators can be built for application in vehicle electromagnetic suspensions with the developments in power electronics, permanent magnet materials, and microelectronic systems in the last ten years (Martins et al., 2006). However, their mass and bulky size with respect to the desired suspension geometry and the unsprung mass dynamics will limit their practical applications in the current state. On the contrary, electrohydraulic actuators have been considered as one of the most viable choices for an active suspension due to their high power-to-weight ratio and low cost. Therefore, in recent years, many researches have been focused on electrohydraulic active suspensions and various control algorithms have been proposed (Alleyne and Hedrick, 1995; Tuan et al., 2001; Chantranuwathana and Peng, 2004; Zhang and Alleyne, 2006; Huang and Chen, 2006; Chen and Huang, 2006; Kaddissi, Kenne and Saad, 2007). Nevertheless, as proved by Alleyne and Liu (1999), pure PID-like controllers are not capable of giving satisfactory performance in the actuator force tracking problem, and more sophisticated control schemes should be employed. Therefore, some attempts, for example by Alleyne and Liu (1999, 2000a,b), Alleyne, Neuhaus and Hedrick (1993), and Thompson and Davis (2001), have been made to compensate for this shortcoming through advanced inner loop force control algorithms. In addition, to overcome the difficulty in achieving a desired force for an active suspension, Zhang and Alleyne (2006) made effort to transform the force tracking problem to a displacement tracking problem. Recently, the adaptive sliding control algorithm based on the function approximation technique was presented (Huang and Chen, 2006; Chen and Huang, 2006) for electrohydraulic active suspensions. However, as pointed in Tseng, Chen and Vang (2001), chattering phenomenon is inevitable in the sliding mode control, and it may excite unmodelled high-frequency dynamics, which degrades the performance of the system and may even lead to instability. It is noted that, due to the highly nonlinear dynamics of electrohydraulic actuator, using the electrohydraulic actuators to track the desired forces is fundamentally limited in its ability when interacting with an environment possessing dynamics (see Alleyne and Liu, 1999). Therefore, developing direct controller design approach for improving performance of electrohydraulic active suspensions is becoming necessary.

In this paper, a new fuzzy control strategy is presented to improve the ride comfort performance of active suspensions using electrohydraulic actuators. In the past decades, fuzzy logic control has been proposed as an alternative approach to conventional control techniques for complex nonlinear systems. It was originally introduced and developed as a model-free control design approach. However, it suffers from criticism of lacking systematic stability analysis and controller design. In the recent ten years or so, prevailing research efforts on fuzzy logic control have been devoted to model-based fuzzy control systems that guarantee not only stability but also performance of closed-loop fuzzy control systems (Feng, 2006). The Takagi-Sugeno (T-S) fuzzy system is one of the most popular fuzzy systems in the model-based fuzzy control. It is described by fuzzy IF-THEN rules, which represent local linear input-output relations of a nonlinear system. The overall fuzzy model of the nonlinear system is obtained by fuzzy "blending" of the linear models. The T-S model is capable of approximating many real nonlinear systems, e.g., mechanical systems and chaotic systems. As it employs linear model in the consequent part, linear control theory can be applied for the system analysis and synthesis accordingly based on the parallel distributed compensation (PDC) scheme. And hence, the T-S fuzzy models are becoming powerful engineering tools for modelling and control of complex dynamic systems.

To apply the model-based fuzzy control strategy to the electrohydraulic active suspensions, in this paper, the nonlinear dynamics of a suspension system with electrohydraulic actuator is represented by a T-S fuzzy model at first. Then, a fuzzy H_∞ controller is designed for the fuzzy T-S model to improve the ride comfort performance with consideration of the control input voltage saturation problem. The sufficient conditions for the existence of the controller are given in terms of linear matrix inequalities (LMIs), which can be solved very efficiently by means of the most powerful tools available to date, e.g., Matlab LMI Toolbox. The proposed control strategy is validated by simulations on a quarter-car suspension model. Results of a comparison show that the designed controller can achieve better performance than the passive suspension and the active suspension with optimal skyhook damper.

The rest of this paper is organised as follows. Section 2 presents the model of a quarter-car suspension system with electrohydraulic actuator. The T-S fuzzy model of the nonlinear suspension model is given in Section 3. In Section 4, the fuzzy H_∞ state feedback controller is obtained based on the solvability of LMIs. Section 5 presents the design result and simulations. Finally, we summarise our findings in Section 6.

The notation used throughout the paper is fairly standard. For a real symmetric matrix W, the notation of $W > 0$ ($W < 0$) is used to denote its positive- (negative-) definiteness. $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. I is used to denote the identity matrix of appropriate dimensions. To simplify notation, $*$ is used to represent a block matrix which is readily inferred by symmetry

2. Electrohydraulic suspension model

A quarter-car suspension model shown in Fig. 1 is considered here for the controller design. This model has been used extensively in the literature (see Alleyne and Hedrick, 1995; Chantranuwathana and Peng, 2004; Huang and Chen, 2006; Chen and Huang, 2006) and it captures many important characteristics of more detailed models. In Fig. 1, $z_s(t)$, $z_u(t)$ are the displacements of the sprung and unsprung masses, respectively; $z_r(t)$ is the road displacement input; m_s is the sprung mass, which represents the car chassis; m_u is the unsprung mass, which represents the wheel assembly; c_s, k_s are damping and stiffness, respectively, of the passive suspension system; k_t serves to model the compressibility of the pneumatic tyre; c_t serves to model the damping of the pneumatic tyre; $F_a(t)$ represents the active control force of the suspension system, which is generated by means of an electrohydraulic actuator placed between the two masses. We assume that $z_s(t)$ and $z_u(t)$ are measured from their static equilibrium positions and that the tyre remains in contact with the road at all times.

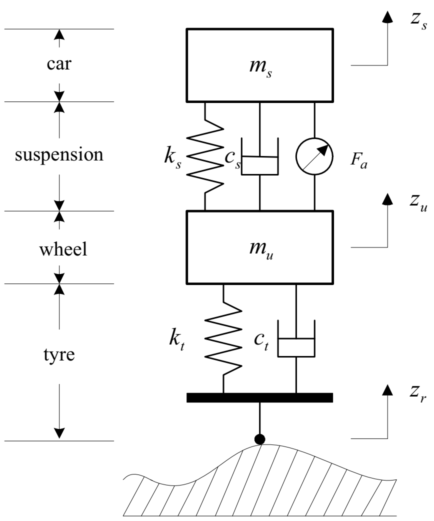

Figure 1. Quarter-car suspension model

In this study, the electrohydraulic actuator dynamics is expressed as (Alleyne and Hedrick, 1995; Chen and Huang, 2006; Alleyne and Liu, 1999, 2000a,b):

$$\dot{F}_a(t) = -\beta F_a(t) - \alpha A_s^2(\dot{z}_s(t) - \dot{z}_u(t)) + \gamma A_s \sqrt{P_s - \frac{\operatorname{sgn}(x_v(t))F_a(t)}{A_s}}\, x_v(t), \quad (1)$$

where $x_v(t)$ is the spool valve displacement; A_s is the actuator ram area; P_s is the hydraulic supply pressure. $\alpha = 4\beta_e/V_t$, where $\beta = \alpha C_{tm}$, $\gamma = \alpha C_d \omega \sqrt{1/\rho}$, and β_e is the effective bulk modulus, V_t is the total actuator volume, C_{tm} is the coefficient of total leakage due to pressure, C_d is the discharge coefficient, ω is the spool valve area gradient, ρ is the hydraulic fluid density.

Considering the electrohydraulic actuator dynamics (1) and the quarter-car suspension model, we define the state variables as follows:

$$\begin{aligned}
x_1(t) &= z_s(t) - z_u(t), \text{ suspension deflection,} \\
x_2(t) &= z_u(t) - z_r(t), \text{ tyre deflection,} \\
x_3(t) &= \dot{z}_s(t), \text{ sprung mass velocity,} \\
x_4(t) &= \dot{z}_u(t), \text{ unsprung mass velocity,} \\
x_5(t) &= F_a(t), \text{ actuator force,} \\
x_6(t) &= x_v(t), \text{ spool valve displacement.}
\end{aligned} \tag{2}$$

Then, if the actuator friction is ignored, the state-space equation of a quarter-car suspension system installed with electrohydraulic actuator can be represented as:

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t), \tag{3}$$

where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) & x_5(t) & x_6(t) \end{bmatrix}^T$ is the state vector, $u(t)$ is the input voltage to actuator servovalve, $w(t) = \dot{z}_r(t)$ is the road disturbance, and the matrices are

$$A = \begin{bmatrix}
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
-\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} & \frac{1}{m_s} & 0 \\
\frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s+c_t}{m_u} & -\frac{1}{m_u} & 0 \\
0 & 0 & -\alpha A_s^2 & \alpha A_s^2 & -\beta & \gamma A_s f(x_5, x_6, t) \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau}
\end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \\ 0 \\ 0 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{K_c}{\tau} \end{bmatrix},$$

where τ is the time constant of the spool valve dynamics, K_c is the conversion gain, and $f(x_5, x_6, t)$ is the nonlinear function given as

$$f(x_5, x_6, t) = \sqrt{P_s - \frac{\operatorname{sgn}(x_6(t)) x_5(t)}{A_s}}. \tag{4}$$

3. Takagi-Sugeno fuzzy modelling

In order to design a controller for the nonlinear suspension model (3) through fuzzy approach, the T-S fuzzy modelling technique will be applied. In this paper, the idea of "sector nonlinearity" (Tanaka and Wang, 2001) is employed to construct an exact T-S fuzzy model for the nonlinear suspension system (3).

Suppose the actuator force $F_a(t)$ $(x_5(t))$ is bounded by its minimum value $F_{a\min}$ and its maximum value $F_{a\max}$ in practice, the nonlinear function $f(x_5,x_6,t)$ will be bounded by its minimum value $f_{\min}$ and its maximum value $f_{\max}$. Thus, the nonlinear function $f(x_5,x_6,t)$ can be represented by

$$f(x_5,x_6,t) = M_1(\xi(t))f_{\min} + M_2(\xi(t))f_{\max},$$

where $\xi(t) = f(x_5,x_6,t)$ is the premise variable, $M_1(\xi(t))$ and $M_2(\xi(t))$ are membership functions, and

$$M_1(\xi(t)) = \frac{f_{\max} - f(x_5,x_6,t)}{f_{\max} - f_{\min}},\ M_2(\xi(t)) = \frac{f(x_5,x_6,t) - f_{\min}}{f_{\max} - f_{\min}}. \tag{5}$$

It can be seen from (5) that $M_1(\xi(t)) \geqslant 0$, $M_2(\xi(t)) \geqslant 0$, and $M_1(\xi(t))+M_2(\xi(t)) = 1$.

Thus, the nonlinear suspension model (3) can be represented by the following fuzzy models.

Model Rule 1:

IF $\xi(t)$ is $M_1(\xi(t))$,
THEN $x(t) = A_1x(t) + B_1w(t) + B_2u(t)$.

Model Rule 2:

IF $\xi(t)$ is $M_2(\xi(t))$,
THEN $x(t) = A_2x(t) + B_1w(t) + B_2u(t)$.

Where, $A_1 = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} & \frac{1}{m_s} & 0 \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s+c_t}{m_u} & -\frac{1}{m_u} & 0 \\ 0 & 0 & -\alpha A_s^2 & \alpha A_s^2 & -\beta & \gamma A_s f_{\min} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}$, and

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} & \frac{1}{m_s} & 0 \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s+c_t}{m_u} & -\frac{1}{m_u} & 0 \\ 0 & 0 & -\alpha A_s^2 & \alpha A_s^2 & -\beta & \gamma A_s f_{\max} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}.$$

And then, the T-S fuzzy model, which exactly represents the nonlinear suspension model (3) under the assumption on bounds of the actuator force $F_a(t) \in [F_{a\,\min}, F_{a\,\max}]$ is obtained as:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\xi(t))A_i x(t) + B_1 w(t) + B_2 u(t) \tag{6}$$

where

$$h_i(\xi(t)) = M_i(\xi(t)),\ h_i(\xi(t)) \geqslant 0,\ i = 1, 2, \text{ and } \sum_{i=1}^{2} h_i(\xi(t)) = 1.$$

In practice, the actuator force $F_a(t)$ $(x_5(t))$ and the spool valve position $x_v(t)$ $(x_6(t))$ can be measured, thus, the nonlinear function $f(x_5, x_6, t)$ can be calculated, and the T-S fuzzy model (6) can be realised.

Furthermore, in a real application, the input voltage to servovalve, $u(t)$, will be bounded. Denote by $\bar{u}(t)$ =sat$(u(t))$ the saturating control input, where sat$(u(t))$ is defined as

$$\bar{u}(t) = \text{sat}(u(t)) = \begin{cases} -u_{\lim} & \text{if } u(t) < -u_{\lim} \\ u(t) & \text{if } -u_{\lim} \leqslant u(t) \leqslant u_{\lim} \\ u_{\lim} & \text{if } u(t) > u_{\lim} \end{cases} \tag{7}$$

where $u_{\lim}$ is known saturated input bound, then, from (6) and (7), the fuzzy system (6) becomes

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{2} h_i(\xi(t))A_i x(t) + B_1 w(t) + B_2 \bar{u}(t) \\ &= \sum_{i=1}^{2} h_i(\xi(t))A_i x(t) + B_1 w(t) + B_2 \frac{1+\varepsilon}{2} u(t) + B_2 \left(\bar{u}(t) - \frac{1+\varepsilon}{2} u(t)\right) \\ &= A_h x(t) + B_1 w(t) + B_2 \frac{1+\varepsilon}{2} u(t) + B_2 \left(\bar{u}(t) - \frac{1+\varepsilon}{2} u(t)\right) \end{aligned} \tag{8}$$

where $A_h = \sum_{i=1}^{2} h_i(\xi(t))A_i$, $0 < \varepsilon < 1$.

REMARK 1 *It is well-known that* $\left\|\bar{u}(t) - \frac{1+\varepsilon}{2}u(t)\right\| \leqslant \frac{1-\varepsilon}{2}\left\|u(t)\right\|$ *as long as* $|u(t)| \leqslant \frac{u_{\lim}}{\varepsilon}$. *So, we have* $\left[\bar{u}(t) - \frac{1+\varepsilon}{2}u(t)\right]^T \left[\bar{u}(t) - \frac{1+\varepsilon}{2}u(t)\right] \leqslant \left(\frac{1-\varepsilon}{2}\right)^2 u^T(t)u(t)$ *for* $|u(t)| \leqslant \frac{u_{\lim}}{\varepsilon}$.

4. Fuzzy $\mathbf{H}_\infty$ controller design

The fuzzy controller design for the T-S fuzzy model (8) will be carried out based on the so-called parallel distributed compensation (PDC) scheme (Tanaka and Wang, 2001). In the PDC design, each control rule is designed from the corresponding rule of a T-S fuzzy model, and the designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts.

For the T-S fuzzy model (8), we construct the fuzzy state feedback controller via the PDC as:

Control Rule 1:

IF $\xi(t)$ is $M_1(\xi(t))$,
THEN $u(t) = K_1 x(t)$.

Control Rule 2:

IF $\xi(t)$ is $M_2(\xi(t))$,
THEN $u(t) = K_2 x(t)$.

The overall fuzzy controller is represented by

$$u(t) = \sum_{i=1}^{2} h_i(\xi(t)) K_i x(t) = K_h x(t), \tag{9}$$

where $K_h = \sum_{i=1}^{2} h_i(\xi(t)) K_i$, K_i is the state feedback gain matrix to be designed.

For vehicle suspension design, it is well-known that ride comfort is an important aspect of performance. Ride comfort usually can be quantified by the sprung mass acceleration. Therefore, in the controller design, the sprung mass acceleration is chosen as the control output, i.e.,

$$z(t) = \ddot{z}_s(t) = Cx(t), \tag{10}$$

where $C = \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} & \frac{1}{m_s} & 0 \end{bmatrix}$.

In order to design an active suspension to perform adequately in a wide range of shock and vibration environments, the L_2 gain of the system (8) with (10), which is defined as

$$\|T_{zw}\|_\infty = \sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2}, \tag{11}$$

where $\|z\|_2^2 = \int_0^\infty z^T(t) z(t) dt$ and $\|w\|_2^2 = \int_0^\infty w^T(t) w(t) dt$, and the supremum is taken over all non-zero trajectories of the system (8) with $x(0) = 0$, is chosen as the performance measure. Our goal is to design a fuzzy controller (9) such that the fuzzy system (8) with controller (9) is quadratically stable and the L_2 gain (11) is minimised.

To design the controller, the following lemma from Zhou and Khargonekar (1988) will be used.

LEMMA 1 *For any matrices (or vectors) X and Y with appropriate dimensions, we have*

$$X^T Y + Y^T X \leqslant \epsilon X^T X + \epsilon^{-1} Y^T Y$$

where $\epsilon > 0$ is any scalar.

Let us define a Lyapunov function for the system (8) as

$$V(x(t)) = x^T(t) P x(t) \tag{12}$$

where P is a positive definite matrix.

By differentiating (12), we obtain

$$\begin{aligned}&\dot{V}(x(t)) = \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t)\\
= &\left[A_h x(t) + B_1 w(t) + B_2 \frac{1+\varepsilon}{2} u(t) + B_2 \left(\bar{u}(t) - \frac{1+\varepsilon}{2} u(t)\right)\right]^T P x(t)\\
+ &x^T(t) P \left[A_h x(t) + B_1 w(t) + B_2 \frac{1+\varepsilon}{2} u(t) + B_2 \left(\bar{u}(t) - \frac{1+\varepsilon}{2} u(t)\right)\right].\end{aligned} \tag{13}$$

By Lemma 1, Remark 1, and definition (9), we have

$$\begin{aligned}\dot{V}(x(t)) \leqslant\ & x^T(t) \left[A_h^T P + P A_h + \left(B_2 \frac{1+\varepsilon}{2} K_h\right)^T P + P B_2 \frac{1+\varepsilon}{2} K_h\right] x(t)\\
&+ w^T(t) B_1^T P x(t) + x^T(t) P B_1 w(t)\\
&+ \epsilon \left(\bar{u}(t) - \frac{1+\varepsilon}{2} u(t)\right)^T \left(\bar{u}(t) - \frac{1+\varepsilon}{2} u(t)\right) + \epsilon^{-1} x^T(t) P B_2 B_2^T P x(t)\\
\leqslant\ & x^T(t) \Theta x(t) + w^T(t) B_1^T P x(t) + x^T(t) P B_1 w(t),\end{aligned} \tag{14}$$

where $\Theta = \left[A_h^T P + P A_h + \left(B_2 \frac{1+\varepsilon}{2} K_h\right)^T P + P B_2 \frac{1+\varepsilon}{2} K_h + \epsilon \left(\frac{1-\varepsilon}{2}\right)^2 K_h^T K_h + \epsilon^{-1} P B_2 B_2^T P\right]$, and ϵ is any positive scalar.

Adding $z^T(t) z(t) - \gamma^2 w^T(t) w(t)$ to two sides of (14) yields

$$\begin{aligned}&\dot{V}(x(t)) + z^T(t) z(t) - \gamma^2 w^T(t) w(t)\\
\leqslant\ & \begin{bmatrix} x^T(t) & w^T(t) \end{bmatrix} \begin{bmatrix} \Theta + C^T C & P B_1 \\ B_1^T P & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}.\end{aligned} \tag{15}$$

Let us consider

$$\Pi = \begin{bmatrix} \Theta + C^T C & P B_1 \\ B_1^T P & -\gamma^2 I \end{bmatrix} < 0 \tag{16}$$

then, $\dot{V}(x(t)) + z^T(t) z(t) - \gamma^2 w^T(t) w(t) \leqslant 0$, and the L_2 gain defined in (11) is less than $\gamma > 0$ with the initial condition $x(0) = 0$ (Boyd et al., 1994). When the disturbance is zero, i.e., $w(t) = 0$, it can be inferred from (15) that if $\Pi < 0$, then $\dot{V}(x(t)) < 0$, and the fuzzy system (8) with the controller (9) is quadratically stable.

Pre- and post-multiplying (16) by $\operatorname{diag}\begin{pmatrix} P^{-1} & I \end{pmatrix}$ and its transpose, respectively, and define $Q = P^{-1}, Y_h = K_h P^{-1}$, the condition $\Pi < 0$ is equivalent to

$$\Sigma = \begin{bmatrix} QA_h^T + A_h Q + \frac{1+\varepsilon}{2} Y_h^T B_2^T + \frac{1+\varepsilon}{2} B_2 Y_h \\ +\epsilon\left(\frac{1-\varepsilon}{2}\right)^2 Y_h^T Y_h + \epsilon^{-1} B_2 B_2^T + QC^T CQ & B_1 \\ B_1^T & -\gamma^2 I \end{bmatrix} < 0. \tag{17}$$

By Schur complement, $\Sigma < 0$ is equivalent to

$$\Psi = \begin{bmatrix} QA_h^T + A_h Q + \frac{1+\varepsilon}{2}\left[Y_h^T B_2^T + B_2 Y_h\right] + \epsilon^{-1} B_2 B_2^T & Y_h^T & QC^T & B_1 \\ * & -\epsilon^{-1}\left(\frac{2}{1-\varepsilon}\right)^2 I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{18}$$

By the definitions $A_h = \sum\limits_{i=1}^{2} h_i(\xi(t)) A_i$ and $Y_h = \sum\limits_{i=1}^{2} h_i(\xi(t)) Y_i$, and the fact that $h_i(\xi(t)) \geqslant 0$ and $\sum_{i=1}^{2} h_i(\xi(t)) = 1$, $\Psi < 0$ is equivalent to

$$\Phi = \begin{bmatrix} QA_i^T + A_i Q + \frac{1+\varepsilon}{2}\left[Y_i^T B_2^T + B_2 Y_i\right] + \epsilon^{-1} B_2 B_2^T & Y_i^T & QC^T & B_1 \\ * & -\epsilon^{-1}\left(\frac{2}{1-\varepsilon}\right)^2 I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0,$$
$$i = 1,\ 2. \tag{19}$$

On the other hand, from (9), the constraint $|u(t)| \leqslant \frac{u_{\lim}}{\varepsilon}$ can be expressed as

$$\left|\sum_{i=1}^{2} h_i(\xi(t)) K_i x(t)\right| \leqslant \frac{u_{\lim}}{\varepsilon}. \tag{20}$$

It is obvious that if $|K_i x(t)| \leqslant \frac{u_{\lim}}{\varepsilon}$, then (20) holds. Let $\Omega(K) = \left\{x(t) \middle| \, |x^T(t) K_i^T K_i x(t)| \leqslant \left(\frac{u_{\lim}}{\varepsilon}\right)^2\right\}$, the equivalent condition for an ellipsoid $\Omega(P, \rho_p) = \left\{x(t) \middle| \, x^T(t) P x(t) \leqslant \rho_p\right\}$ being a subset of $\Omega(K)$ is (Cao and Lin, 2003):

$$K_i \left(\frac{P}{\rho_p}\right)^{-1} K_i^T \leqslant \left(\frac{u_{\lim}}{\varepsilon}\right)^2. \tag{21}$$

By the Schur complement, inequality (21) can be written as

$$\begin{bmatrix} \left(\frac{u_{\lim}}{\varepsilon}\right)^2 I & K_i \left(\frac{P}{\rho_p}\right)^{-1} \\ \left(\frac{P}{\rho_p}\right)^{-1} K_i^T & \left(\frac{P}{\rho_p}\right)^{-1} \end{bmatrix} \geqslant 0. \tag{22}$$

Using the definitions $Q = P^{-1}$ and $Y_i = K_i P^{-1}$, we get that inequality (22) is equivalent to

$$\begin{bmatrix} \left(\frac{u_{\lim}}{\varepsilon}\right)^2 I & Y_i \\ Y_i^T & \rho_p^{-1} Q \end{bmatrix} \geqslant 0. \tag{23}$$

In summary, for given numbers $0 < \varepsilon < 1$ and $\rho_p > 0$, if there exist matrices $Q > 0$ and Y_i, $i = 1, 2$, scalars $\epsilon > 0$ and $\gamma > 0$ such that LMIs (19) and (23) are satisfied, then the closed-loop system (8) with controller (9) is quadratically stable and the L_2 gain defined by (11) is less than γ. Moreover, the controller gain can be obtained as $K_i = Y_i Q^{-1}$. If γ is minimised, then the optimal fuzzy H_∞ controller is obtained.

5. Numerical example

In this section, we will apply the proposed approach to design a fuzzy H_∞ state feedback controller for a quarter-car suspension model as described in Section 2. The quarter-car suspension model parameter values are listed in Table 1 (Alleyne and Hedrick, 1995; Chen and Huang, 2006) and the values of the hydraulic actuator parameters used in the simulation are given in Table 2.

Table 1. Parameter values of the quarter-car suspension model

Parameter	m_s	m_u	c_s	k_s	k_t	c_t
Unit	kg	kg	Nm/s	N/m	N/m	Nm/s
Value	290	59	1000	16812	190000	0

Table 2. Parameter values of the hydraulic actuator

Parameter	α	β	γ	p_s	A_s	τ	K_c
Unit	$\mathrm{N/m^5}$	$\mathrm{s^{-1}}$	$\mathrm{N/m^{5/2}/kg^{1/2}}$	Pa	$\mathrm{m^2}$	s	m/V
Value	4.515×10^{13}	1	1.545×10^9	10342500	3.35×10^{-4}	0.003	0.001

In this study, we suppose that the input voltage of the spool valve is limited as $u_{\lim} = 2.5$ V, and the applied value of the actuator output force is limited to 800 N such that the bounds of the nonlinear function (4) are given as $f_{\min} = 2800$ and $f_{\max} = 3600$. The fuzzy H_∞ controller is implemented based on the assumption that all the state variables defined in (2) can be measured. It is seen from previous section that the controller gain can be determined by $K_i = Y_i Q^{-1}$, $i = 1, 2$, if matrices Q and Y_i are known. As matrices Q and Y_i are required to satisfy conditions (19) and (23) simultaneously, they can be obtained by solving inequalities (19) and (23). It is noticed that in (19) and (23), ε and ρ_p are two positive scalar parameters that can be chosen *a priori*. Once ε and ρ_p are given, inequalities (19) and (23) become LMIs for matrices Q and Y_i and scalars ϵ and γ, and we can use software like Matlab LMI Toolbox to solve matrices Q and Y_i without much difficulties. Note that the selection of ε and ρ_p will definitely affect the solutions of LMIs (19) and (23). For some chosen values of ε and ρ_p, LMIs (19) and (23) are feasible to find solutions, and the controller gain K_i, calculated with the feasible solutions of Q and Y_i could make the control system performance good or bad, which will be further validated through simulations. However, for some values of ε and ρ_p, LMIs (19) and (23) may not be feasible to find solutions. Choosing appropriate values for ε and ρ_p is a trial and error process, and certainly, some search algorithms like genetic algorithms (GAs) can be used to search for these two parameters. In general, small value of ε will make the control input bigger such that it may reach the saturation limit and big value of ρ_p will make the controller gain smaller so that it takes less affection on the control performance and even may not allow for finding feasible solutions in terms of the minimal solution for γ. By choosing $\varepsilon = 0.516$, $\rho_p = 3.98 \times 10^{-5}$, and solving LMIs (19) and (23) for matrices $Q > 0$, Y_i, $i = 1, 2$, scalar $\epsilon > 0$, with minimising the scalar $\gamma > 0$, we obtain the controller gains as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} 702.2128 & 318.4156 & 91.0434 & -84.6707 \\ -0.0426 & -2.1854 \times 10^4 & & \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 746.8704 & 335.1144 & 86.4629 & -79.7999 \\ -0.0453 & -2.3622 \times 10^4 & & \end{bmatrix}. \end{aligned} \tag{24}$$

More design examples will be discussed in the end of this section.

Fig. 2 shows the schematic diagram of the suspension control system with an electrohydraulic actuator. In the diagram, the electrohydraulic actuator is used to provide active force $F_a(t)$ to the suspension model. This electrohydraulic actuator is simulated by the model (1). The box of Controller denotes the fuzzy H_∞ controller, which is designed according to the obtained T-S fuzzy model (8). This controller is used to calculate the control input voltage sent to servovalve. The measured variables $x_5(t)$ and $x_6(t)$ are used to calculate the nonlinear function $f(x_{5,}x_6, t)$ defined in (4) and accordingly the coefficient h_i. The measured state feedback variables $x_1(t) \sim x_6(t)$ are used to calculate the voltage signal according to h_i and K_i, as given above. With the control input voltage, the electrohydraulic actuator will generate appropriate force to improve the ride comfort performance under external road disturbance.

For comparison, a desired active suspension that is a passive suspension with a "skyhook damper" attached (Alleyne and Hedrick, 1995) is also presented. The optimal value of the skyhook damping coefficient is given as 3000 N/m/s.

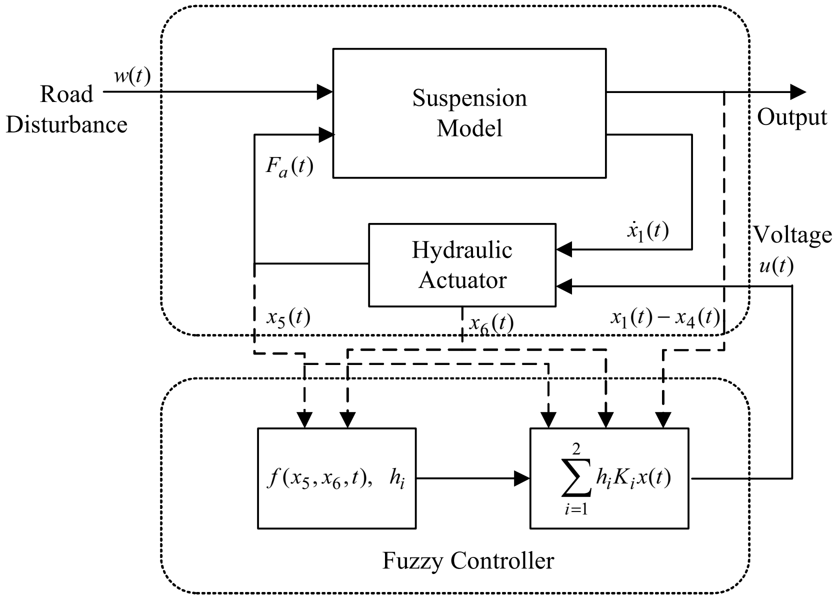

Figure 2. Schematic diagram of closed-loop control system

In Chen and Huang (2006), an adaptive sliding controller is designed to make the actuator force following the desired skyhook dynamics so that better ride comfort performance can be achieved. To compare the results, the same road disturbance as given in Chen and Huang (2006) is used in the simulation, where the road disturbance is given as:

$$z_r(t) = 0.0254\sin 2\pi t + 0.005\sin 10.5\pi t + 0.001\sin 21.5\pi t\ (m). \tag{25}$$

It can be seen from (25) that the road disturbance is close to the car body resonance frequency (1 Hz) with high frequency disturbance to simulate the rough road surface. The simulation program is realised by Matlab/Simulink.

Fig. 3 shows sprung mass acceleration for three kinds of suspensions, i.e., passive suspension (Passive), active suspension with skyhook damper (Skyhook), and active suspension with electrohydraulic actuator and fuzzy H_∞ controller (Fuzzy) as given in (24). It is clearly shown that the proposed fuzzy H_∞ control strategy improves the sprung mass acceleration magnitude significantly compared to the passive suspension as well as the active suspension with skyhook damper. It is noted that for the tracking control strategy, it can only realise the ride comfort performance which is close to the desired skyhook dynamics. However, the proposed fuzzy H_∞ control strategy can improve the desired suspension performance even with the highly nonlinear electrohydraulic actuator.

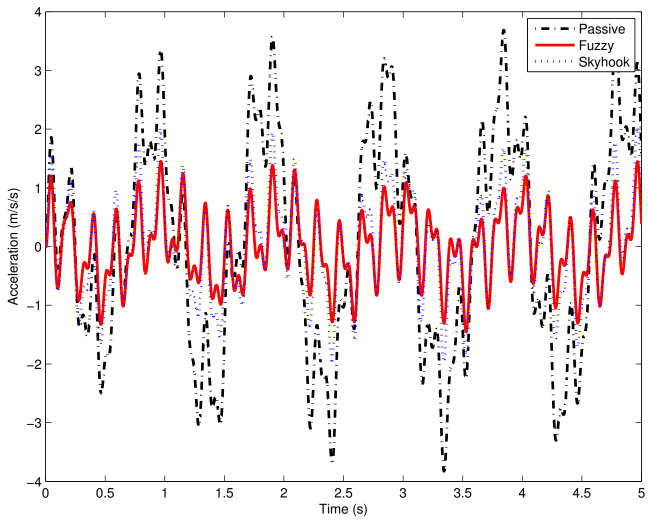

Figure 3. Sprung mass acceleration

Fig. 4 shows the sprung mass displacement. It also shows clearly that the proposed design achieves much less sprung mass displacement compared to the passive suspension and the active suspension with desired skyhook damper.

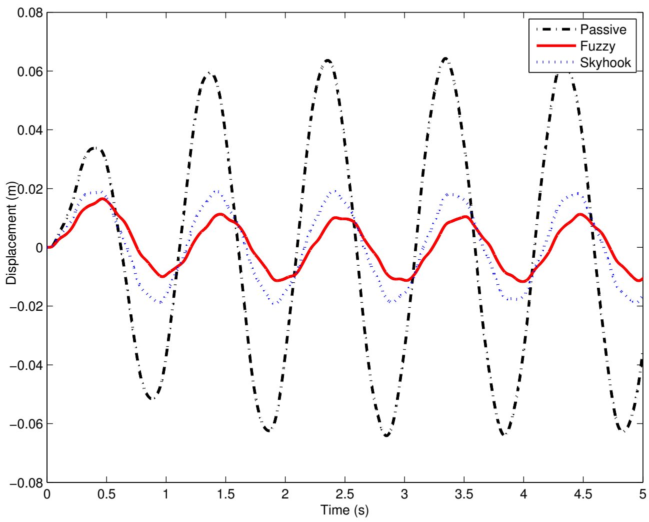

Figure 4. Sprung mass displacement

Fig. 5 shows the actuator forces. For the tracking control approach, it can only follow the desired skyhook actuator force. On the contrary, the proposed control strategy will provide more effective actuator force which directly aims to minimise the sprung mass acceleration.

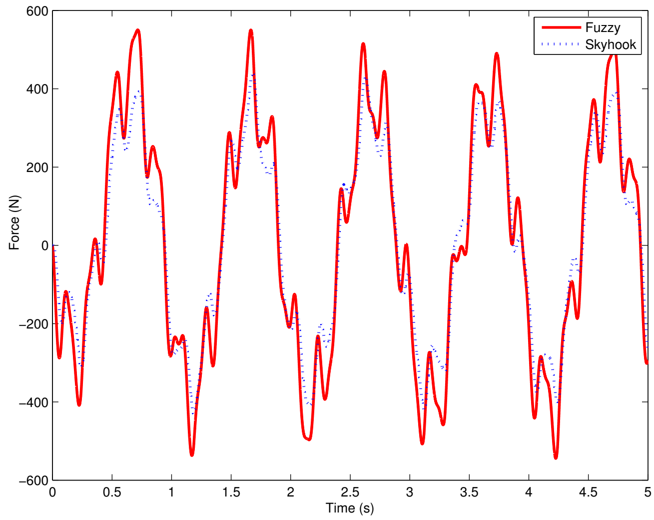

Figure 5. Actuator output force

Fig. 6 shows the control input voltage, which is within the defined input voltage range. Fig. 7 shows the nonlinear function output. It can be seen from Figure 7 that the nonlinear function output is located within the estimated bounds. In terms of the value of nonlinear function output, the grade of membership function h_i (or M_i) can be calculated using equation (5). As the final controller gain is determined by $K_h = \sum_{i=1}^{2} h_i(\xi(t))K_i$, it can be seen that this gain is time-varying with respect to the time-varying value of h_i. In fact, the values of h_1 and h_2 can be regarded as weights for K_1 and K_2, respectively, at each time instant. To see this clearly, the values for h_1 and h_2 are shown in Fig. 8, where the effort done by each controller K_i on the plant can be observed.

As mentioned earlier, choosing different values for ε and ρ_p will affect the possible solutions to the proposed controller design scheme. The above presented example actually shows a good design with $\varepsilon = 0.516$ and $\rho_p = 3.98 \times 10^{-5}$ because the obtained suspension performance is validated as better than the passive suspension and the active suspension with desired skyhook damper, according to illustrations from Figs. 3 and 4. If we choose the values for ε and ρ_p as $\varepsilon = 0.9$ and $\rho_p = 10^{-5}$, by solving LMIs (19) and (23) for matrices $Q > 0$,

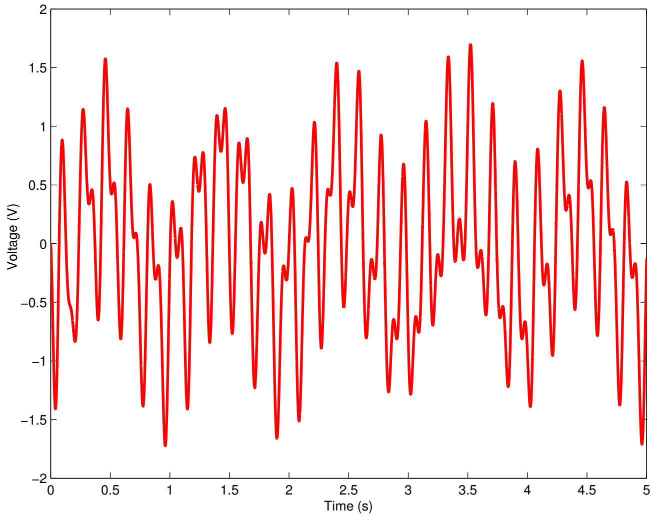

Figure 6. Actuator control input voltage

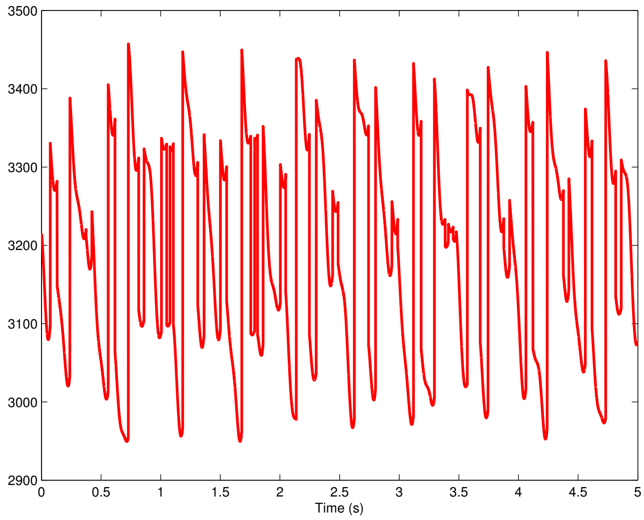

Figure 7. Nonlinear function value

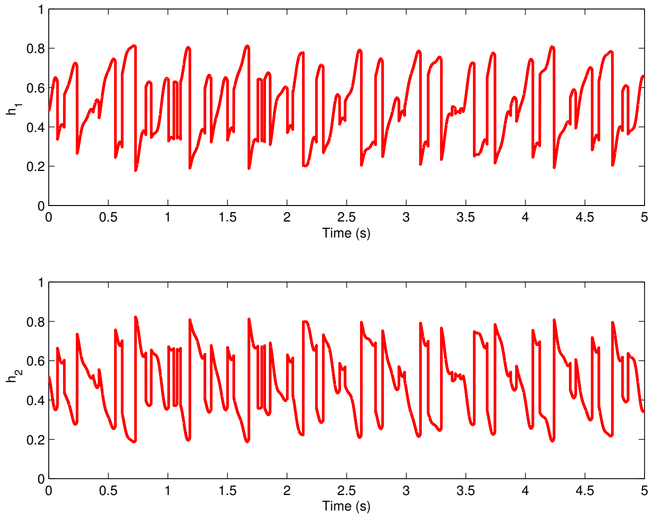

Figure 8. Grade of membership function

Y_i, $i = 1, 2$, scalar $\epsilon > 0$, with minimising scalar $\gamma > 0$, we can obtain the controller gains equal

$$\begin{aligned} K_1 &= \left[\begin{array}{llll} 2249.0975 & 2461.8372 & -618.7120 & -51.7475 \\ -0.7287 & -7.1856 \times 10^4 & & \end{array} \right], \\ K_2 &= \left[\begin{array}{llll} 2285.6666 & 2513.4587 & -651.1889 & -31.1786 \\ -0.7414 & -8.0378 \times 10^4 & & \end{array} \right]. \end{aligned} \tag{26}$$

The suspension performance done by this newly designed controller (26) is validated through the same simulation program and the road disturbance as given in (25). For a clear comparison, and to save space, we only compare the response on sprung mass displacement for three suspensions in Fig. 9 (left). It can be seen that the newly designed controller achieves smaller sprung mass displacement than passive suspension, however, its sprung mass displacement is bigger than the active suspension with desired skyhook damper. Furthermore, as the chosen value $\varepsilon = 0.9$ is close to 1, it is expected that the control input voltage realised by this controller could be less saturated. In fact, it is observed from Fig. 9 (right) that the control input voltage is really far less than the saturation limit. We now modify ε as $\varepsilon = 0.1$ and keep $\rho_p = 10^{-5}$, the controller gains obtained are

$$\begin{aligned} K_1 &= \left[\begin{array}{llll} 6379.0068 & 5337.0005 & 671.4731 & -437.4303 \\ -0.3854 & -1.5696 \times 10^5 & & \end{array} \right], \\ K_2 &= \left[\begin{array}{llll} 6594.7211 & 5510.5861 & 667.1314 & -425.3791 \\ -0.3985 & -1.6503 \times 10^5 & & \end{array} \right]. \end{aligned} \tag{27}$$

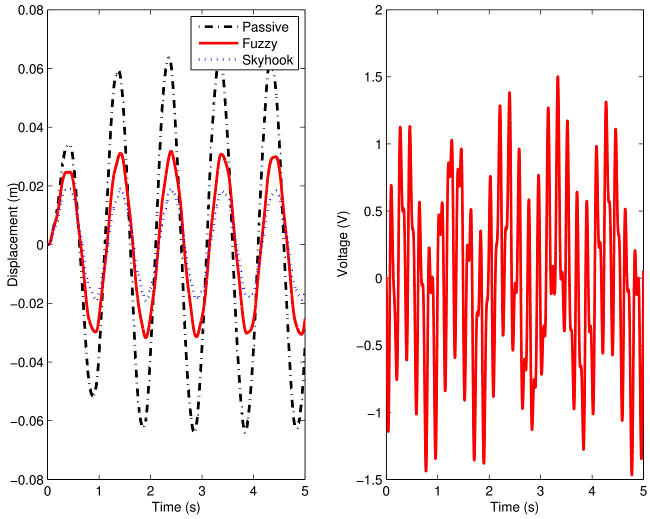

Figure 9. Controller design result for $\varepsilon = 0.9$ and $\rho_p = 10^{-5}$

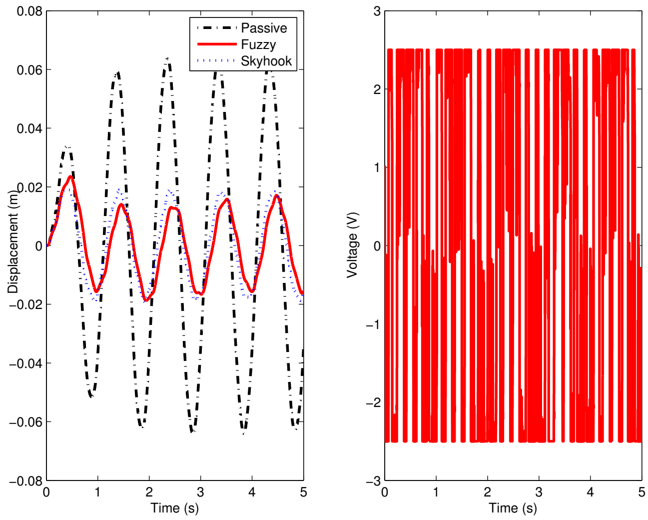

Figure 10. Controller design result for $\varepsilon = 0.1$ and $\rho_p = 10^{-5}$

Similarly, we validate this newly designed suspension performance using the same simulation program and compare the response on sprung mass displacement in Fig. 10 (left) for three suspensions. We observe from Fig. 10 (left) that the this newly designed suspension performance is close to the active suspension with desired skyhook damper. The control input voltage is shown in Fig. 10 (right), and this time, the control input voltage is highly saturated, as expected. Compared to the suspension performance realised by controller (24), the newly designed controllers, given in (26) and (27) cannot be accepted for a good design in terms of their performance. In addition, for some values of ε and ρ_p, for example, $\varepsilon = 0.5$ and $\rho_p = 10^8$, no feasible solutions can be found which indicates an infeasible design.

6. Conclusions

In this paper, we present a new fuzzy control strategy for electrohydraulic active suspensions. Using the idea of "sector nonlinearity", the nonlinear dynamics of electrohydraulic actuator can be represented by a T-S fuzzy model in a defined region. Thus, a fuzzy H_∞ controller can be designed for the obtained T-S fuzzy model by means of PDC scheme. In this study, the control objective is to optimise the ride comfort performance in terms of the L_2 gain from the road disturbance to the sprung mass acceleration. At the same time, the actuator input voltage saturation problem is considered. The sufficient conditions for designing such a controller is expressed by LMIs. The advantage of the proposed control strategy is that the optimal suspension performance is obtained directly. It does not need an inner control loop for electrohydraulic actuator to track the desired force (pressure, or displacement, etc.), does not need setting up fuzzy rules according to expert experience. Since the T-S fuzzy model can exactly represent the original model in a defined region, when the designed fuzzy controller is applied to the original system, stability can be guaranteed. The designed controller is simple in structure and can be easily realised in practice.

Acknowledgments

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